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LAB 2 REPORT: SYNTHESIS OF DIFFERENT PID CONTROLLERS USING ASTA

The aim of this study is to analyze the behaviour of the following system, with different types of controllers, where:

$$G(s) = \frac{1}{(1+s)^2(1+10s)}$$

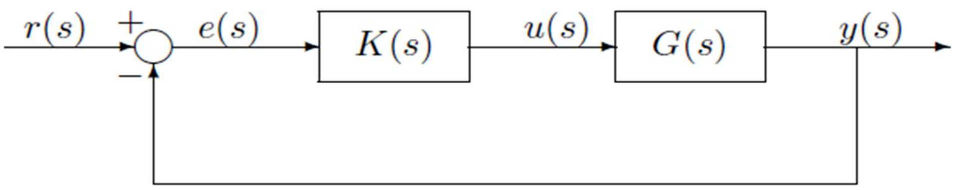


Figure 1: Closed loop architecture and the transfer function G(s).

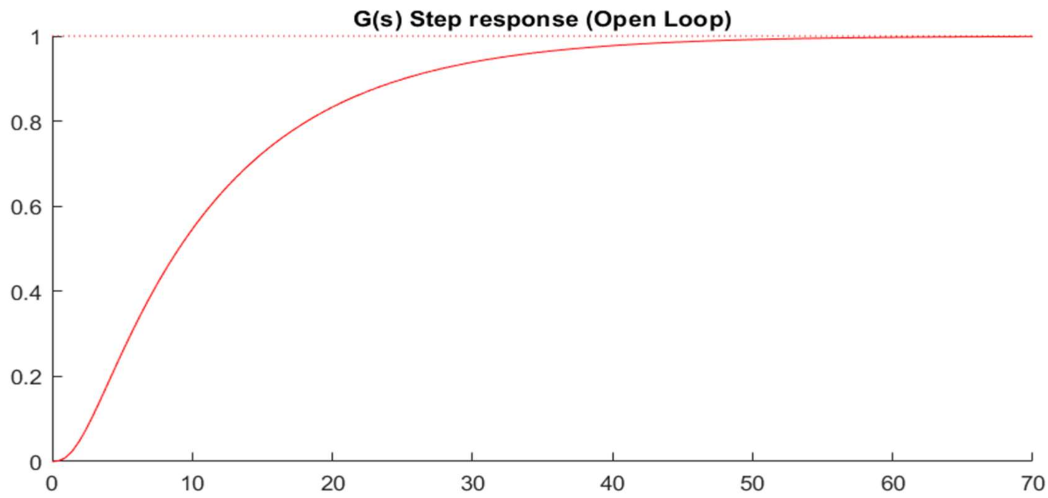


Figure 2: Step Response of L(s) = G(s)*K(s) with K(s) = 1.

The open loop system is stable as it can be observed in the figure, since all the poles of the transfer function of the open loop are in the left half plane.

1. Proportional Controller

Being K(s)=P, we want the maximum of the complementary sensitivity function to be 2.3dB.
Firstly, the value P=1 is given so that to analyze how far we are from the instruction to be satisfied. The following closed loop plot is obtained. The unitary negative feedback has the objective of reducing the effect of disturbances, but the dc-gain is changed as we can see in the figure: $y_\infty = 0.5$.

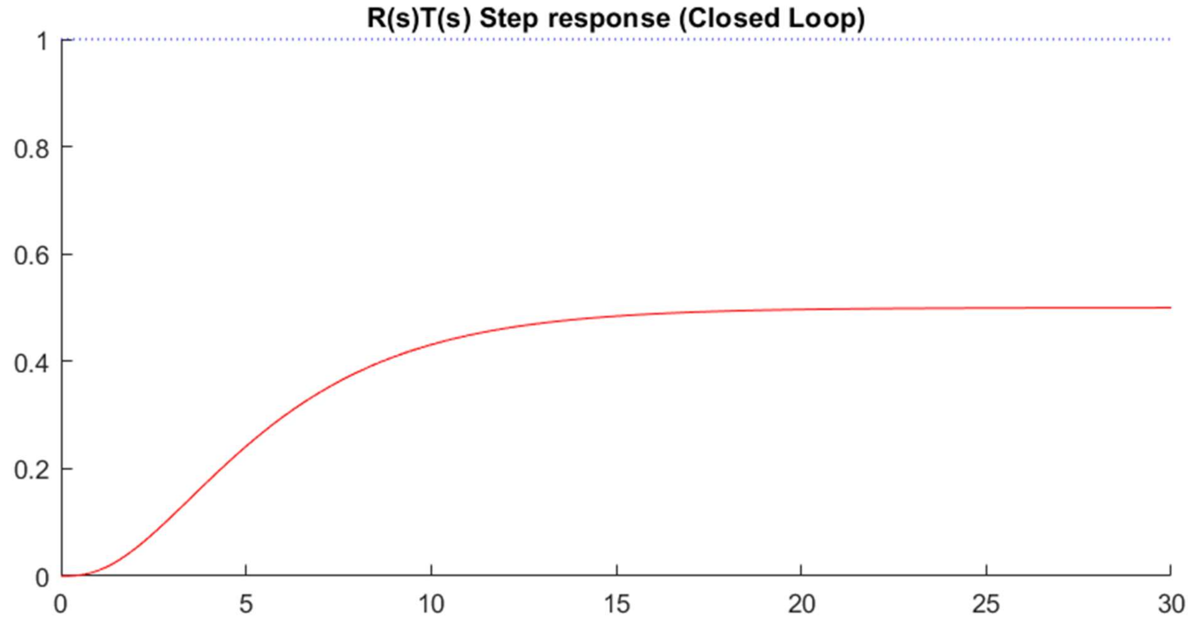


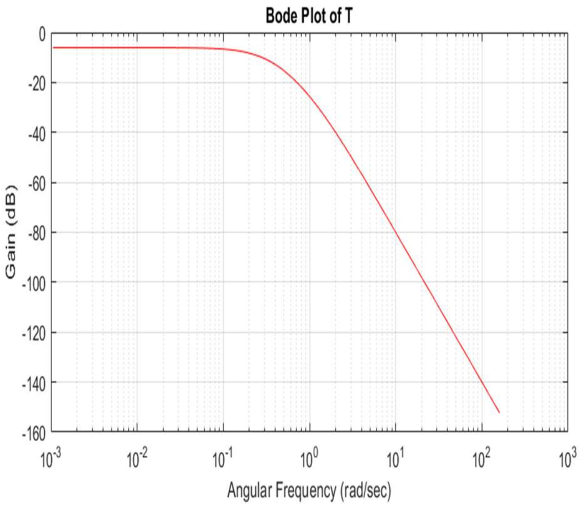
Figure 3: Step Response of the closed loop with P=1.

	M_p (%)	t_p (s)	t_s 5% (s)	t_s 2% (S)	y_∞	ε_s (%)
K = 1	-	-	13.47	16.51	0.5	50

Table 1: Time domain statistics of the response for P=1.

The system has no overshoot, so there is no peak time. We can see that the error is 50% because the gain of the response is half the gain of the input, so we can conclude that the regulation is not well done.
Taking a look at the frequency response (complementary sensitivity function |T(jw)| and Nichols plots), we can say the following: there is no any resonant peak in the |T(jw)| plot because the system is far away from the critical point (-1,0) or (-180º). This can be seen in Nichols plot. In this part, we want the maximum of the complementary sensitivity to be 2.3dB. In other words, when this is obtained, we will see that L(jw) is tangent to the M-

circle of 2.3dB in Nichols plot, and we will also see a resonant peak with a value of 2.3dB in the |T(jw)| plot.



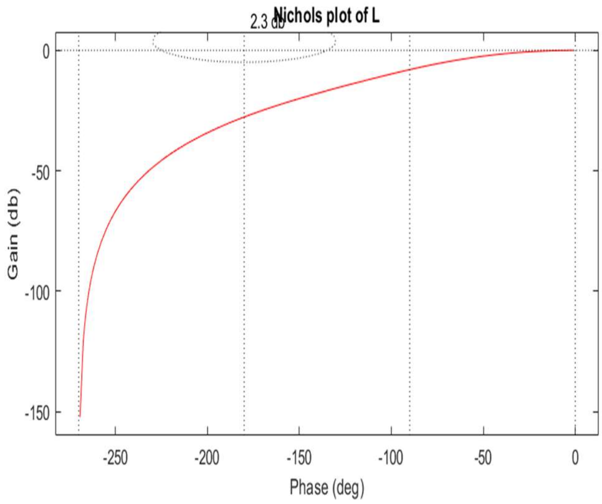


Figure 4: |T(jw)| for P=1.

Figure 5: Nichols plot of L(s).

For the moment, lets glanced at the frequency properties. As we previously said, there is no resonant peak, so Mr = 0.

	$\lvert T(jw)\rvert_{max}$ (dB)	w_r (rad/s)	$\lvert T(0)\rvert$ (dB)	Mr (dB)	w_c (rad/s)
K = 1	-6.02067	0.0010557	-6.0206	0	0.2376

Table 2: Frequency domain statistics (|T(jw)|) for K(s)=1.

	M_g (dB)	M_Φ (º)	M_d (s)	M_{sd} (dB)
K = 1	27.6765	-180	∞	-0.887

Table 3: Stability margins for K(s)=1.

Regarding the stability margins, all of them are satisfied:

- M_g > 6dB.
- M_φ > 40º.
- $M_d = M_\varphi / w = \pi / 0 = \infty$.

Having done all the analysis for P=1, we satisfy the imposed rule: the complementary sensitivity must have a maximum at 2.3dB, in other words, L(jw) must be tangent to the 2.3dB circle. If we take a quick look to the figure 5, we can see that we need to increase P so that L(jw) goes up. After some trial and error, we found that this is satisfied for a value of P = 6.565.

The whole process is done again, obtaining the following results:

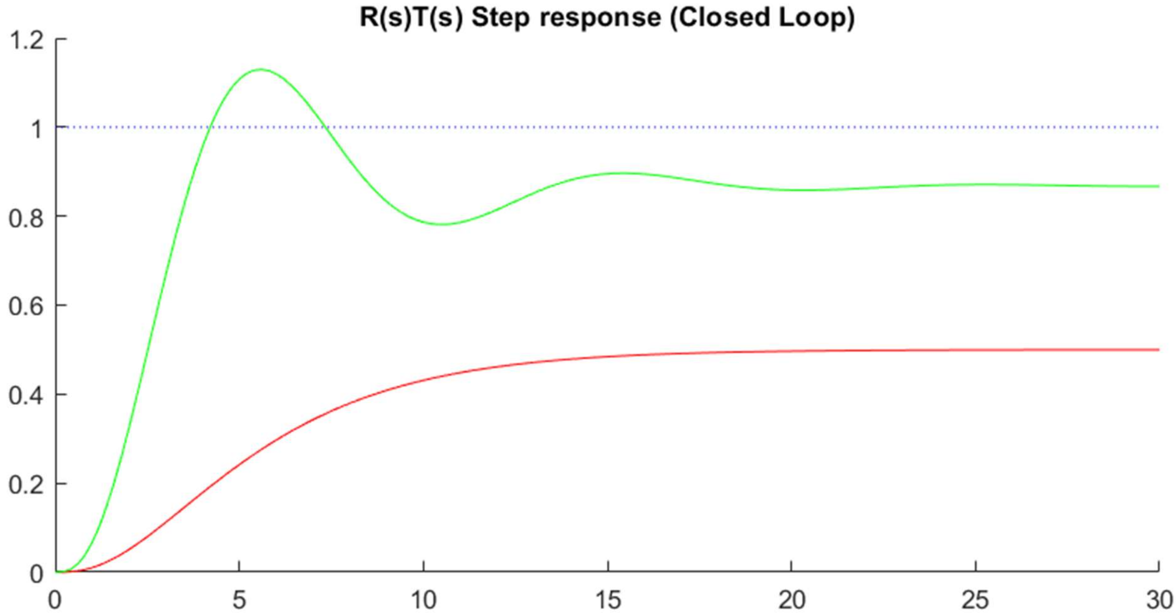


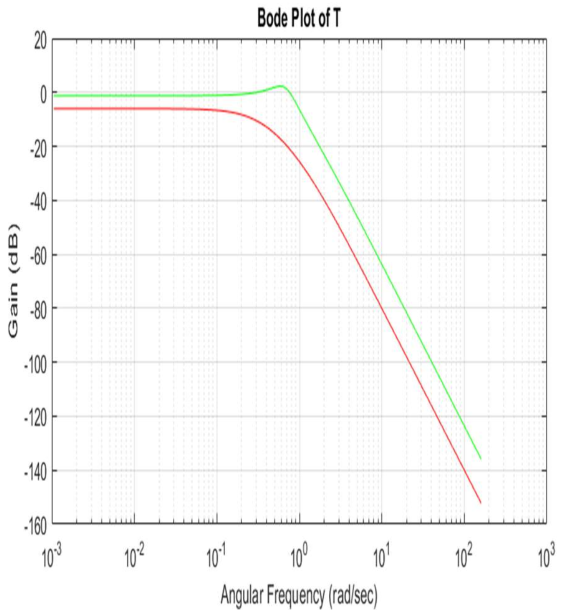
Figure 6: Step Response of the closed loop with K(s)=6.565 (Green).

Now the dc-gain is closer to 1, but still some error exists. We could continue increasing the P) making it faster so that the dc-gain gets closer and closer to 1, but there is a high risk of destabilizing the system, so if we want a perfect regulation (error = 0, which could be obtained if P = infinite), we will have to introduce a controller with integral action, which is done in the next part.

	M_p (%)	t_p (s)	t_s 5% (s)	t_s 2% (S)	y_∞	ε_s (%)
K = 1	-	-	13.47	16.51	0.5	50
K = 6.565	30.16	5.579	12.267	16.944	0.8678	13.22

Table 4: Time domain statistics of the response for K(s)=6.565.

We could also say that the effect of the control signal is not asked to be analyzed, but it should be taken into account when we increase P, because the control signal is much higher. In this case, we could suffer saturation in the actuators.
For this new P, we reach a resonant peak in the time domain plot. When we increased P, the system is faster, however, it takes a little bit longer to reach the final value (2%) due to the oscillation.
In this new case, M_p = 30.16% is far away from the 23%, which would be ideal. This is going to be analyzed after seeing the frequency properties.



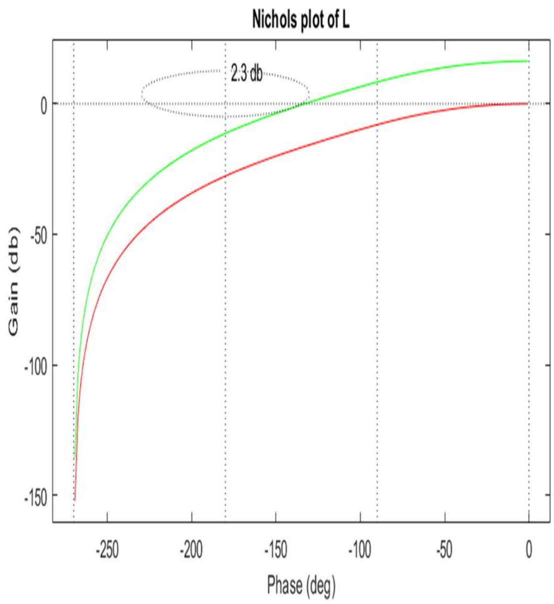


Figure 7: |T(jw)| for P = 0.656.

Figure 8: Nichols plot for P = 0.656.

	$\lvert T(jw)\rvert_{max}$ (dB)	w_r (rad/s)	$\lvert T(0)\rvert$ (dB)	Mr (dB)	w_c (rad/s)
K = 1	-6.02067	0.001055	-6.0206	0	0.2376
K = 0.656	2.298	0.5735	-1.23	3.529	0.9256

Table 5: Frequency domain statistics (|T(jw)|) for P=0.656.

Now we have a maximum $|T(jw)|_{max}$ = 2.3dB at w_r = 0.57 rad/s. M_r is expected not to be 2.3dB due to the fact that the static error is not null, what implies that |T(0)| is different from 0: M_r = 2.3 - (-1.23) = 3.53dB. So taking into consideration the relation between M_r and M_p (the higher M_r is, higher is M_p) and knowing that when there is no static error, if M_r = 2.3dB, then M_p = 23%, we can conclude that M_p must be higher than 23% because in this case M_r > 2.3dB.
When we increase P, we can conclude that the system is faster, so the angular cut-off frequency (w_c) must be higher, in other words, higher must be the bandwidth. Concluding, when we have a system which is closer to the critical point, it is a faster system, but the overshoot is higher and the stability margins will be smaller, as we will see now.

	M_g (dB)	M_ϕ (º)	M_d (s)	M_{sd} (dB)
K = 1	27.6765	-180	∞	-0.887
K = 6.565	11.33	46.96	1.604	-5.4

Table 6: Stability margins for P=6.565.

Not surprisingly all the stability margins are now smaller, as we are closer to the critical point.

- M_g > 6dB.
- M_φ > 40º.
- $M_d = M_\varphi / w$. Where $M_\varphi = 46.96\pi/180$ and w = 0.513rad/s
  M_d = 1.604, so we could have 1.6 seconds of delay before the system getting unstable.
- Finally, we can study the shortest distance to the critical point by means of the Theorem of Bode Freudenberg. The plot of the Sensitivity function helps us.

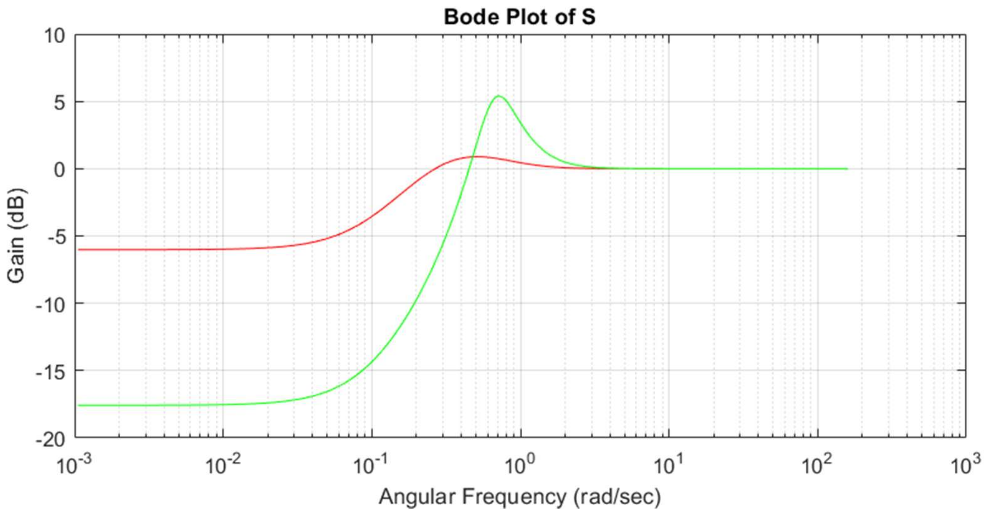


Figure 9: Sensitivity function |S(jw)|.

As we can see, when P = 6.565, S(0) is smaller, what implies a better performance of the system, referring to disturbances. Thanks to the theorem, we know that as the system is stable, the areas S- and S+ (defined by horizontal axis (0dB)) are equal. This implies that with a higher P, we have a higher maximum value of S(jw). Remembering the formula when w>0 of the distance to the critical point = 1/max(|S(jw)|, we can say that in this case, the new distance to the critical point must be smaller, as it can be seen in the properties in the table. So it always exits this unavoidable trade-off: when we increase P, we obtain a better performance but the stability margins decreases, so we will have to be cautious when increasing this proportional action.

2. Proportional and Integral Controller

With the P controller we were able to modify the sensitivity until 2.3dB with the gain having a value of 6.565, but as we can see in figure 10, the dc gain of the closed loop in steady state is around 0.87. As we are looking at a value equal to 1, in other words, we are looking for the perfect tracking performance, a PI controller must be introduced, since the system in open loop L(s) is of type 0 (no integrators) and the input is a step.

With the PI controller we have to tune a new variable T, that we first establish it with the ideal relation $10/w_r$ where w_r is the resonant frequency of our P controller (w_r=0.5735). At the moment that we implement the controller, the error tends to zero but in a very slow way. This is the reason why we should choose a smaller Ti.

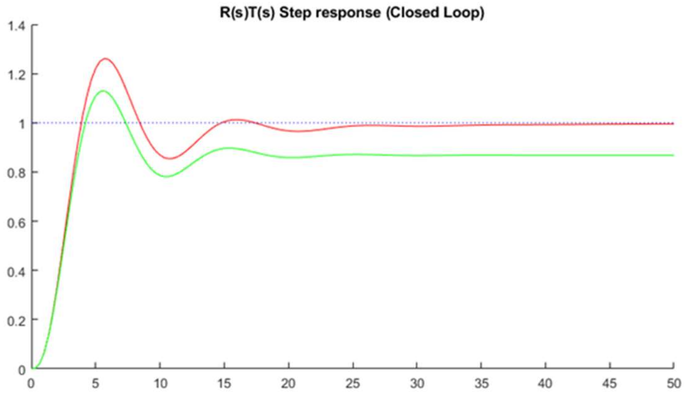


Figure 10: Comparison between P controller and PI controller with $10/w_r$ and without modifying P.

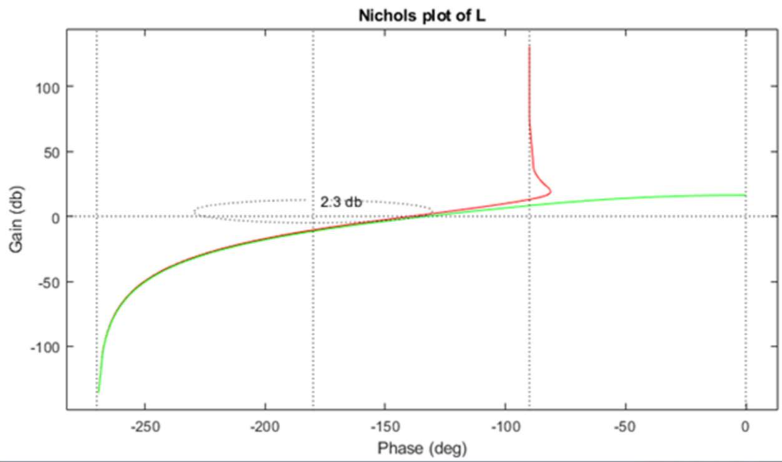


Figure 11: Frequency response between P controller and first tune PI controller.

To increase the speed of our system we decrease the value of T. As it is recommended to have a value of T between $1/w_r$ and $10/w_r$, we use a value of $5/w_r$. Now we have a faster response but there is still a problem to be solved. When we decrease the value of T, the stability margins are not satisfied, and the plot |L(jw)| gets inside the 2.3db circle. Thus, we need to modify our P value in order to obtain be tangent to the 2.3dB-circle, finishing in a value of P=4.5.

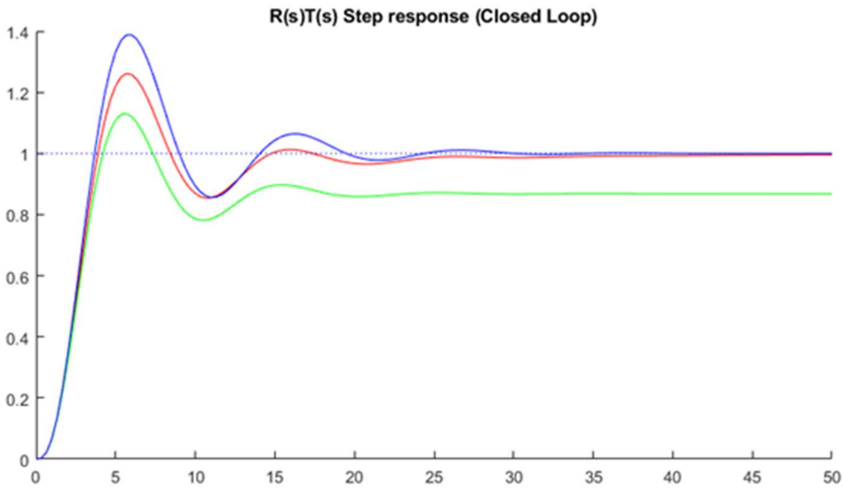


Figure 12: In green, P controller. In red, PI controller with $10/w_r$. In blue PI controller with $5/w_r$.

In figure 13 is shown the frequency response before and after changing P. Now we can see that |L(jw)| is tangent to the 2.3dB-circle.

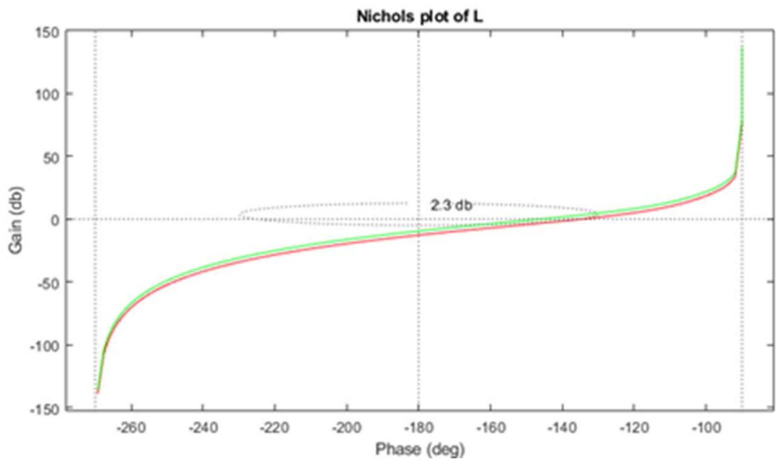


Figure 13: Frequency response of PI controller with P=6.565 (green) and PI controller with P=4.5(red).

Comparing the PI response, we can observe that the error is eliminated in comparison with the P controller (ε_s= 13.22%). The percentage of overshoot is a little bit lower for the PI tuned (M_p=24.7284) comparing it with the no tuned PI (M_p=26.1553). This is the reason why this last one reaches the steady state later, because it has more oscillations. The proportional system should be faster than the PI tuned, but it has a high overshoot that slows it down.

Control	T		M_p (%)	t_p (s)	t_s 5% (s)	t_s 2% (S)	y_∞	ε_s (%)
P	.-	K=6.565	30.16	5.579	12.267	16.944	0.8678	13.22
PI	T= 10/0.573	K = 6.565	26.1553	5.753	13.394	23.59	1	0
PI	T= 5/0.573	K = 4.5	24.7284	7.17	10.65	16.1	1	0

Table 7: Time domain response comparison between P, no-tuned PI and tuned PI controller.

For the tuned PI controller, we can see in figure 13 that |T(jw)| is almost tangent to the 2.3dB-circle ($|T(jw)|_{max}$=2.3544). Taking into account the values of Table 8, we see that the PI controller with P=6.565 (the same value of the P controller) is inside the 2.3dB-circle with a value of $|T(jw)|_{max}$ =3.51576, so we can conclude that we are closer to the critical point. In the two PI controllers we have |T(0)|=0, in other words, we can confirm that there is no steady state error in both systems.

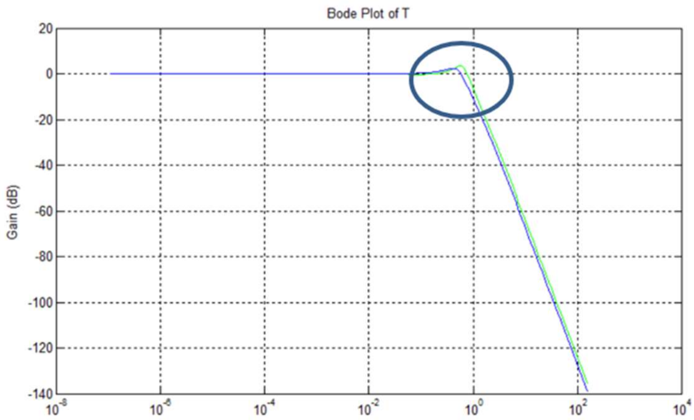


Figure 14: Bode diagram of PI with T=5/w_r (blue) and PI with T=10/w_r (red)

Control	T		$\lvert T(jw)\rvert_{max}$ (dB)	w_r (rad/s)	$\lvert T(0)\rvert$ (dB)	Mr (dB)	w_c (rad/s)
P	.-	K=6.565	2.298	0.5735	-6.0206	0	0.2376
PI	T= 10/0.573	K = 6.565	3.51576	0.5594	0	3.5157	0.88
PI	T= 5/0.573	K = 4.5	2.35441	0.4156	0	2.3544	0.7

Table 8: Frequency response.

For the stability margins (see Table 9) all of the controllers accomplish the conditions for gain margin and phase margin. For no tuned PI:

- M_g=10.4173dB > 6dB.
- M_φ=40.33º> 40º.

In the phase margin of the no tuned PI we have a value of 40.33º so we are very close to the critical point.

For the tuned PI:

- M_g=12.6868 > 6dB.
- M_φ=45º> 40º.
- The delay margins allow us to have a delay between 1.3 and 2 before reaching instability.

Control	T		M_g (dB)	M_ϕ (º)	M_d (s)	M_{sd} (dB)
P	.-	K=6.565	11.33	46.96	1.604	-5.4
PI	T= 10/0.573	K = 6.565	10.4173	40.33	1.372	-6.13
PI	T= 5/0.573	K = 4.5	12.6868	45	2.0006	-5.03

Table 9: Stability margins for P, no tuned PI and tuned PI.

In conclusion, with PI controller |S(0)| at w=0 will tend to -∞ since $\mathcal{E}_s$=0(%). The decrease of T value will accelerate the system but it will also rise the negative phase, which can destabilize the system, so it has to be modified depending on the necessities of the user. So as a negative phase was introduced when using 5/w_r, we were obliged to reduce the value of P=4.5 to get away from the critical point.

This is one of the important drawbacks that exists when working with a PI controller: the pole created at zero can have a destabilizing effect. So this is the reason why using a Lag controller, which is an approximation of the PI controller, can be a good idea.

3. Lag Controller

In this case, the increase of gain at low frequencies is going to be high but not infinite as before. Besides, the problem about the negative phase is somehow fixed although not totally fixed. In the Bode plot, we can see how at low frequencies the phase of L(jw) using a lag controller starts on 0(blue colour), whereas using a pure PI, it starts in -90º(green).

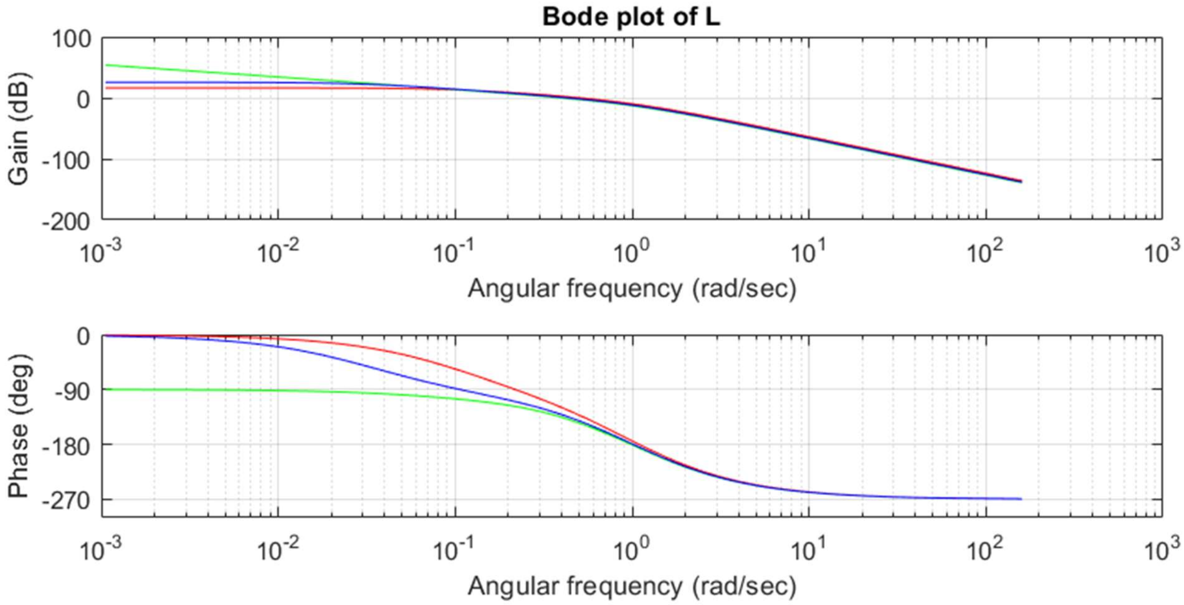


Figure 15: Bode diagram of L(s) using PI controller(green) and lag controller (blue).

For tuning the lag controller, firstly we have to notice that they are asking for the steady state error to be 5%. This way, we can calculate the value of P that should be introduced in the system. Using the formula of the error we obtain P=19.

For tuning b, we can make an approximation with the value we used before(Po = 6.565, gain with the proportional controller) and the P we obtained in this new case:

$$b = \frac{P}{P_o} = 2.9$$

This formula is used when T ≈ 10 / wr. However, as we previously explained, we are using half of this value, so the plot L(jw) gets inside the M circle. This is the reason why we have to increase b until the value of 3.75, so that it is again tangent to the 2.3dB circle.

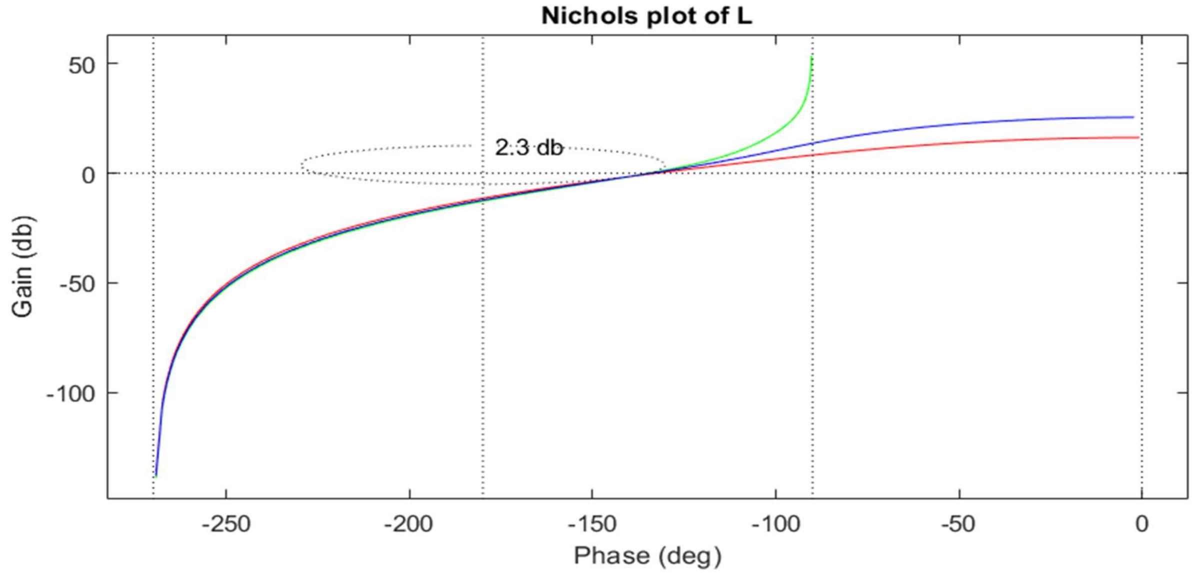


Figure 16: Nichols plot of L(jw) using PI (green) and Lag controller (blue).

In Nichols plot, we can clearly see how with the PI controller we have an infinite gain at low frequencies, which is something good because it means that the steady state error is null, but at the same time, the phase is -90º, what can cause instabilities. So with the lag controller we don't have this drawback although we could never reach the null error because we would need a very huge gain that would make the system unstable.

We can also make some comparisons in the time response with the following figure and table.

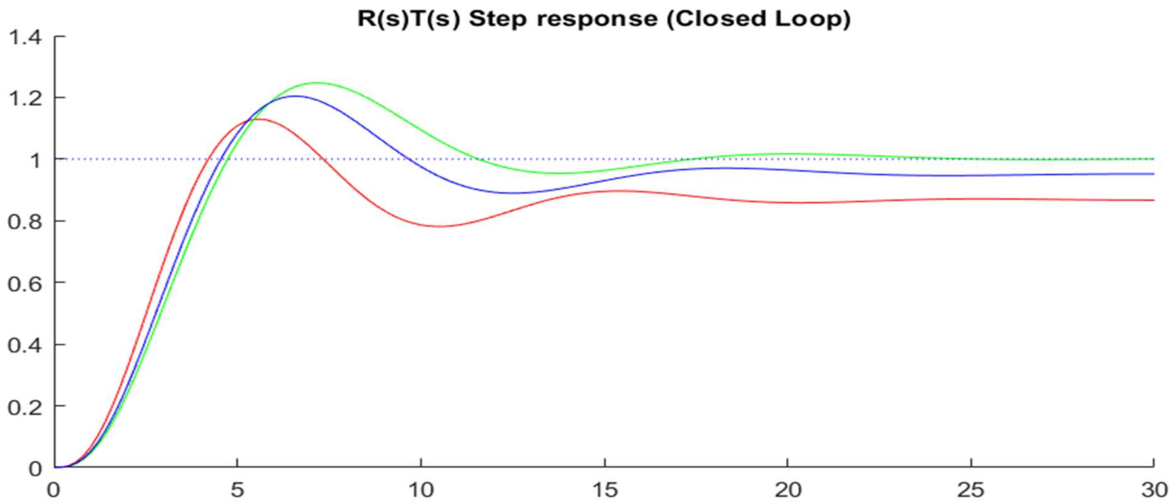


Figure 17: Step response of closed loop using PI(green) and lag controller (blue).

Control	b	T		M_p (%)	t_p (s)	t_s 5% (s)	t_s 2% (S)	y_∞	ε_s (%)
P	.-	.-	K=6.565	30.16	5.579	12.267	16.944	0.8678	13.22
PI	.-	T= 5/0.573	K = 4.5	24.7284	7.17	10.65	16.1	1	0
Lag Controller	3.75	T= 5/0.574	K = 19	26.7459	6.58	13.74	19.08	0.95	5

Table 10: Time domain statistics comparison between P, PI and Lag controller.

The lag controller is the slowest reaching the steady state due to a higher oscillation, which is produced by a higher gain K=19, but it is faster than the PI system, thus, it reaches the peak in less time. The system using a P controller is fast, but it reaches very late the steady state due to high oscillations.

If we take a look at the properties in the frequency domain, we can highlight that $|T(0)| \neq 0$, since the error is not null. This also can be seen in the plot of $|S(jw)|$. In the case of the PI, $|S(0)| \rightarrow -\infty$, whereas in the lag controller, this value is finite. So, we can conclude thanks to the Theorem of Bode Freudenberg and the unavoidable trade-off explained before that with the lag controller the stability margins are higher but the performance is worse.

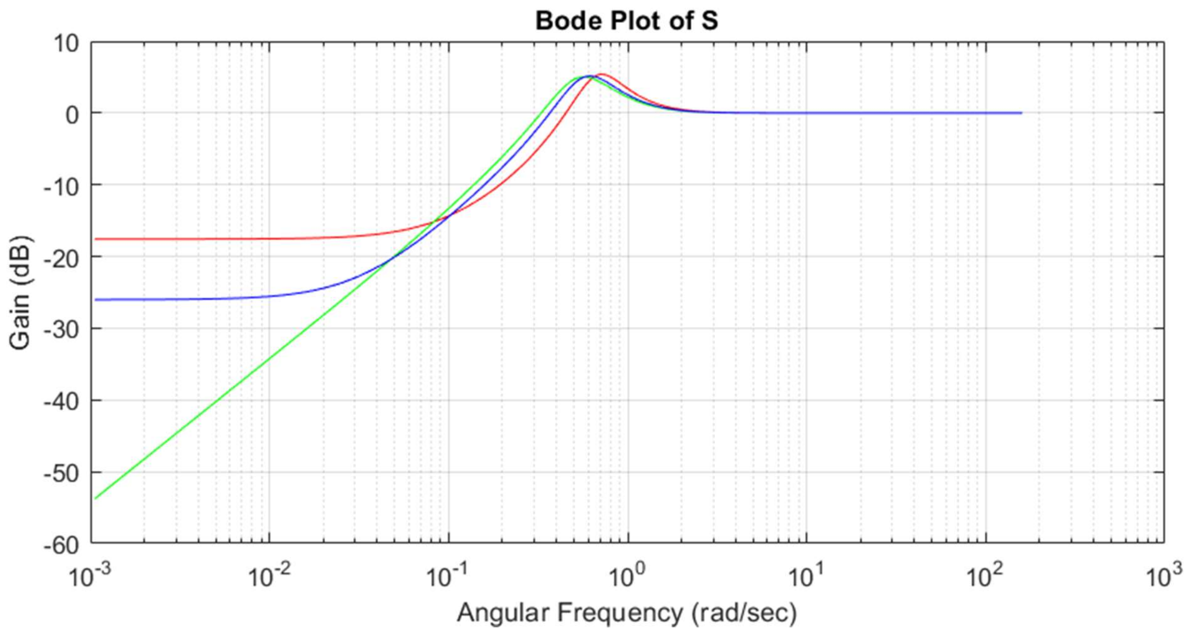


Figure 18: $|S(jw)|$ using PI(green) and Lag controller (blue).

We can see that the values of $|T(jw)|$ are not the same between the controllers and different from 2.3dB because they are not perfect tangent to the 2.3dB circle. Due to the increase of K, the system is a little bit faster and this can be observed in the higher values of the angular frequencies. Referring to the overshoot, M_r must be higher in the lag controller, so the relation with M_p is verified again.

Control	b	T		$\lvert T(jw)\rvert_{max}$ (dB)	w_r (rad/s)	$\lvert T(0)\rvert$ (dB)	Mr (dB)	w_c (rad/s)
P	.-	.-	K=6.565	2.298	0.5735	-6.0206	0	0.2376
PI	.-	T= 5/0.573	K = 4.5	2.35441	0.4156	0	2.3544	0.7
Lag Controller	3.75	T= 5/0.574	K = 19	2.31361	0.4695	-0.4455	2.7591	0.7689

Table 11: Frequency domain statistics comparison between P, PI and Lag controller.

What we said before about the stability margins being higher in the lag controller seems not to be true, although they are quite similar (see Table 12). This could be due to the fact that in this case we have a higher gain K, which destabilize the system and it gets compensated with the trade-off for giving at the end lower stability margins. However, all stability margin values satisfy the stability of the system:

- M_g=12.1937dB > 6dB.
- M_φ=45.7162º> 40º.
- M_d=1.85 so we could have 1.85 seconds of delay before the system getting unstable.
- We are slightly closer to the critical point.

Control	b	T		M_g (dB)	M_ϕ (º)	M_d (s)	M_{sd} (dB)
P	.-	.-	K=6.565	11.33	46.96	1.604	-5.4
PI	.-	T= 5/0.573	K = 4.5	12.6868	45	2.0006	-5.03
Lag Controller	3.75	T= 5/0.574	K = 19	12.1937	45.7162	1.8563	-5.15

Table 12: Stability margins for P, PI and Lag controller

4. Conclusions

Using a controller is usually a necessity for the regulation or the tracking of the system (here we have studied the tracking problem, as we always had a step input). When a perfect tracking is not required, we can use a proportional controller with high gain to make the system faster and to approximate the proportional controller to the inverse of the model (so that the outcome in the steady state gets closer to the reference). But, is obvious that we can not introduce whatever energy to the system (when proportional action equals to infinite, outcome y equals to reference r) , because it gets unbalanced. This is the reason why there is a maximum proportional action that can be introduced into the system.

So in case of willing the outcome to be equal in the steady state to the reference, we need to use a controller with integral action, which introduces a pole in the origin. Steady state error will be null (if the input is a step), however our system will be really slow. Another drawback is that this controller introduces a negative phase that could destabilize the system.

So sometimes the best option is to use an intermediate controller as the Lag controller. We can not have null steady state error, but very close to it. We will also have a faster system than the one with the PI controller.

Sometimes, in some applications, you can afford a little error in the tracking and a Lag controller could be used instead of the PI controller so that not to slow down the global system. In other cases, you could want just to slightly speed up the system and a proportional controller will be ideal. All in all, the election of the controller depends on the application and its requirements.