

## Lecture 3a

# Valuation

## *Financial & Real Investments*

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## Topics

- Valuation of Financial Assets
  - Bonds or Fixed Income Securities
  - Common Stock
  - Preferred Stock
- Efficient Market Hypothesis
- Investment Projects

# What is A Bond?

A **Long-term Debt** instrument in which a borrower agrees to make payments of interest and principal on “**specific**” dates to the holders of the bond.

## Features of a bond

- Secured versus Unsecured
- Par Value – Also called face value
- Coupon rate – The interest rate of the bond stated as % of par value
- Maturity Date – Determine time to maturity or existence
- Bond Ratings – Indicate the risk of default
- Yield to Maturity – Also called “**promised**” (misleading) yield
- Pecking Order of Claims
- Contingent Properties – Callable, convertible or Puttable

## Examples of Bonds Quoted in Singapore

Issuer	Curr- ency	Amount (\$m)	Issued Year	Coupon	Maturity dd-mm-yy	Payment Freq
Centrepoint	SGD	150	2010	3.430	15-04-15	2
Citigroup	SGD	100	2005	3.500	08-04-20	2
DBS	USD	750	2004	5.000	15-11-19	2
F&N	SGD	200	2009	3.410	12-08-16	2
ABN Amro	AUD	916	2011	9.160	29-06-14	4
Ford	USD	5,000	2000	7.375	28-10-15	2
HDB	SGD	500	2005	3.375	21-04-15	2
JTC	SGD	300	1999	5.000	23-06-09	2
LTA	SGD	500	2007	4.810	09-06-17	2

# International Government Bonds

Issuer	Currency	Coupon	Maturity yyyy-mm-dd	Price	Current Yield <sup>¶</sup>
Denmark	DKK	5.00	2013-11-15	104.0000	4.81
Germany	EUR	2.00	2006-03-10	98.9535	2.02
Netherlands	EUR	5.00	2011-07-15	105.9563	4.72
Netherlands	EUR	3.75	2014-07-15	95.1800	3.94
France	EUR	5.50	2010-04-25	108.8400	5.05
Austria	EUR	3.80	2013-10-20	95.9600	3.96
Finland	EUR	5.75	2011-02-23	110.4288	5.21
Portugal	EUR	5.85	2010-05-20	110.3900	5.30
Singapore	SGD	4.00	2018-09-01	95.7900	4.18

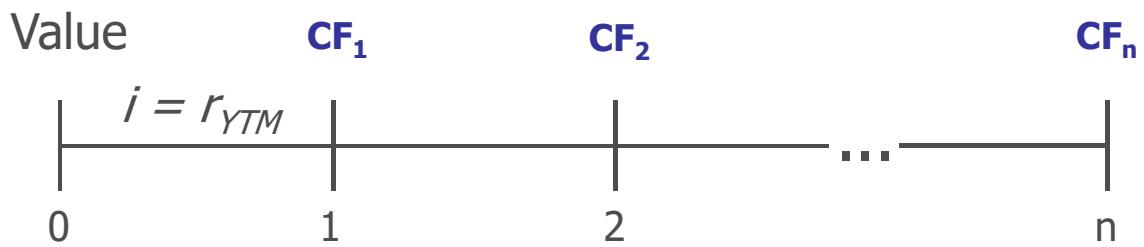
<sup>¶</sup>*Current Yield* is a technical term used in the bond markets and defined as Annual Coupon divided by the current price.

## Where Are Bonds Traded?

- Primarily traded in the over-the-counter (OTC) market.
- Most bonds are owned by and traded among large financial institutions.
- Full information on bond trades in the OTC market is not published, but a representative group of bonds is listed and traded on the bond division of the NYSE.
- For more details on Asian debt markets, click the link:

<https://asianbondsonline.adb.org/abm.php>

# Simple Valuation of A Straight Bond



- If you know the appropriate discount of a bond, the value of the bond is just the sum of the discounted cash flows.

$$Value = PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_n}{(1+r)^n}$$

Note that for bonds, CF<sub>1</sub> to CF<sub>n-1</sub> = Coupon and CF<sub>n</sub> = Coupon + Par Value

- Inversely if you know the value, you can find the discount rate, which is the **IRR**, commonly known as the *yield to maturity*.

## Yield to Maturity (YTM)

- What is the YTM on a 10-year, 9% annual coupon, \$1,000 par value bond, selling for \$887?
- Must find the  $r_{YTM}$  that solves this model.

$$V_B = \frac{\text{INT}}{(1+r_{YTM})^1} + \dots + \frac{\text{INT} + \text{Par Value}}{(1+r_{YTM})^N}$$
$$\$887 = \frac{90}{(1+r_{YTM})^1} + \dots + \frac{90 + 1,000}{(1+r_{YTM})^{10}}$$

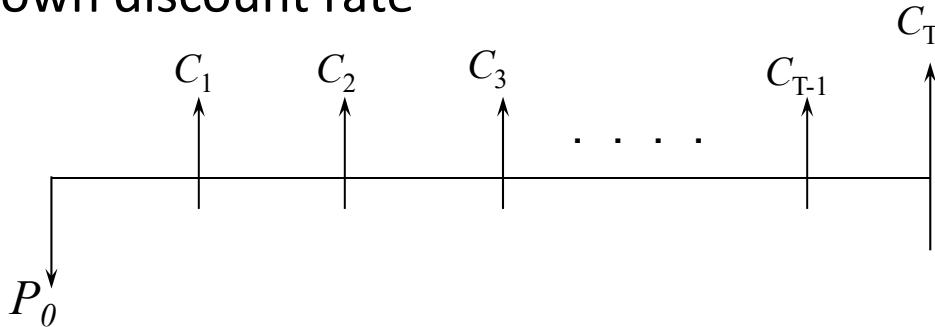
where INT = annual coupon payment

- The above calculation is based on the assumption that the coupon is paid once a year.
- What is the  $r_{YTM}$  if the coupon is paid twice a year?

# Bond Valuation

## A Demanding Approach

- This approach allows each cash flow to have its own discount rate



$$\begin{aligned}
 \text{Value} &= PV \\
 &= \frac{C_1}{(1+y_1)} + \frac{C_2}{(1+y_2)^2} + \cdots + \frac{C_T}{(1+y_T)^T} \\
 &= A \cdot b
 \end{aligned}$$

where  $A = [C_1 \quad \cdots \quad C_T]$

$$b = \begin{bmatrix} 1 \\ \frac{1}{(1+y_1)} \\ \vdots \\ 1 \\ \frac{1}{(1+y_1)^T} \end{bmatrix}$$

## Finding $b$ or $y_t$

- The data below is for three treasury bonds

Bond	Price	Year 1	Year 2	Year 3
1	99.50	105	0	0
2	101.25	6	106	0
3	100.25	7	7	107

$$b = A^{-1}P$$

$$\begin{aligned}
 &= \begin{bmatrix} 105 & 0 & 0 \\ 6 & 106 & 0 \\ 7 & 7 & 107 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 99.50 \\ 101.25 \\ 100.25 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{(1+y_1)} \\ \frac{1}{(1+y_2)^2} \\ \frac{1}{(1+y_3)^3} \end{bmatrix} = \begin{bmatrix} \frac{1}{b_1} \\ \frac{1}{b_2} \\ \frac{1}{b_3} \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{b_1}\right)^{\frac{1}{2}} - 1 \\ \left(\frac{1}{b_2}\right)^{\frac{1}{3}} - 1 \\ \left(\frac{1}{b_3}\right)^{\frac{1}{3}} - 1 \end{bmatrix}
 \end{aligned}$$

Note: If  $y_t$  are defined as continuously compounded,

$$\begin{aligned}
 y_t &= \ln(1/b_t) / t \\
 &= -\ln(b_t) / t
 \end{aligned}$$

# Finding $b$ or $y_t$ by Regression Analysis

- When the number of bonds is more than the number of maturities, we can estimate  $b$  or  $y_t$  by minimizing the squared error or using the regression analysis without intercept (through origin).

$$P_i = C_{i,1} \cdot b_1 + C_{i,2} \cdot b_2 + \dots + C_{i,T} \cdot b_T + e_i$$

In matrix form, the equation can be expressed as

$$\mathbf{P} = \mathbf{A} \cdot \mathbf{b} + \mathbf{e}$$

Set  $\mathbf{b} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P}$ , then  $\mathbf{e}^T \mathbf{e}$  will be minimized

Note that

$$\mathbf{P} = \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}_{N \times 1}, \mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_T \end{bmatrix}_{T \times 1}$$
$$\mathbf{A} = \begin{bmatrix} C_{1,1} & \cdots & C_{1,T} \\ \vdots & \ddots & \vdots \\ C_{N,1} & \cdots & C_{N,T} \end{bmatrix}_{N \times T}$$

## Mathematics of Regression

- First express the equation of the sum of the squared error as follows:

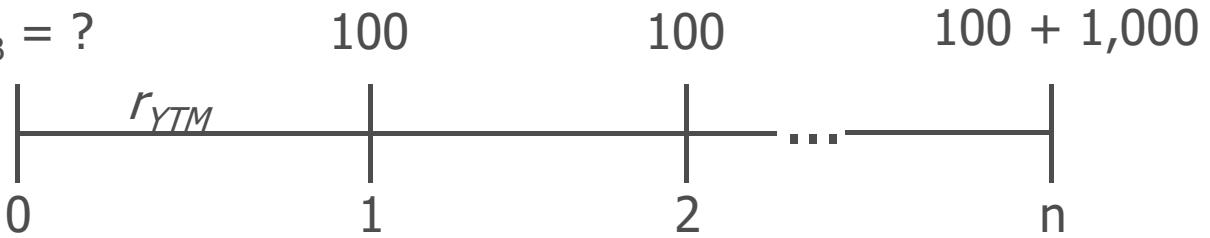
$$\begin{aligned} P &= A \cdot b + e \Rightarrow e = P - A \cdot b \\ e^T e &= (P - A \cdot b)^T (P - A \cdot b) \\ &= (P^T - b^T \cdot A^T)(P - A \cdot b) \\ &= P^T P - 2b^T A^T P + b^T A^T A b \end{aligned}$$

- To minimize the sum of the squared error, find the 1<sup>st</sup> order condition

$$\begin{aligned} \frac{\partial(e^T e)}{\partial b} &= -2A^T P + 2A^T A b \\ 0 &= -2A^T P + 2A^T A b^* \\ A^T A b^* &= A^T P \\ b^* &= (A^T A)^{-1} A^T P \end{aligned}$$

# What is the value of a 10-year bond, 10% coupon with annual payment, if $r_{YTM} = 10\%$ ?

$$V_B = ?$$



$$V_B = \frac{\$100}{(1.10)^1} + \dots + \frac{\$100}{(1.10)^{10}} + \frac{\$1,000}{(1.10)^{10}}$$

$$V_B = \$90.91 + \dots + \$38.55 + \$385.54$$

$$V_B = \$1,000$$

- If the coupon and the yield to maturity are stated as the nominal annual rate, compounded semiannually, what should be the value of the bond?

4 October 2019

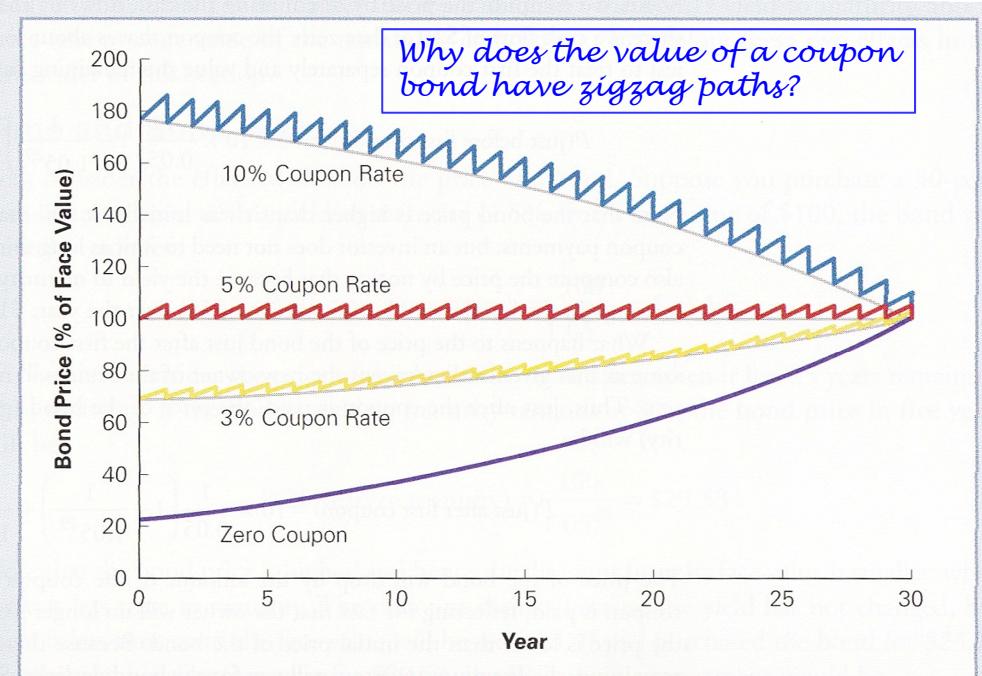
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## Changes in Value over Time of A Coupon Bond

### The Effect of Time on Bond Prices

The graph illustrates the effects of the passage of time on bond prices when the yield remains constant. The price of a zero-coupon bond rises smoothly. The price of a coupon bond also rises between coupon payments, but tumbles on the coupon date, reflecting the amount of the coupon payment. For each coupon bond, the gray line shows the trend of the bond price just after each coupon is paid.



Note: Yield to maturity or YTM is equal to 5%.

**Would you prefer to buy a 10-year, 10% annual coupon bond or a 10-year, 10% semiannual coupon bond, if both are trading at the same price?**

**The semiannual bond's effective annual rate is:**

$$i_{\text{EFF}} = \left(1 + \frac{k_m}{m}\right)^m - 1 = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 10.25\%$$

**10.25% > 10% (The Bond's Annual Effective Rate)**

→ Hence the semiannual coupon bond is preferred.

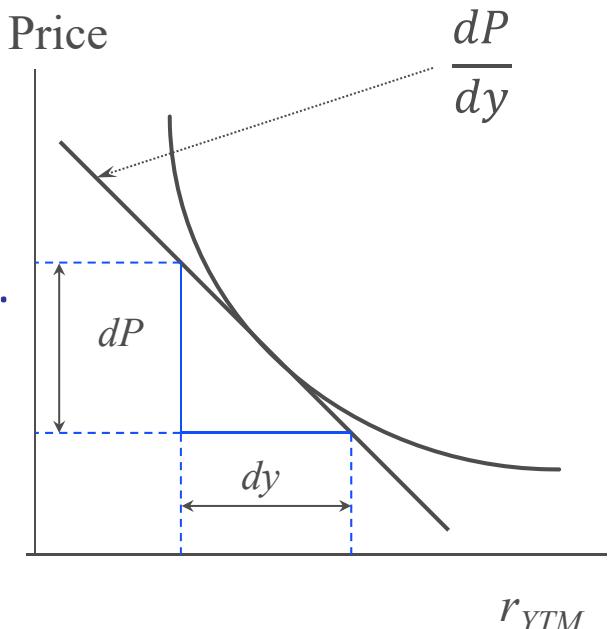
## **Yield-to-Call (YTC)**

- Consider a 10-year, 10% semiannual coupon bond selling for \$1,135.90. The bond can be called in 4 years for \$1,050.
  - The bond's YTM can be determined to be 8% (verify).
  - Solving for the YTC is identical to solving for YTM, except the time to call is used for  $N$  and the call premium is  $FV$ .
    - Answer :  $YTC = 3.568 \times 2 = 7.136\%$
    - $\text{Rate}(\text{nper}=8, \text{PMT}=50, \text{PV}=-1135.9, \text{FV}=1050) = 3.568\%$
- Which Rate, YTM or YTC, Would be More Relevant to investors?

# Further Extension

- Can you differentiate the bond price with respect to its discount rate or YTM? If yes, you will find interesting results.

$$P = \sum_{t=1}^T \frac{CF_t}{(1 + r_{YTM})^t}$$



# Common Stocks

## □ Features of Common Stock

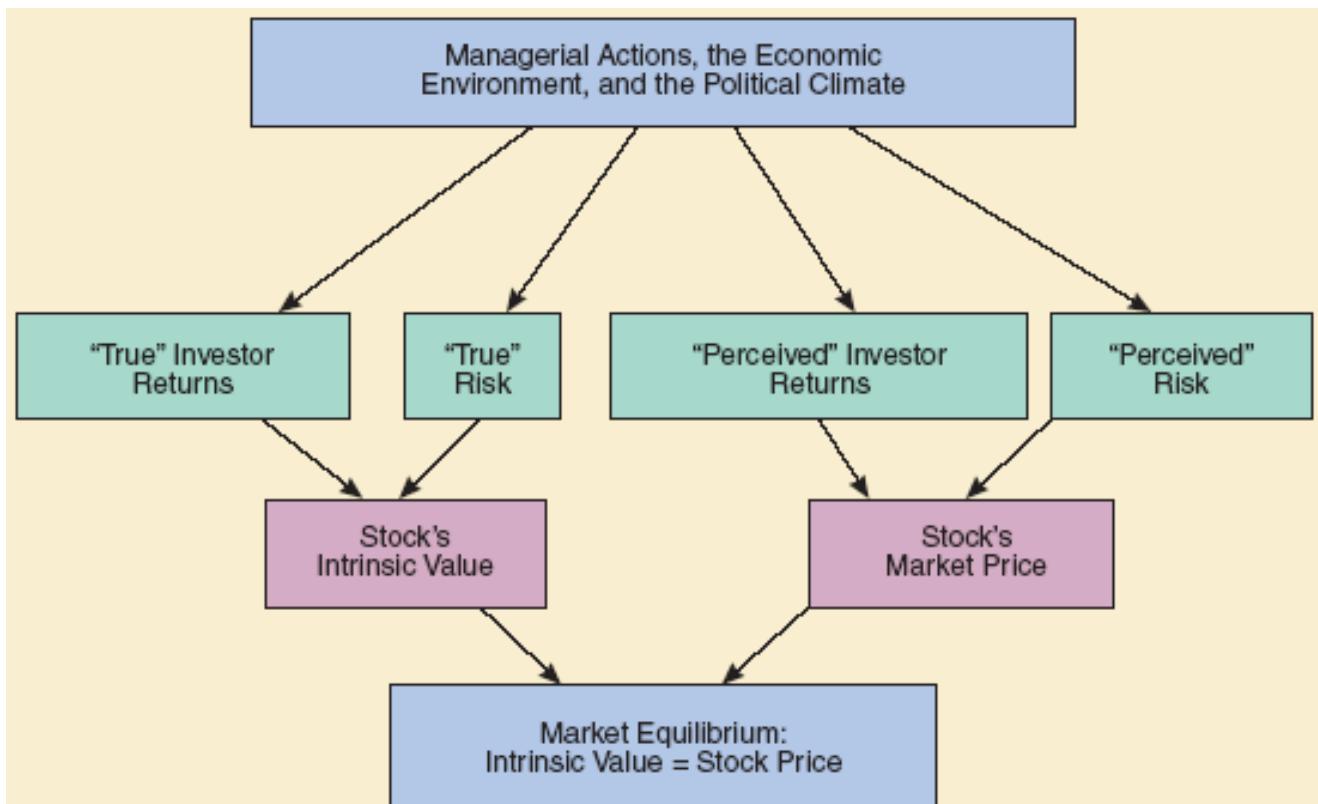
- Represents ownership
- Ownership implies control
- Stockholders elect directors
- Directors elect management
- Management's goal is to maximize the value of the firm.

## □ How to Determine the Value of Common Stocks?

# Value vs Price

- Outside investors, corporate insiders, and analysts use a variety of approaches to estimate the *intrinsic* value of a stock, which is *unobservable*.
- If the market is informationally efficient, the *observable* price of a stock should, on average, be equal to the intrinsic value.
  - Outsiders estimate the intrinsic value of a stock to find investment opportunities.
    - Given a reliable intrinsic value, they consider the stocks to be undervalued (overvalued) if the observed price below (above) its intrinsic value.

**Figure 1-1: Determinants of Intrinsic Value & Price of A Stock**



# Estimating the Intrinsic Value of A Stock

*Dividend Model, Discounted FCF, Market Multiples*

## □ Dividend Growth (Gordon) Model

- Value of a stock is the present value of the future dividends expected to be generated by the stock.
- A stock whose dividends are expected to grow forever at a constant rate,  $g$ .
- $D_1 = D_0 \times (1+g)^1, D_2 = D_0 \times (1+g)^2, \dots, D_t = D_0 \times (1+g)^t$

$$P_0 = \frac{D_0(1+g)}{(1+r)} + \frac{D_0(1+g)^2}{(1+r)^2} + \dots + \frac{D_0(1+g)^n}{(1+r)^n}$$

## Dividend Growth (Gordon) Model

- If  $g$  is constant, the dividend growth formula converges to:

$$P_0 = \frac{D_0(1+g)}{r_s - g} = \frac{D_1}{r_s - g}$$

$r_s$  must be greater than  $g$ .

Note that  $P_0$  is identical to the PV of growing perpetuities

- Derivation

$$\begin{aligned} P_0 &= \frac{D_0(1+g)}{(1+r_s)} + \frac{D_0(1+g)^2}{(1+r_s)^2} + \dots \\ \frac{P_0(1+g)}{(1+r_s)} &= \frac{D_0(1+g)^2}{(1+r_s)^2} + \frac{D_0(1+g)^3}{(1+r_s)^3} + \dots \\ P_0 \left\{ 1 - \frac{(1+g)}{(1+r_s)} \right\} &= P_0 \left\{ \frac{(1+r_s) - (1+g)}{(1+r_s)} \right\} = \frac{D_0(1+g)}{(1+r_s)} \end{aligned}$$

# Dividend Constant-Growth Model

## Implications

□ The model implies that

$$r_s = \frac{D_1}{P_0} + g$$

Example: A company just paid a dividend of \$2 per share. If  $r_s = 13\%$  and the dividend growth rate is equal to 6% and constant, estimate the price of the stock and verify the components of  $r_s$ .

$$\begin{aligned} P_0 &= \frac{D_1}{r_s - g} = \frac{2 \times 1.06}{0.13 - 0.06} \\ &= \frac{2.12}{0.07} = 30.29 \end{aligned}$$

$$\begin{aligned} P_1 &\Rightarrow \frac{D_2}{r_s - g} = \frac{2 \times (1.06)^2}{0.13 - 0.06} = 32.10 \\ &\Rightarrow P_0 \times (1 + g) = 30.29 \times 1.06 = 32.10 \end{aligned}$$

❖ **Dividend Yield**

$$= D_1 / P_0 = 2.12 / 30.29 = 7.0\%$$

❖ **Capital Gains Yield**

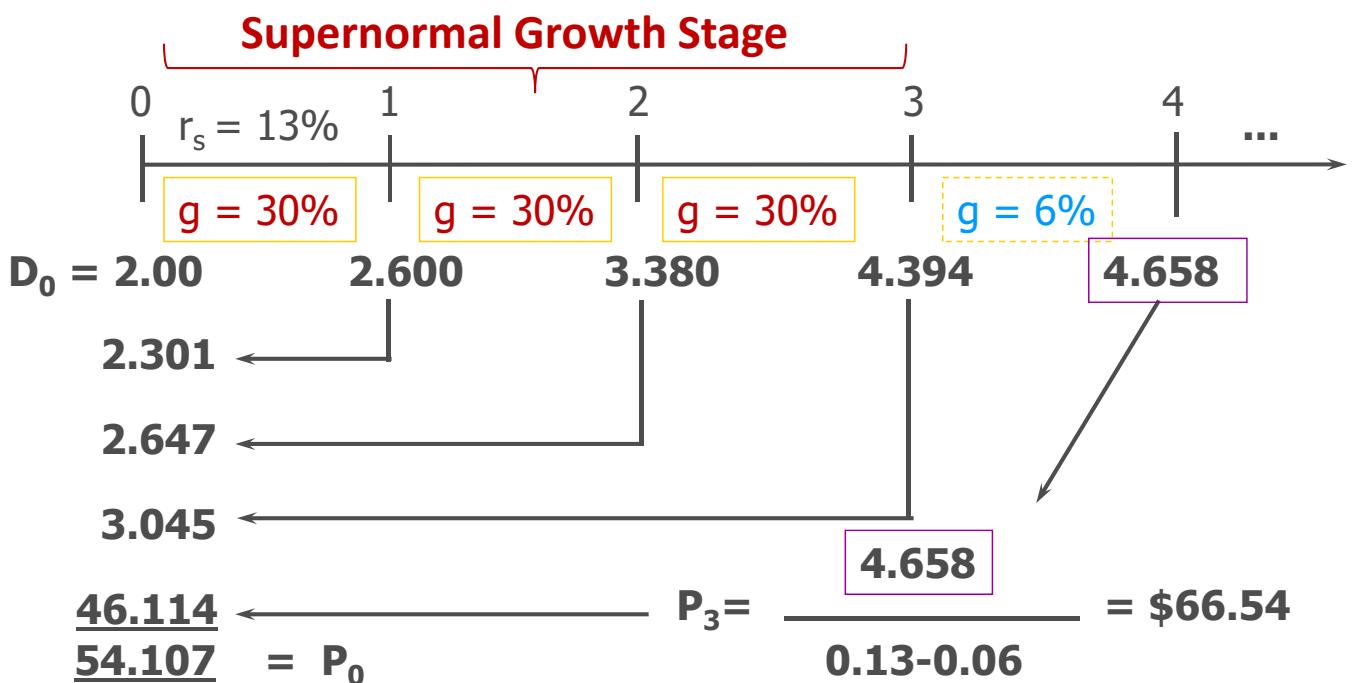
$$\begin{aligned} &= (P_1 - P_0) / P_0 = (D_2 - D_1) / D_1 = g \\ &= (32.10 - 30.29) / 30.29 = 6.0\% \end{aligned}$$

❖ **Total Return ( $r_s$ )**

$$\begin{aligned} &= \text{Dividend Yield} + \text{Capital Gains Yield} \\ &= 7.0\% + 6.0\% = 13.0\% \end{aligned}$$

## Dividend Model

### Two-Stage Growth



Note that

- During non-constant growth, dividend yield and capital gains yield are not constant, and capital gains yield  $\neq g$ .
- After  $t = 3$ , the stock has constant growth and dividend yield = 7%, while capital gains yield = 6%.

# Corporate Value Model

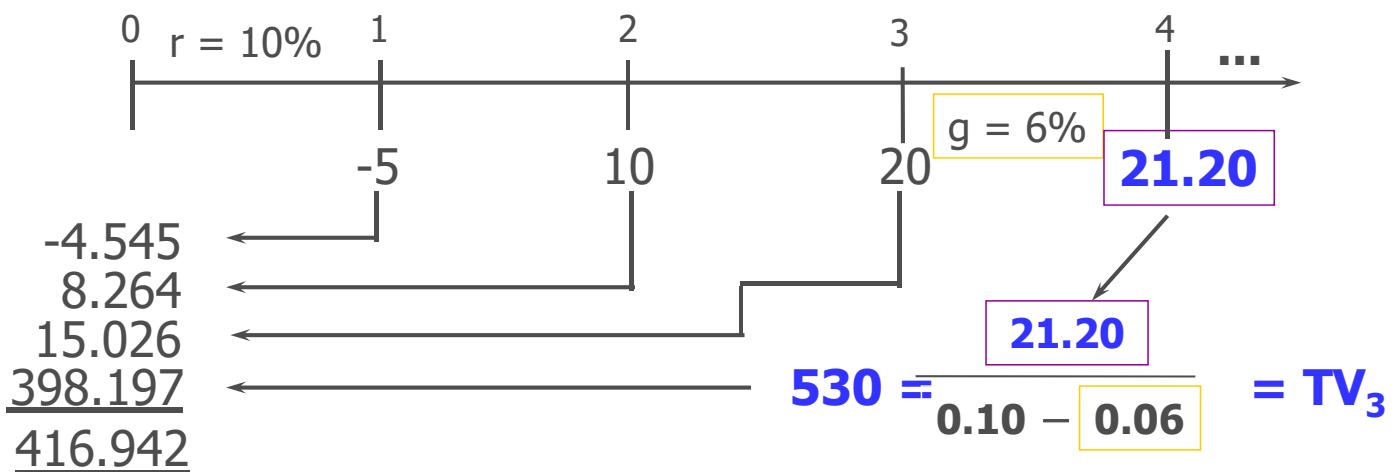
## Discounted FCF

- This approach suggests the value of the “entire firm” equals the present value of the firm’s free cash flows.
- Remember, free cash flow is the firm’s after-tax operating income less the net capital investment
  - $FCF = NOPAT - \text{Net Capital Investment}$ 
    - NOPAT or Net Operating Profit after Taxes =  $EBIT \times (1 - \text{Tax Rate})$
    - EBIT = Earnings before Interest and Taxes
    - Net Capital Investment = Gross Investment - Depreciation

## Discounted FCF Method

- Find the market value (MV) of the firm, by finding the PV of the firm’s future FCFs.
- Subtract MV of firm’s net debt and preferred stock to get MV of common stock.
- Divide MV of common stock by the number of shares outstanding to get intrinsic stock price.

Example: Given the long-run  $g_{FCF} = 6\%$ , and WACC of 10%, use the corporate value model to find the firm’s intrinsic value.



**If the firm has \$40 million in debt and has 10 million shares of stock, what is the firm's intrinsic value per share?**

$$\begin{aligned}\square \text{ MV of Equity} &= \text{MV of Firm} - \text{MV of Debt} \\ &= \$416.94 - \$40 \\ &= \$376.94 \text{ million}\end{aligned}$$

$$\begin{aligned}\square \text{ Value per Share} &= \text{MV of Equity} / \# \text{ of shares} \\ &= \$376.94 / 10 \\ &= \$37.69\end{aligned}$$

## The Discounted FCF Method

### *Some Interesting Observations*

- The discounted FCF method is often preferred to the dividend growth model, especially when considering number of firms that don't pay dividends or when dividends are hard to forecast.
- The FCF method is somewhat similar to the dividend growth model; assuming at some point the FCF will grow at a constant rate.
- Terminal value ( $TV_N$ ) represents the value of the firm at the point that growth becomes constant.
  - The growth rate used in determining  $TV_n$  must be less than the discount rate.

# Market Multiple Analysis

- Analysts often use the following multiples to value stocks.

- P / E Ratio
- P / CF Ratio
- P / Sales Ratio
- EV / EBITDA
- EV / EBIT
- EV / NOPAT

Note:

EBITDA = Earnings before interest, taxes, depreciation & amortization

EV = Enterprise value of the firm

- For example, estimate the P/E of comparable firms. Multiply this estimate by the expected earnings to back out an estimate of the stock price.
- The key limitation of multiples is that comparable firms must be *identical*. In reality, no firm is identical to any other firm.

## What is Market Equilibrium?

- Since demand is equal to supply in equilibrium, stock prices should be relatively stable.
- In equilibrium, two conditions hold:
  - The stock price equals its *intrinsic* value .
  - *The expected* return must equal *the required* return.
- Recall that
  - Neither the intrinsic value nor the required return is observable.
  - To estimate them, we need a set of assumptions or a pricing model.

# How is the Equilibrium Established?

- If price is below intrinsic value ...
  - The current price ( $P_0$ ) is “too low” and offers a bargain.
  - Buy orders will be greater than sell orders.
  - $P_0$  will be bid up until expected return equals required return.
- It takes the force of competition to equate the price and the value.
- Hence the equilibrium requires competition.

## How Is the Equilibrium Value Determined?

- Are the equilibrium intrinsic value and the expected return estimated by managers?
- Or are they determined by something else?
- Equilibrium levels are based on the market's estimate of intrinsic value and the market's required rate of return, which are both dependent upon the attitudes of the *marginal* investor.

Note: The *marginal* investor is the investor who determines the market prices of the securities under consideration.

# Preferred Stock

- Hybrid Security between Debt and Equity
- Like bonds, preferred stockholders receive a fixed dividend that must be paid before dividends are paid to common stockholders.
- However, companies can omit preferred dividend payments without fear of pushing the firm into bankruptcy.

Example: If preferred stock with an annual dividend of \$5 sells for \$50, what is the preferred stock's expected return?

$$\begin{aligned} V_p &= D / r_p \\ \$50 &= \$5 / r_p \\ r_p &= \$5 / \$50 \\ &= 0.10 \text{ or } 10\% \end{aligned}$$

## Efficient Market Hypothesis

Since securities are **fairly priced** in equilibrium, one cannot beat the market except through good luck or inside information.

- **Weak Form:** Can't profit by looking at past trends. A recent decline is no reason to think stocks will go up (or down) in the future. Since evidence supports weak-form EMH, "Technical Analysis" would be futile.
- **Semi-strong Form:** All publicly available information is reflected in stock prices, so it doesn't pay to pore over annual reports looking for undervalued stocks. Largely true.
- **Strong Form:** All information, even inside information, is embedded in stock prices. Not true--insiders can gain by trading on the basis of insider information, but that's illegal.