

Find Last Occurrence

$O(\log N)$

arr =

0	1	2	3	4	5	6	7	8
1	1	1	2	3	3	3	3	4

↑
si

↑
mid

↑
ei

target = 3

dry run

si = 0, ei = 8, mid = 4

si = 5, ei = 8, mid = 6

si = 7, ei = 8, mid = 7

arr =

0	1	2	3	4	5	6	7	8
1	1	1	2	2	3	3	3	3

target = 3

pseudo
code

```
int si = 0, ei = n-1;
while (si <= ei) {
    int mid = (si + ei) / 2;
    if (arr[mid] == target) {
        if (arr[mid] == arr[mid+1]) {
            si = mid+1;
        } else {
            return mid;
        }
    } else if (arr[mid] > target) {
        ei = mid-1;
    } else {
        si = mid+1;
    }
}
```



dry run

si = 0, ei = 8, mid = 4

si = 5, ei = 8, mid = 6

si = 7, ei = 8, mid = 7

return 7

Binary Search Upper Bound (BSUB)

code

```
public static int BSUB(int n, int[] arr, int target) {  
    int si = 0;  
    int ei = n - 1;  
    while ( si <= ei ) {  
        int mid = (si + ei) / 2;  
        if ( arr[mid] == target ) {  
  
            if ( mid < arr.length - 1 && arr[mid] == arr[mid + 1] ) {  
                si = mid + 1;  
            } else {  
                return mid;  
            }  
  
        } else if ( arr[mid] < target ) {  
            si = mid + 1;  
        } else {  
            ei = mid - 1;  
        }  
    }  
    return -1;  
}
```

arr =

0	1	2	3	4	5	6	7	8
1	1	1	2	2	3	3	3	4

tar=3

↑↑
si ei

↑
mid

si=0, ei=8, mid=4

si=5, ei=8, mid=6

si=5, ei=5, mid=5

Binary Search
Lower Bound
(BSLB)

```
public static int BSUB(int n, int[] arr, int target) {  
    int si = 0;  
    int ei = n - 1;  
    while ( si <= ei ) {  
        int mid = (si + ei) / 2;  
        if ( arr[mid] == target ) {  
            if ( mid - 1 >= 0 && arr[mid] == arr[mid - 1] ) {  
                ei = mid - 1;  
            } else {  
                return mid;  
            }  
        } else if ( arr[mid] < target ) {  
            si = mid + 1;  
        } else {  
            ei = mid - 1;  
        }  
    }  
    return -1;  
}
```

Note:- Binary search need not to applied on given sequence.

means.

it can be applied on any imaginary sequence.

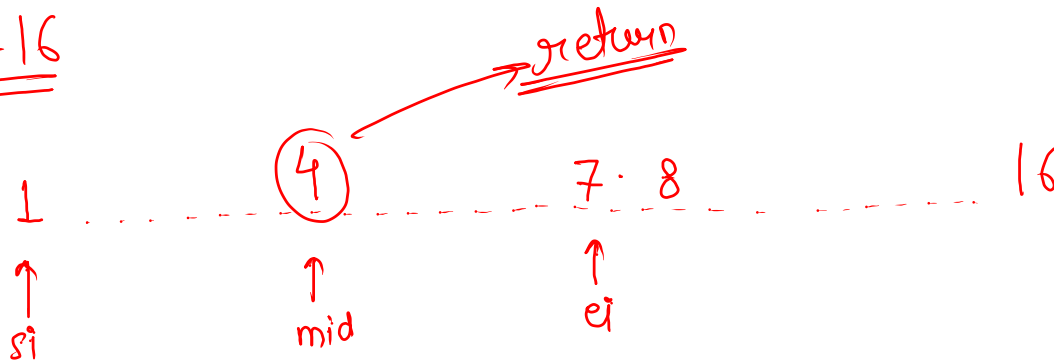
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$$\underline{\underline{n = 20}}$$



Find Square Root

$$\underline{\underline{n = 16}}$$



$$si = 1, ei = 16, mid = 8$$

$$si = 1, ei = 7, mid = 4$$

$$\underline{\underline{8 \times 8 == 16}}$$

$$4 \times 4 == 16$$

Input

n = 30

1 3 4 5 6 7 ... 14 15 ... 30

↑ ei
↑ si
↑ mid

while(si <= ei)

si = 1, ei = 30, mid = 15

si = 1, ei = 14, mid = 7

si = 1, ei = 6, mid = 3

si = 4, ei = 6, mid = 5

si = 6, ei = 6, mid = 6

6 x 6 == 30
6 x 6 < 30
6 x 6 > 30

5 x 5 == 30
5 x 5 < 30
5 x 5 > 30

15 x 15 >= (30)
target

7 x 7 > 30

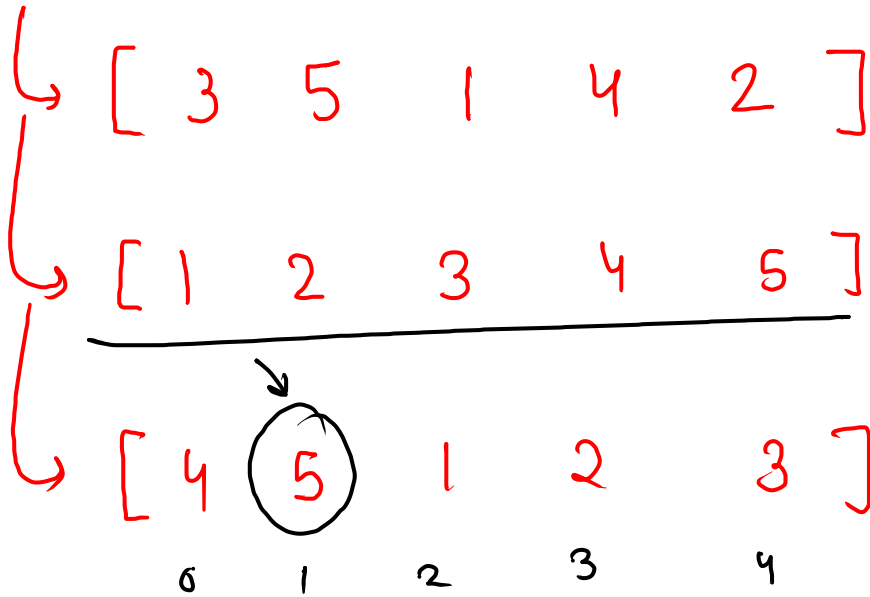
3 x 3 == 30
2 x 3 > 30
3 x 2 < 30

Code

```
public static void main(String[] args) {  
    Scanner scn = new Scanner(System.in);  
    int n = scn.nextInt();  
    System.out.println(squareRoot(n));  
}
```

```
public static int squareRoot(int n) {  
    int si = 1;  
    int ei = n;  
    while ( si <= ei ) {  
        int mid = (si + ei) / 2;  
        if ( mid * mid <= n ) {  
            si = mid + 1;  
        } else if ( mid * mid > n ) {  
            ei = mid - 1;  
        }  
    }  
    return ei;  
}
```


Find The Index of Rotation



using B.S

$O(\log N)$

arr = [8 9 10 11 12 13 1 2 3 4 5 6 7]

\uparrow s_i
 \uparrow mid
 \uparrow e_i

