HW5

8.1

A real life linear regression example could be agricultural researchers using regression to understand relationship for effect of fertilizer on yield of crops. Some predictors could be 1. Amount of fertilizer 2. amount of water 3. Rainfall 4. moisture of soil.

The response variable is crop yield.

8.2

```
crime <- read.delim("http://www.statsci.org/data/general/uscrime.txt")
head(crime)
```

```
##      M So   Ed  Po1  Po2    LF   M.F Pop   NW    U1  U2 Wealth Ineq     Prob
## 1 15.1  1  9.1  5.8  5.6 0.510  95.0  33 30.1 0.108 4.1   3940 26.1 0.084602
## 2 14.3  0 11.3 10.3  9.5 0.583 101.2  13 10.2 0.096 3.6   5570 19.4 0.029599
## 3 14.2  1  8.9  4.5  4.4 0.533  96.9  18 21.9 0.094 3.3   3180 25.0 0.083401
## 4 13.6  0 12.1 14.9 14.1 0.577  99.4 157  8.0 0.102 3.9   6730 16.7 0.015801
## 5 14.1  0 12.1 10.9 10.1 0.591  98.5  18  3.0 0.091 2.0   5780 17.4 0.041399
## 6 12.1  0 11.0 11.8 11.5 0.547  96.4  25  4.4 0.084 2.9   6890 12.6 0.034201
##      Time Crime
## 1 26.2011   791
## 2 25.2999  1635
## 3 24.3006   578
## 4 29.9012  1969
## 5 21.2998  1234
## 6 20.9995   682
```

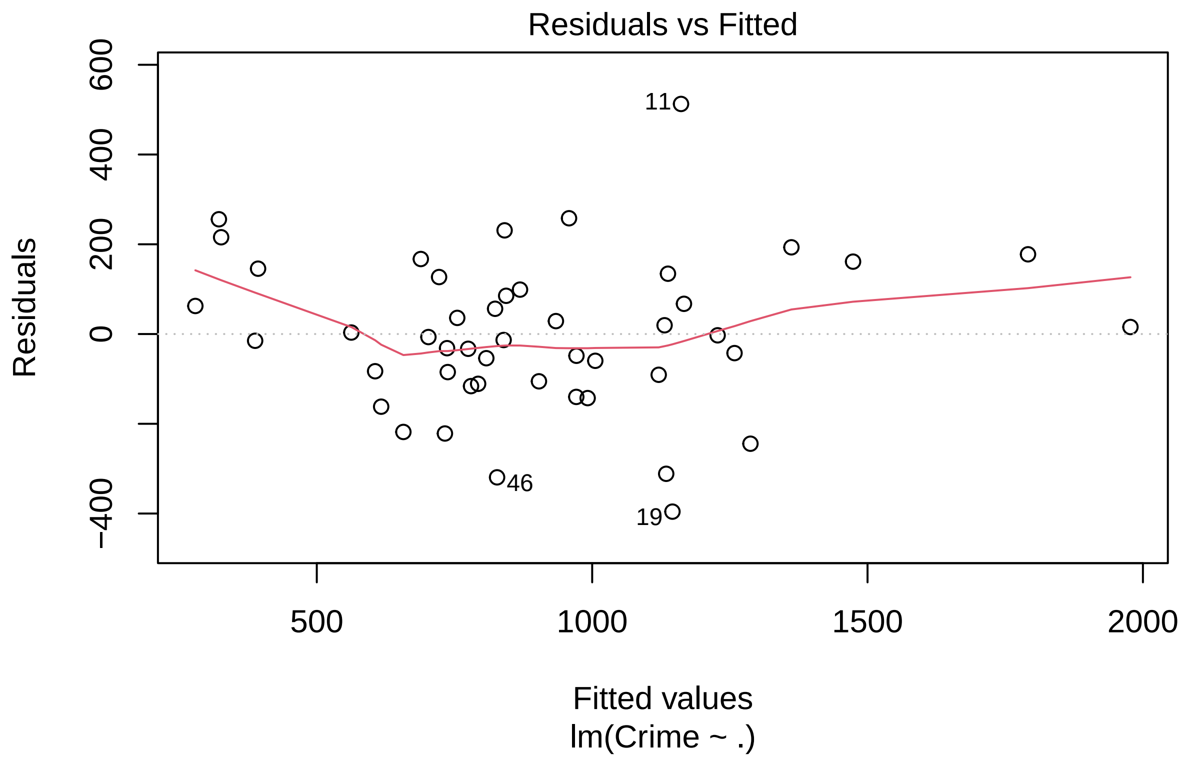
Including Plots

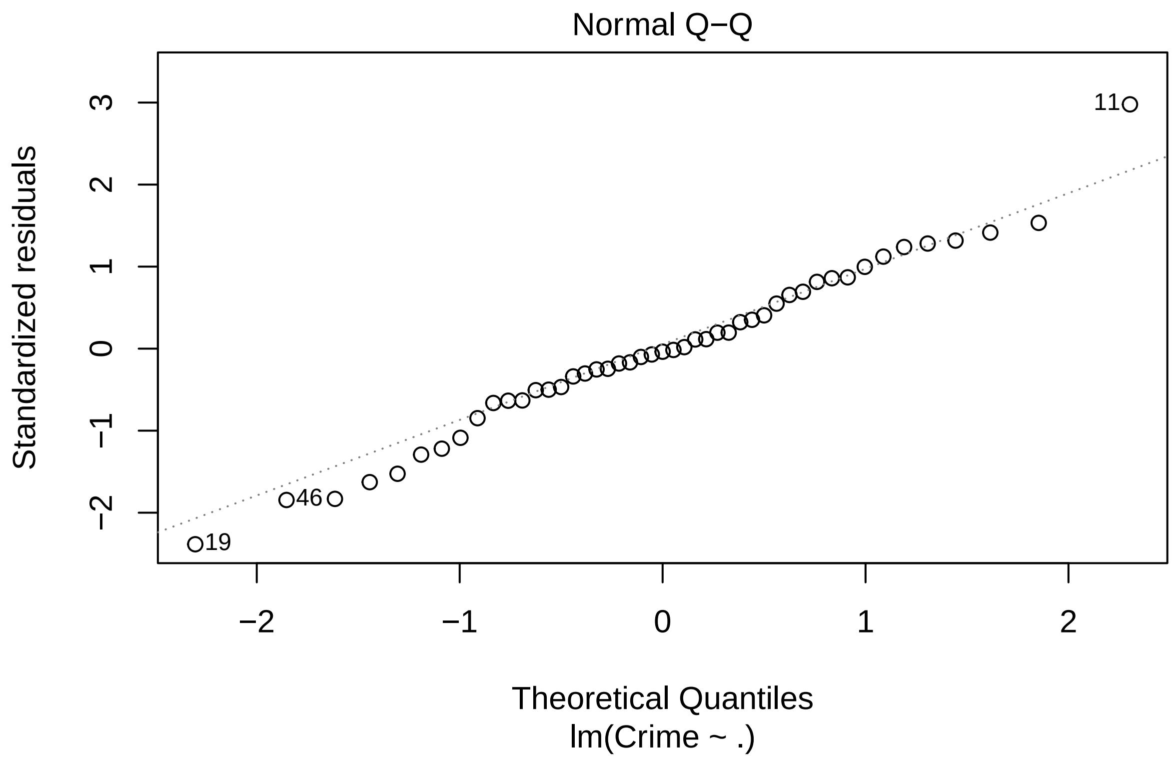
You can also embed plots, for example:

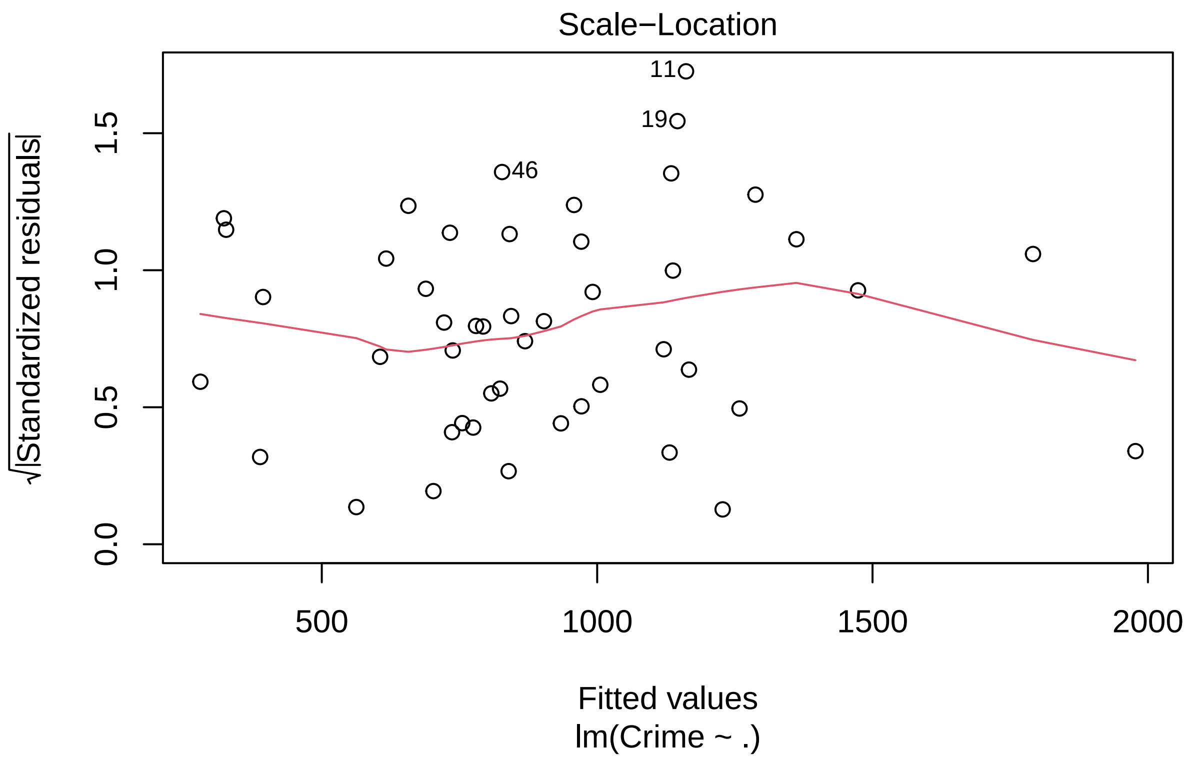
```
a <- data.frame(M = 14.0,So = 0,Ed = 10.0, Po1 = 12.0,Po2 = 15.5,
                 LF = 0.640, M.F = 94.0,Pop = 150,NW = 1.1,U1 = 0.120,
                 U2 = 3.6, Wealth = 3200,Ineq = 20.1,Prob = 0.04, Time = 39.0)

#This is the test data for regression model
```

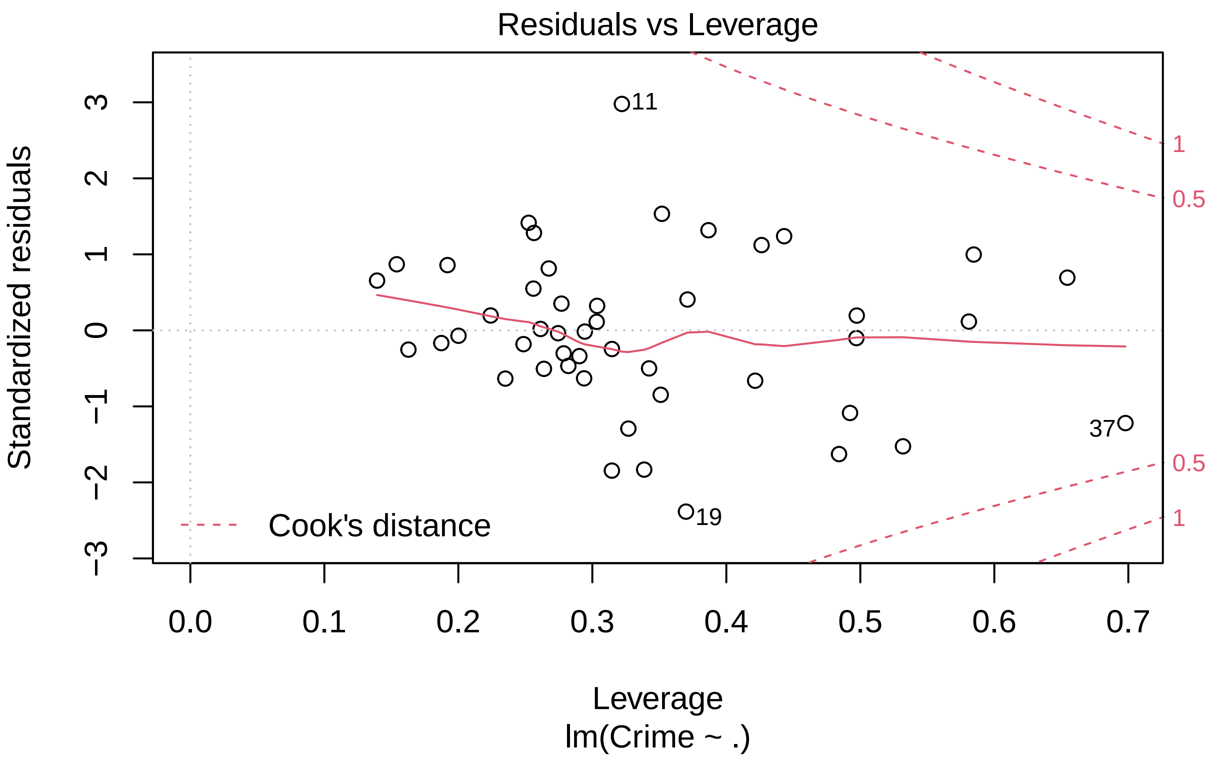
```
model1 <- lm(Crime~., data = crime)
```

```
plot(model1)
```


Residuals vs Fitted
Residuals
600
400
200
0
−400
11
46
19
500
1000
1500
2000
Fitted values
lm(Crime ~ .)


Normal Q−Q
Standardized residuals
3
2
1
0
−1
−2
11
46
19
−2
−1
0
1
2
Theoretical Quantiles
lm(Crime ~ .)


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
11
19
46
500
1000
1500
2000
Fitted values
lm(Crime ~ .)



You can see from the plot there a couple outliers but not hugely influential in data set.

```
predict(model1, a)
```

```
##        1
## 155.4349
```

```
min(crime$Crime) # minimum value in response variable crime
```

```
## [1] 342
```

Using the predict model we see that through linear regression that this value is way out of the range of data. It is far below the minimum. This model more likely not to predict accurately because of this.

```
summary(model1)
```

```
##
## Call:
## lm(formula = Crime ~ ., data = crime)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -395.74  -98.09   -6.69  112.99  512.67
##
```

```
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.984e+03  1.628e+03  -3.675 0.000893 ***
## M            8.783e+01  4.171e+01   2.106 0.043443 *
## So          -3.803e+00  1.488e+02  -0.026 0.979765
## Ed           1.883e+02  6.209e+01   3.033 0.004861 **
## Po1          1.928e+02  1.061e+02   1.817 0.078892 .
## Po2         -1.094e+02  1.175e+02  -0.931 0.358830
## LF          -6.638e+02  1.470e+03  -0.452 0.654654
## M.F          1.741e+01  2.035e+01   0.855 0.398995
## Pop         -7.330e-01  1.290e+00  -0.568 0.573845
## NW           4.204e+00  6.481e+00   0.649 0.521279
## U1          -5.827e+03  4.210e+03  -1.384 0.176238
## U2           1.678e+02  8.234e+01   2.038 0.050161 .
## Wealth       9.617e-02  1.037e-01   0.928 0.360754
## Ineq         7.067e+01  2.272e+01   3.111 0.003983 **
## Prob        -4.855e+03  2.272e+03  -2.137 0.040627 *
## Time        -3.479e+00  7.165e+00  -0.486 0.630708
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 209.1 on 31 degrees of freedom
## Multiple R-squared:  0.8031, Adjusted R-squared:  0.7078
## F-statistic: 8.429 on 15 and 31 DF,  p-value: 3.539e-07
```

Looking at the summary we can see what features have a higher significant impact on the response variable. This is determined by p value. We can just use variable with a p value less than 0.05.

The null hypothesis is basically that the predictor is not meaningful in the model. So if the p value is less than 0.05, we reject null and conclude that the variable is significant.

```r
model2 <- lm(Crime~M+Ed+Ineq+Prob, data = crime)
summary(model2)
```

```
##
## Call:
## lm(formula = Crime ~ M + Ed + Ineq + Prob, data = crime)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -532.97 -254.03  -55.72  137.80  960.21
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1339.35    1247.01  -1.074  0.28893
## M              35.97      53.39   0.674  0.50417
## Ed            148.61      71.92   2.066  0.04499 *
## Ineq           26.87      22.77   1.180  0.24458
## Prob        -7331.92    2560.27  -2.864  0.00651 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 347.5 on 42 degrees of freedom
## Multiple R-squared:  0.2629, Adjusted R-squared:  0.1927
```

```
## F-statistic: 3.745 on 4 and 42 DF,  p-value: 0.01077
```

Looking at the summary of model 2 we now see that Ed and Prob are the only variables with p value less than 0.05. Lets see what model 2 predicts.

```r
predict(model2, a)
```

```
##        1
## 897.2307
```

This is the prediction for model 2. Lets look at the max value and see if the value is in range.

```r
print(max(crime$Crime)) #max is 1993
```

```
## [1] 1993
```

```r
print(min(crime$Crime)) #min is 342
```

```
## [1] 342
```

Model 2 value is within the range so the predicted value makes sense. Looking at R squared values between model1 and model2 you can see model1 has a better R squared value at 0.7 while model 2 is at 0.19. The higher the R squared value, it shows the model fits the observed data better. Since we are taking all variables into account in model1, this is most likely overfitting.

```r
model3 <- lm(Crime~M+Ed+Ineq+Prob+Po1+U2, data = crime)
summary(model3)
```

```
##
## Call:
## lm(formula = Crime ~ M + Ed + Ineq + Prob + Po1 + U2, data = crime)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -470.68  -78.41  -19.68  133.12  556.23
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5040.50     899.84  -5.602 1.72e-06 ***
## M             105.02      33.30   3.154  0.00305 **
## Ed            196.47      44.75   4.390 8.07e-05 ***
## Ineq           67.65      13.94   4.855 1.88e-05 ***
## Prob        -3801.84    1528.10  -2.488  0.01711 *
## Po1           115.02      13.75   8.363 2.56e-10 ***
## U2             89.37      40.91   2.185  0.03483 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 200.7 on 40 degrees of freedom
## Multiple R-squared:  0.7659, Adjusted R-squared:  0.7307
## F-statistic: 21.81 on 6 and 40 DF,  p-value: 3.418e-11
```

For model 3 I added in Po1 and U2 and I picked these two variables to be added since in the initial model 1 they were the two variable that still had a p value above 0.05 but only very slighty compared to other variable. Looking at the R squared value for model 3, we can see it is 0.7 which is pretty good so this model fits the data pretty well.

```
predict(model3, a)
```

```
##        1
## 1304.245
```

Model 3 gives a predicted value of 1304 which is within the range of values so it this data point does make sense.

```
AIC(model1)
```

```
## [1] 650.0291
```

```
AIC(model2)
```

```
## [1] 690.0666
```

```
AIC(model3)
```

```
## [1] 640.1661
```

We can also see that the AIC is smallest for model 3 which shows its the best model compared to the others.

```
crossvalidate <- trainControl(method = "repeatedcv", number = 10, repeats = 3)

model4 <- train(Crime~M+Ed+Ineq+Prob+Po1+U2, data = crime, method = "lm", trControl = crossvalidate)

print(model4)
```

```
## Linear Regression
##
## 47 samples
##  6 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold, repeated 3 times)
## Summary of sample sizes: 42, 42, 43, 43, 42, 43, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   203.8866  0.7997959  166.0094
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

We can also use K-fold Cross validation to further validate data and reduce bias. We get a R squared with 0.75 which is also an improved R squared value.