Programming Assignment 2: Priority Queues and Disjoint Sets

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Introduction

In this programming assignment, you will practice implementing priority queues and disjoint sets and using them to solve algorithmic problems. In some cases you will just implement an algorithm from the lectures, while in others you will need to invent an algorithm to solve the given problem using either a priority queue or a disjoint set union.

Learning Outcomes

Upon completing this programming assignment you will be able to:

1. Apply priority queues and disjoint sets to solve the given algorithmic problems.
2. Convert an array into a heap.
3. Simulate a program which processes a list of jobs in parallel.
4. Simulate a sequence of merge operations with tables in a database.

Passing Criteria: 2 out of 3

Passing this programming assignment requires passing at least 2 out of 3 code problems from this assignment. In turn, passing a code problem requires implementing a solution that passes all the tests for this problem in the grader and does so under the time and memory limits specified in the problem statement.

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1 Problem: Convert array into heap

Problem Introduction

In this problem you will convert an array of integers into a heap. This is the crucial step of the sorting algorithm called HeapSort. It has guaranteed worst-case running time of $O(n \log n)$ as opposed to QuickSort's average running time of $O(n \log n)$. QuickSort is usually used in practice, because typically it is faster, but HeapSort is used for external sort when you need to sort huge files that don't fit into memory of your computer.

Problem Description

Task. The first step of the HeapSort algorithm is to create a heap from the array you want to sort. By the way, did you know that algorithms based on Heaps are widely used for external sort, when you need to sort huge files that don't fit into memory of a computer?

Your task is to implement this first step and convert a given array of integers into a heap. You will do that by applying a certain number of swaps to the array. Swap is an operation which exchanges elements a_i and a_j of the array a for some i and j. You will need to convert the array into a heap using only $O(n)$ swaps, as was described in the lectures. Note that you will need to use a min-heap instead of a max-heap in this problem.

Input Format. The first line of the input contains single integer n. The next line contains n space-separated integers a_i.

Constraints. $1 \le n \le 100\,000$; $0 \le i, j \le n-1$; $0 \le a_0, a_1, \dots, a_{n-1} \le 10^9$. All a_i are distinct.

Output Format. The first line of the output should contain single integer m — the total number of swaps. m **must satisfy conditions** $0 \le m \le 4n$. The next m lines should contain the swap operations used to convert the array a into a heap. Each swap is described by a pair of integers i, j — the 0-based indices of the elements to be swapped. After applying all the swaps in the specified order the array must become a heap, that is, for each i where $0 \le i \le n-1$ the following conditions must be true:

1. If $2i+1 \le n-1$, then $a_i < a_{2i+1}$.
2. If $2i+2 \le n-1$, then $a_i < a_{2i+2}$.

Note that all the elements of the input array are distinct. Note that any sequence of swaps that has length at most $4n$ and after which your initial array becomes a correct heap will be graded as correct.

Time Limits.

language	C	C++	Java	Python	JavaScript	Scala
time (sec)	1	1	3	3	3	3

Memory Limit. 512MB.

Sample 1.

Input:

```
5
5 4 3 2 1
```

Output:

```
3
1 4
0 1
1 3
```

After swapping elements 4 in position 1 and 1 in position 4 the array becomes 5 1 3 2 4.

After swapping elements 5 in position 0 and 1 in position 1 the array becomes 1 5 3 2 4.
After swapping elements 5 in position 1 and 2 in position 3 the array becomes 1 2 3 5 4, which is already a heap, because $a_0 = 1 < 2 = a_1, a_0 = 1 < 3 = a_2, a_1 = 2 < 5 = a_3, a_1 = 2 < 4 = a_4$.

Sample 2.

Input:

```
5
1 2 3 4 5
```

Output:

```
0
```

The input array is already a heap, because it is sorted in increasing order.

Starter Files

There are starter solutions only for C++, Java and Python3, and if you use other languages, you need to implement solution from scratch. Starter solutions read the array from the input, use a quadratic time algorithm to convert it to a heap and use $\Theta(n^2)$ swaps to do that, then write the output. You need to replace the $\Theta(n^2)$ implementation with an $O(n)$ implementation using no more than $4n$ swaps to convert the array into heap.

What to Do

Change the `BuildHeap` algorithm from the lecture to account for min-heap instead of max-heap and for 0-based indexing.

Need Help?

Ask a question or see the questions asked by other learners at this forum thread.

2 Problem: Parallel processing

Problem Introduction

In this problem you will simulate a program that processes a list of jobs in parallel. Operating systems such as Linux, MacOS or Windows all have special programs in them called schedulers which do exactly this with the programs on your computer.

Problem Description

Task. You have a program which is parallelized and uses n independent threads to process the given list of m jobs. Threads take jobs in the order they are given in the input. If there is a free thread, it immediately takes the next job from the list. If a thread has started processing a job, it doesn't interrupt or stop until it finishes processing the job. If several threads try to take jobs from the list simultaneously, the thread with smaller index takes the job. For each job you know exactly how long will it take any thread to process this job, and this time is the same for all the threads. You need to determine for each job which thread will process it and when will it start processing.

Input Format. The first line of the input contains integers n and m.
The second line contains m integers t_i — the times in seconds it takes any thread to process i-th job. The times are given in the same order as they are in the list from which threads take jobs.
Threads are indexed starting from 0.

Constraints. $1 \leq n \leq 10^5$; $1 \leq m \leq 10^5$; $0 \leq t_i \leq 10^9$.

Output Format. Output exactly m lines. i-th line (0-based index is used) should contain two space-separated integers — the 0-based index of the thread which will process the i-th job and the time in seconds when it will start processing that job.

Time Limits.

language	`C`	`C++`	`Java`	`Python`	`JavaScript`	`Scala`
time (sec)	1	1	4	6	6	6

Memory Limit. 512MB.

Sample 1.

Input:

```
2 5
1 2 3 4 5
```

Output:

```
0 0
1 0
0 1
1 2
0 4
```

1. The two threads try to simultaneously take jobs from the list, so thread with index 0 actually takes the first job and starts working on it at the moment 0.
2. The thread with index 1 takes the second job and starts working on it also at the moment 0.
3. After 1 second, thread 0 is done with the first job and takes the third job from the list, and starts processing it immediately at time 1.

4. One second later, thread 1 is done with the second job and takes the fourth job from the list, and starts processing it immediately at time 2.
5. Finally, after 2 more seconds, thread 0 is done with the third job and takes the fifth job from the list, and starts processing it immediately at time 4.

Sample 2.

Input:

```
4 20
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Output:

```
0 0
1 0
2 0
3 0
0 1
1 1
2 1
3 1
0 2
1 2
2 2
3 2
0 3
1 3
2 3
3 3
0 4
1 4
2 4
3 4
```

Explanation:
Jobs are taken by 4 threads in packs of 4, processed in 1 second, and then the next pack comes. This happens 5 times starting at moments 0, 1, 2, 3 and 4. After that all the $5 \times 4 = 20$ jobs are processed.

Starter Files

The starter solutions for C++, Java and Python3 in this problem read the input, apply an $\Theta(n^2)$ algorithm to solve the problem and write the output. You need to replace the $\Theta(n^2)$ algorithm with a faster one. If you use other languages, you need to implement the solution from scratch.

What to Do

Think about the sequence of events when one of the threads becomes free (at the start and later after completing some job). How to apply priority queue to simulate processing of these events in the required order? Remember to consider the case when several threads become free simultaneously.

Beware of integer overflow in this problem: use type `long long` in C++ and type `long` in Java wherever the regular type `int` can overflow given the restrictions in the problem statement.

Need Help?

Ask a question or see the questions asked by other learners at this forum thread.

3 Problem: Merging tables

Problem Introduction

In this problem, your goal is to simulate a sequence of merge operations with tables in a database.

Problem Description

Task. There are n tables stored in some database. The tables are numbered from 1 to n. All tables share the same set of columns. Each table contains either several rows with real data or a symbolic link to another table. Initially, all tables contain data, and i-th table has r_i rows. You need to perform m of the following operations:

1. Consider table number $destination_i$. Traverse the path of symbolic links to get to the data. That is,

```
while $destination_i$ contains a symbolic link instead of real data do
    $destination_i \leftarrow \texttt{symlink}(destination_i)$
```

2. Consider the table number $source_i$ and traverse the path of symbolic links from it in the same manner as for $destination_i$.
3. Now, $destination_i$ and $source_i$ are the numbers of two tables with real data. If $destination_i \neq source_i$, copy all the rows from table $source_i$ to table $destination_i$, then clear the table $source_i$ and instead of real data put a symbolic link to $destination_i$ into it.
4. Print the maximum size among all n tables (recall that size is the number of rows in the table). If the table contains only a symbolic link, its size is considered to be 0.

See examples and explanations for further clarifications.

Input Format. The first line of the input contains two integers n and m — the number of tables in the database and the number of merge queries to perform, respectively.
The second line of the input contains n integers r_i — the number of rows in the i-th table.
Then follow m lines describing merge queries. Each of them contains two integers $destination_i$ and $source_i$ — the numbers of the tables to merge.

Constraints. $1 \leq n, m \leq 100\,000$; $0 \leq r_i \leq 10\,000$; $1 \leq destination_i, source_i \leq n$.

Output Format. For each query print a line containing a single integer — the maximum of the sizes of all tables (in terms of the number of rows) after the corresponding operation.

Time Limits.

language	C	C++	Java	Python	JavaScript	Scala
time (sec)	2	2	14	6	6	14

Memory Limit. 512MB.

Sample 1.

Input:

```
5 5
1 1 1 1 1
3 5
2 4
1 4
5 4
5 3
```

Output:

```
2
2
3
5
5
```

In this sample, all the tables initially have exactly 1 row of data. Consider the merging operations:

1. All the data from the table 5 is copied to table number 3. Table 5 now contains only a symbolic link to table 3, while table 3 has 2 rows. 2 becomes the new maximum size.
2. 2 and 4 are merged in the same way as 3 and 5.
3. We are trying to merge 1 and 4, but 4 has a symbolic link pointing to 2, so we actually copy all the data from the table number 2 to the table number 1, clear the table number 2 and put a symbolic link to the table number 1 in it. Table 1 now has 3 rows of data, and 3 becomes the new maximum size.
4. Traversing the path of symbolic links from 4 we have $4 \to 2 \to 1$, and the path from 5 is $5 \to 3$. So we are actually merging tables 3 and 1. We copy all the rows from the table number 1 into the table number 3, and now the table number 3 has 5 rows of data, which is the new maximum.
5. All tables now directly or indirectly point to table 3, so all other merges won't change anything.

Sample 2.

Input:

```
6 4
10 0 5 0 3 3
6 6
6 5
5 4
4 3
```

Output:

```
10
10
10
11
```

Explanation:
In this example tables have different sizes. Let us consider the operations:

1. Merging the table number 6 with itself doesn't change anything, and the maximum size is 10 (table number 1).
2. After merging the table number 5 into the table number 6, the table number 5 is cleared and has size 0, while the table number 6 has size 6. Still, the maximum size is 10.

3. By merging the table number 4 into the table number 5, we actually merge the table number 4 into the table number 6 (table 5 now contains just a symbolic link to table 6), so the table number 4 is cleared and has size 0, while the table number 6 has size 6. Still, the maximum size is 10.
4. By merging the table number 3 into the table number 4, we actually merge the table number 3 into the table number 6 (table 4 now contains just a symbolic link to table 6), so the table number 3 is cleared and has size 0, while the table number 6 has size 11, which is the new maximum size.

Starter Files

The starter solutions in C++, Java and Python3 read the description of tables and operations from the input, declare and partially implement disjoint set union, and write the output. You need to complete the implementation of disjoint set union for this problem. If you use other languages, you will have to implement the solution from scratch.

What to Do

Think how to use disjoint set union with path compression and union by rank heuristics to solve this problem. In particular, you should separate in your thinking the data structure that performs union/find operations from the merges of tables. If you're asked to merge first table into second, but the rank of the second table is smaller than the rank of the first table, you can ignore the requested order while merging in the Disjoint Set Union data structure and join the node corresponding to the second table to the node corresponding to the first table instead in you Disjoint Set Union. However, you will need to store the number of the actual second table to which you were requested to merge the first table in the parent node of the corresponding Disjoint Set, and you will need an additional field in the nodes of Disjoint Set Union to store it.

Need Help?

Ask a question or see the questions asked by other learners at this forum thread.

4 Solving a Programming Challenge in Five Easy Steps

4.1 Reading Problem Statement

Start by reading the problem statement that contains the description of a computational task, time and memory limits, and a few sample tests. Make sure you understand how an output matches an input in each sample case.

If time and memory limits are not specified explicitly in the problem statement, the following default values are used.

Time Limits.

language	`C`	`C++`	`Java`	`Python`	`JavaScript`	`Scala`
time (sec)	1	1	1.5	5	5	3

Memory limit: 512 Mb.

4.2 Designing an Algorithm

After designing an algorithm, prove that it is correct and try to estimate its expected running time on the most complex inputs specified in the constraints section. If you laptop performs roughly 10^8–10^9 operations per second, and the maximum size of a dataset in the problem description is $n = 10^5$, then an algorithm with quadratic running time is unlikely to fit into the time limit (since $n^2 = 10^{10}$), while a solution with running time $O(n \log n)$ will. However, an $O(n^2)$ solution will fit if $n = 1\,000$, and if $n = 100$, even an $O(n^3)$ solutions will fit. Although polynomial algorithms remain unknown for some hard problems in this book, a solution with $O(2^n n^2)$ running time will probably fit into the time limit as long as n is smaller than 20.

4.3 Implementing an Algorithm

Start implementing your algorithm in one of the following programming languages supported by our automated grading system: `C`, `C++`, `Haskell`, `Java`, `JavaScript`, `Python2`, `Python3`, or `Scala`. For all problems, we provide starter solutions for `C++`, `Java`, and `Python3`. For other programming languages, you need to implement a solution from scratch. The grading system detects the programming language of your submission automatically, based on the extension of the submission file.

We have reference solutions in `C++`, `Java`, and `Python3` (that we don't share with you) which solve the problem correctly under the given constraints, and spend at most 1/3 of the time limit and at most 1/2 of the memory limit. You can also use other languages, and we've estimated the time limit multipliers for them. However, we have no guarantee that a correct solution for a particular problem running under the given time and memory constraints exists in any of those other languages.

In the Appendix, we list compiler versions and flags used by the grading system. We recommend using the same compiler flags when you test your solution locally. This will increase the chances that your program behaves in the same way on your machine and on the testing machine (note that a buggy program may behave differently when compiled by different compilers, or even by the same compiler with different flags).

4.4 Testing and Debugging

Submitting your implementation to the grading system without testing it first is a bad idea! Start with small datasets and make sure that your program produces correct results on all sample datasets. Then proceed to checking how long it takes to process a large dataset. To estimate the running time, it makes sense to implement your algorithm as a function like `solve(dataset)` and then implement an additional procedure `generate()` that produces a large dataset. For example, if an input to a problem is a sequence of integers of length $1 \leq n \leq 10^5$, then generate a sequence of length 10^5, pass it to your `solve()` function, and ensure that the program outputs the result quickly.

Check the boundary values to ensure that your program processes correctly both short sequences (e.g., with 2 elements) and long sequences (e.g., with 10^5 elements). If a sequence of integers from 0 to, let's say, 10^6 is given as an input, check how your program behaves when it is given a sequence $0, 0, \ldots, 0$ or a sequence $10^6, 10^6, \ldots, 10^6$. Afterwards, check it also on randomly generated data. Check degenerate cases like an empty set, three points on a single line, a tree which consists of a single path of nodes, etc.

After it appears that your program works on all these tests, proceed to stress testing. Implement a slow, but simple and correct algorithm and check that two programs produce the same result (note however that this is not applicable to problems where the output is not unique). Generate random test cases as well as biased tests cases such as those with only small numbers or a small range of large numbers, strings containing a single letter "a" or only two different letters (as opposed to strings composed of all possible Latin letters), and so on. Think about other possible tests which could be peculiar in some sense. For example, if you are generating graphs, try generating trees, disconnected graphs, complete graphs, bipartite graphs, etc. If you generate trees, try generating paths, binary trees, stars, etc. If you are generating integers, try generating both prime and composite numbers.

4.5 Submitting to the Grading System

When you are done with testing, submit your program to the grading system! Go to the submission page, create a new submission, and upload a file with your program (make sure to upload a source file rather than an executable). The grading system then compiles your program and runs it on a set of carefully constructed tests to check that it outputs a correct result for all tests and that it fits into the time and memory limits.

The grading usually takes less than a minute, but in rare cases, when the servers are overloaded, it might take longer. Please be patient. You can safely leave the page when your solution is uploaded.

As a result, you get a feedback message from the grading system. You want to see the "`Good job!`" message indicating that your program passed all the tests. The messages "`Wrong answer`", "`Time limit exceeded`", "`Memory limit exceeded`" notify you that your program failed due to one of these reasons. If you program fails on one of the first two test cases, the grader will report this to you and will show you the test case and the output of your program. This is done to help you to get the input/output format right. In all other cases, the grader will *not* show you the test case where your program fails.

5 Appendix: Compiler Flags

C (`gcc 5.2.1`). File extensions: `.c`. Flags:

```
gcc -pipe -O2 -std=c11 <filename> -lm
```

C++ (`g++ 5.2.1`). File extensions: `.cc`, `.cpp`. Flags:

```
g++ -pipe -O2 -std=c++14 <filename> -lm
```

If your `C/C++` compiler does not recognize `-std=c++14` flag, try replacing it with `-std=c++0x` flag or compiling without this flag at all (all starter solutions can be compiled without it). On Linux and MacOS, you most probably have the required compiler. On Windows, you may use your favorite compiler or install, e.g., `cygwin`.

Java (`Open JDK 8`). File extensions: `.java`. Flags:

```
javac -encoding UTF-8
java -Xmx1024m
```

JavaScript (`Node v6.3.0`). File extensions: `.js`. Flags:

```
nodejs
```

Python 2 (`CPython 2.7`). File extensions: `.py2` or `.py` (a file ending in `.py` needs to have a first line which is a comment containing "python2"). No flags:

```
python2
```

Python 3 (`CPython 3.4`). File extensions: `.py3` or `.py` (a file ending in `.py` needs to have a first line which is a comment containing "python3"). No flags:

```
python3
```

Scala (`Scala 2.11.6`). File extensions: `.scala`.

```
scalac
```