# Dp with Bitmask

man ↓ → woman

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 0 |
| 2 | | | 0 | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

$N \times N$

$$a_{ij} \rightarrow \begin{cases} 1 & \rightarrow i^{th} \text{ man and } j^{th} \text{ woman are compat} \\ 0 & \rightarrow \text{not compat} \end{cases}$$

Total possible ways to pair up the

$$f(\overset{2}{1}, \{1, 2, 3, 4, 5\}) \Rightarrow f(2, \{1, 2, 3, 5\})$$

$f(i, W)$ $=$ $\sum_{c[i,x]=1} f(i+1, W - \{x\})$

$\forall$ $x$ which are compatible with the ith man

# of ways to make a valid pairing such that men [i, N] & women in the set W are available

Dynamic Bitmask

{1, 2, 3} → 1 1 1 → 7

1st no → 2nd wcma

{1, 2, 3} → 1 0 1 → 5

1st no → 3rd no

{1, 2, 3} → 1 1 0 → 6

2 variable → 2d dp

$f(i, mask) = \sum_{x} f(i+1, mask - \{x\})$

null

string

(1, 2) (2, 3)

(1, 3) (3, 1)

mask = $\boxed{0\ 1\ 0}$ → 2

$f(1, 7) \rightarrow f(2, 5) \rightarrow f(3, 4)$ - - -

$f(2, 6) \rightarrow f(3, 2)$ . . .

_ _ _ _ _ . . . . . _ _ 21 bits

N bits

N=4 → 1111 → 15 $2^N - 1$

f(1, mask)

$f(1, (1111)_{10})$

$(1 << N) - 1$

110101 & 000100

5 4 3 2 1 0

001000

$\geq 0$

$w = 1 \rightarrow n$

2-1

0

$\left( (1 << (w-1)) \, \& \, mask \right) == 0$

110101 $\rightarrow$ 110001

$$
\begin{array}{cccccccc}
 & 1 & 1 & 0 & 1 & 0 & 1 \\
\wedge & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}
$$

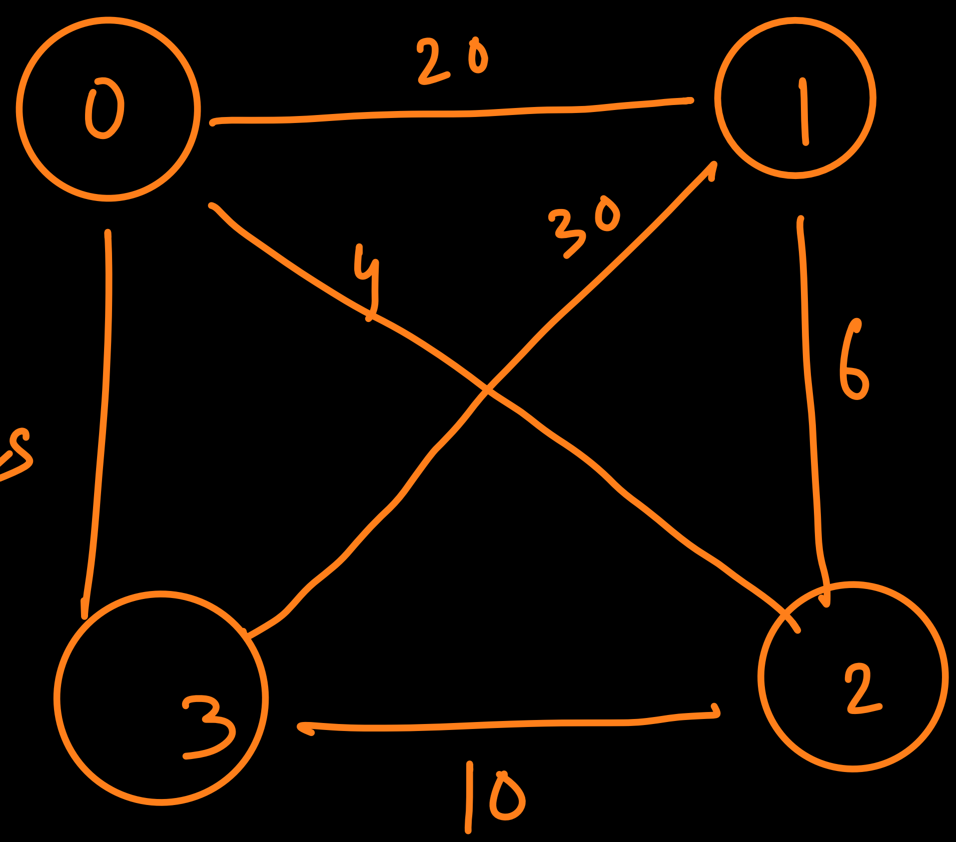
$1 << (2)$

Op unlu Bitmask ← Travelling Salesman Problem → graph

2

Calc the min wt hamiltonian cycle in the graph.

adj matrix




hamiltonian cycle is defined as the set of edges having every node once & after having all the nodes we come back to the source.

Brute force

0 − 1 − 2 − 3 − 0

0 − 2 − 3 − 1 − 0

↳ permutation

1 − 2 − 3 − 0 − 1

1 − 3 − 2 − 0 − 1

Let's pick any node as src node.

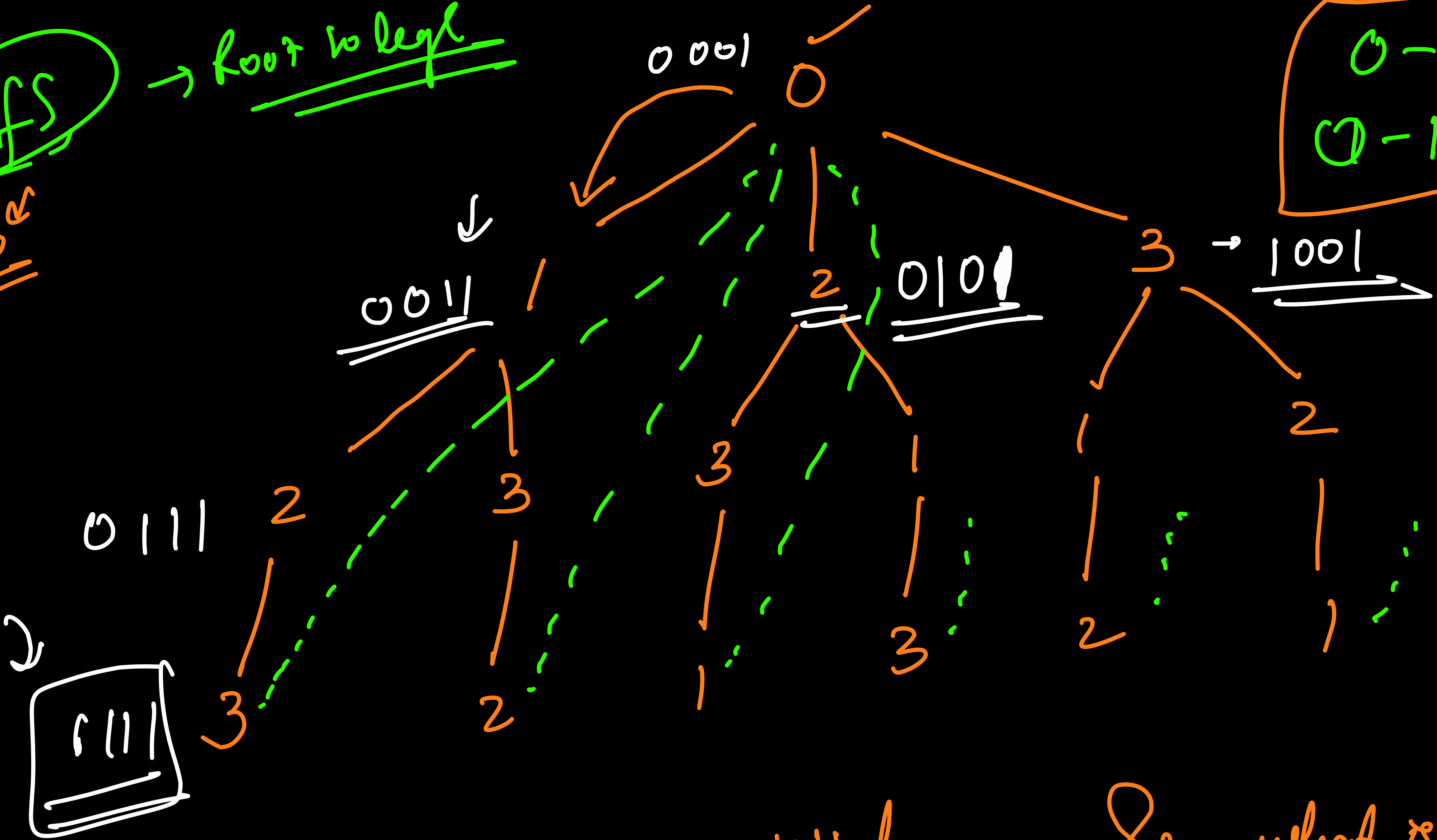
0 − 1 − 2 − 3 − 0

1 − 2 − 3 − 0 − 1

0 − 2 − 3 − 1 − 0

3 − 1 − 0 − 2 − 3

Dfs → Root to leaf

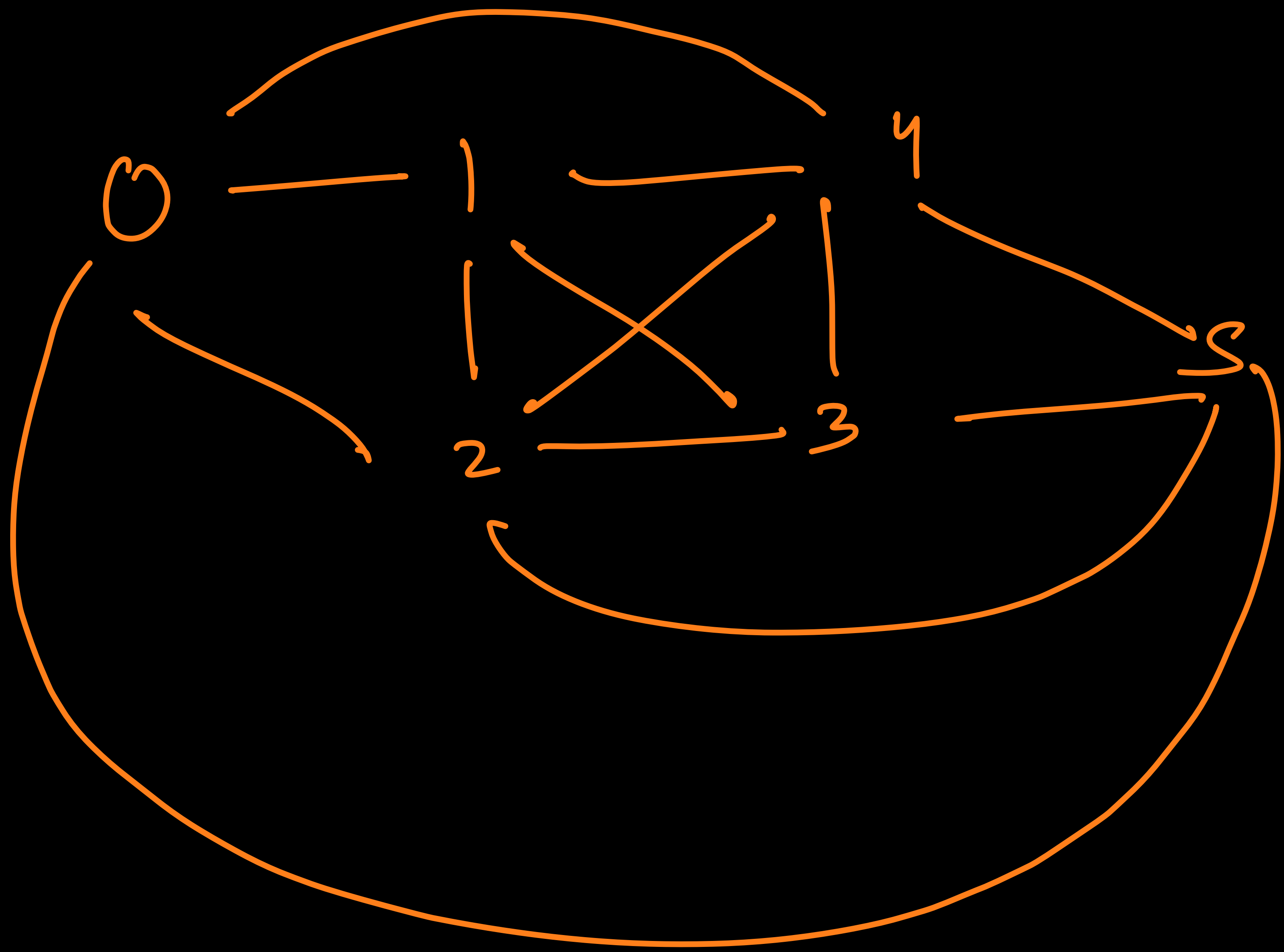


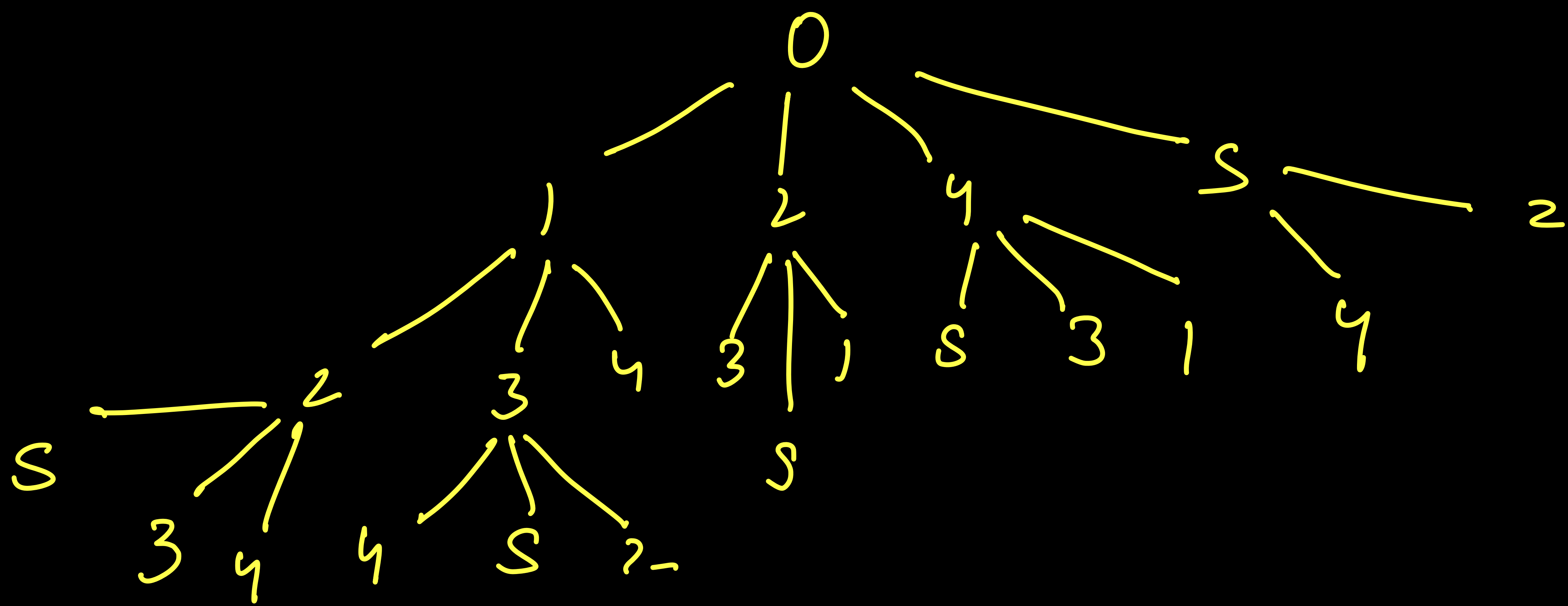

0 - 1 - 2 - 3 - 0

0 - 1 - 3 - 2 - 0

$2^n - 1$

what all nodes are visited, & what is the ans noc


0
1
4
2
3
5


0
1
2
4
5
2
2
3
4
3
1
S
3
1
4
S
3
4
4
S
2
S

$$f(\text{curr}, \underline{\text{visited}\{\}}) = \min(f(\text{neighbour}, \text{visited} + \text{curr}))$$

$\xrightarrow{}$ set

?

min cost if we start from curr, with the given vis nodes

bitmask

binary → int

# Backtracking with Bitmask

N queen →

N bits

0 1 0 1

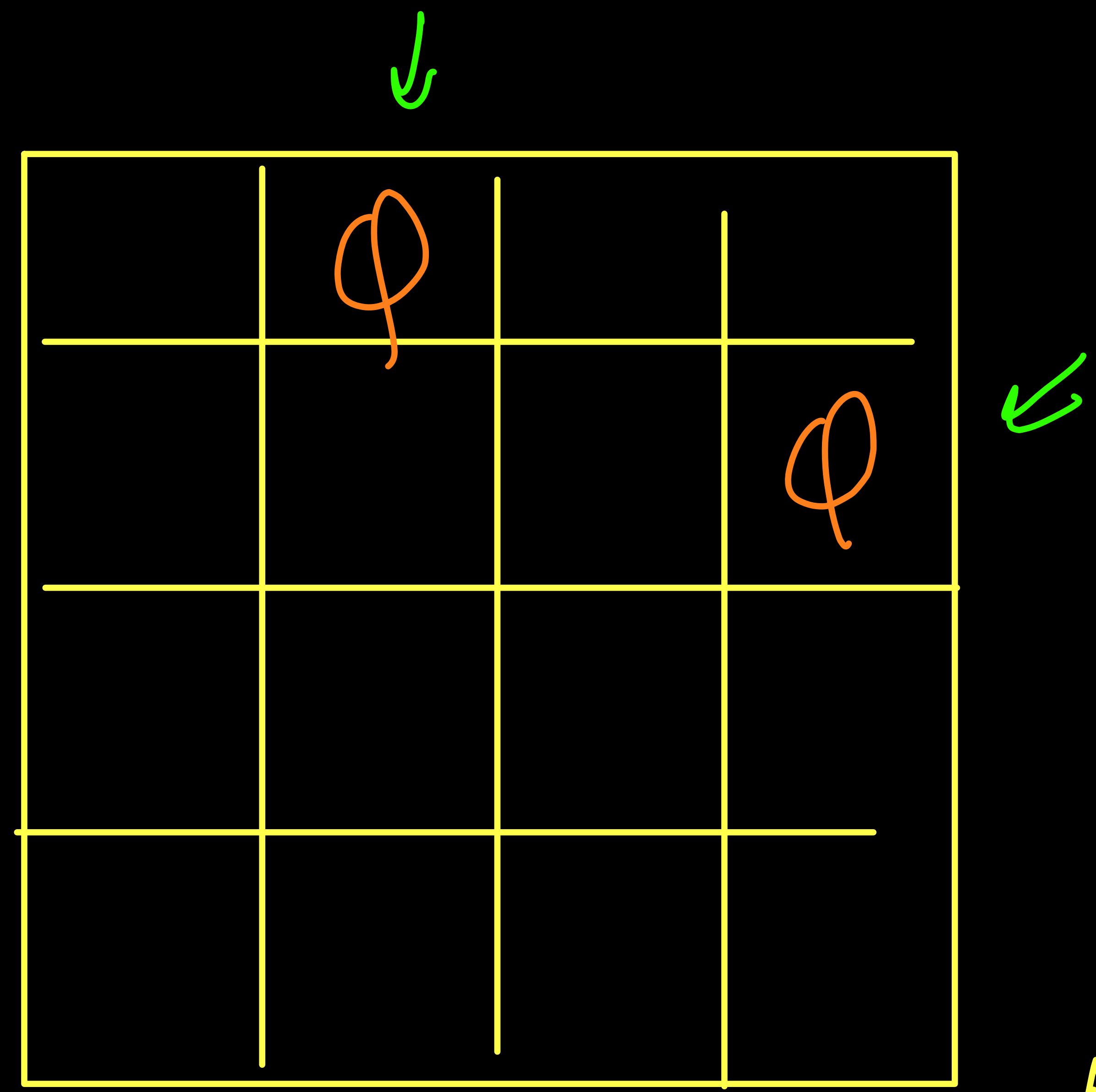
0101

B)

↓

0101

& 0100



N queens

N x N

Total l.d = 2 × N−1 =

Total r.d

R.D

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 2 | 3 | 4 | 5 | 6 |
| 3 | 3 | 4 | 5 | 6 | 7 |
| 4 | 4 | 5 | 6 | 7 | 8 |

→ R.D

r + c

_ _ _ 1 _ _ 1 _ _ _ (9)

5 4 3 2 1 0

$r - c + n - 1$

$1 - 0 + 4$

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 4 | 3 | 2 | 1 | 0 |
| 1 | 5 | 4 | 3 | 0 | 1 |
| 2 | 6 | 5 | 4 | 3 | 2 |
| 3 | 7 | 6 | 5 | 4 | 3 |
| 4 | 8 | 7 | 6 | 5 | 4 |

codes

_ _ _ _ $\overset{1}{\_}$ _ _ _ _

4 3 2 1 0

$x = r + c$

$r.d.mask = rdmask \mid (1 << x)$

$\boxed{x = r - c + n - 1}$

$ldmask = ldmask \mid (1 << x)$

N rabbits

→ group

$g_1 \rightarrow$

$g_2 \rightarrow$

$g_3 \rightarrow$

$N=3 \rightarrow$

$\{1\}\ \{2\}\ \{3\} \rightarrow$ 1 way

$\{1,2\}\ \{3\} \rightarrow$ 1 way

$\{1,3\}\ \{2\}$

$\{2,3\}\ \{1\}$

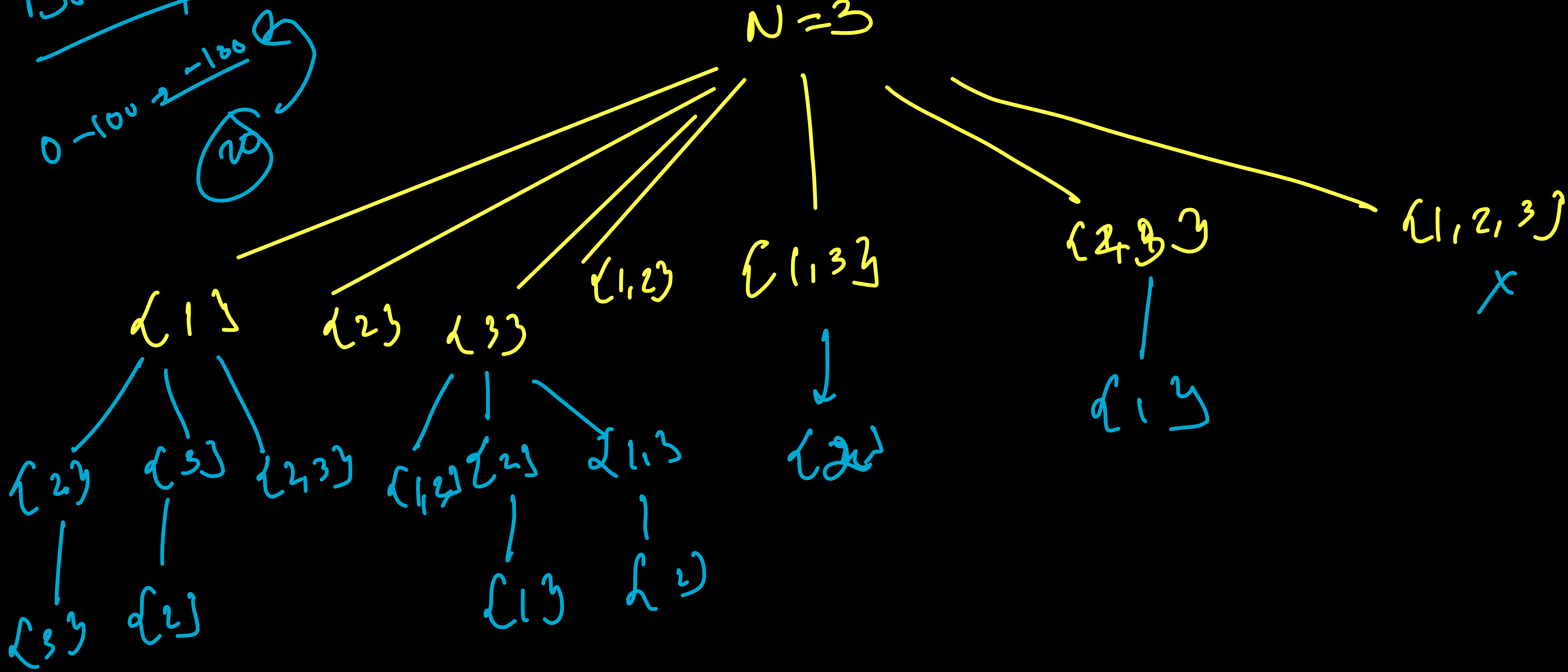
$\{1,2,3\}$

⋮

Brute fa

0 -100 2 -100

20




.10 0 1 h

10101 → $a_{s3} + a_{s1} + a_{31}$

$f(S) = \max\left( f(S - G) + \underline{\text{sum}(G)} \right)$

$S \to$ bit, $2$

$S - G \to$ bitmask

max possible score achievable by grouping rabbits in set $\underline{S}$.

$G \to$ all possible groups

$\{x_1, x_2, x_3\} \to \boxed{a_{12} + a_{23} + a_{13}}$

$\to$ bitmask

$\underline{1111} \to$

$f(2^n - 1)$

$2^3-1 \to (111)_2 \to (7)_{10}$

$R_1R_2R_3$

$R_1$ $R_2$ $R_3$ $R_2R_3$ $R_1R_3$ $R_1R_2$ $R_1R_2R_3$

$10110 \to 10010 \to 10100 \to 00110 -$

2

$g = \text{mask}\ ;\ g \mathrel{!}= 0\ ;\ g = (g-1)\ \&\ \text{mask}$

101010
101001

111
110
110

101
111
101

g = 111 → 111

110 → 110

b = b-1

11010 → 11001

16 + 8 + 2     16 + 8 + 1

26 → 25

1111

1101

→ 00010

11111 → 11110 → 11101 → 11100

11011 → 11010 → 11001

(13) (14) (13) (2)

1111 → 1110 → 1101 → 1100 → 1011

→ 1101 S

S = 1101

1100 & 1101

1100 → 1011 & 1101 ✓

1001

$R_1\ R_2\ R_3\ R_4$

$1\quad 1\quad 1\quad 1$

$R_3$

$\wedge \quad R_1\ R_2 \quad\quad R_4$

$1\quad 1\quad 0\quad 1$

$0\quad 0\quad 1\quad 0$

$R_3$

Q2

10110

10101

10100