

# Non-linearity

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My opinion of how some common non-linear functions relate to each other.  
Dirac delta function

$$\delta(x) = \begin{cases} \infty & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \quad (1)$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \quad (2)$$

Integral of Dirac delta function is a step function

$$\int_{-\infty}^{\infty} \delta(x) dx = \int_{-\infty}^{0^-} \delta(x) dx + \int_{0^+}^{\infty} \delta(x) dx \quad (3)$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \quad (4)$$

$$= \Delta(x) \quad (5)$$

Integral of step function is a rectified linear unit

$$\int_{-\infty}^{\infty} \Delta(x) dx = \int_{-\infty}^{0^-} \Delta(x) dx + \int_{0^+}^{\infty} \Delta(x) dx \quad (6)$$

$$= \int_{-\infty}^{0^-} 0 dx + \int_{0^+}^{\infty} 1 dx \quad (7)$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \quad (8)$$

$$= \max(0, x) \quad (9)$$

Resources

1. Ian Goodfellow, Yoshua Bengio and Aaron Courville. Deep Learning book.  
<https://www.deeplearningbook.org>
2. Adam Taylor. Distributions: What exactly is the Dirac Delta "Function"?  
<https://www.cantorsparadise.com/distributions-what-exactly-is-the-dirac-delta-function-e2af19d6e700>