I tried to find similar modules from google, but didn't find proper module yet. There are many node modules for parsing PDFs and I can content of PDF in backend, but I am not sure we can use it in web browsers.

Your answer could be improved with, so that others can confirm that your answer is correct. You can find more information on how to write good answers in the help center.
– Community Bot CommentedFeb 27 at 410

i∈Rn ∀ i∈(1,…,k) and k<n) The basis vectors can change. I want a method for converting the basis vectors into an basis that spans the same subspace as the original basis vectors with the requirement that the from the subspace spanned by the basis vectors to the orthonormal basis is smooth.

This is not possible even for continuous maps for any k in (0n) because the Stiefel manifold is connected. On the other hand, every fiber of Vk() to Gk(Rn) is disconnected. If we had a section, we could divide Vk(Rn) into two pieces, those which lie in the same connected component of a fibr as the section and those which line in a different connected component. These two pieces never meet in a fibe and therefore never meet at all, so Vk(Rn) has at least two components, which contradicts is connectedness. (More precisely, we use that the fibration is locally so the inverse image of a ball on Gk(Rn) has two components, one containing the section and one not, and since these don't meet in the inverse image of any ball in Gk(Rn) they don't meet anywhere.)

You say you want the map φ:B→Vk(Rn) so that [b]φ(b) is, where [b] is the subspace spanned by the basis b (For this to be well defined, we need (bP)=φ(b) .) Your manifold formulation sugest you want this map to be composition of b→[] and (putative) section G(Rn)→Vk(Rn) ..