

# Dynamic

Video - 83

# Programming



Note :- This playlist is only for explanation of Dns & solutions.

See my "DP Concepts & Dns" playlist for understanding DP from scratch...



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Recursion + Memo → Bottom UP

Optimal Bottom UP

$1, 2, 3, \boxed{3, 4}$

For an integer array `nums`, an **inverse pair** is a pair of integers `[i, j]` where  $0 \leq i < j < \text{nums.length}$  and `nums[i] > nums[j]`.

Given two integers `n` and `k`, return the number of different arrays consist of numbers from `1` to `n` such that there are exactly `k` **inverse pairs**. Since the answer can be huge, return it **modulo**  $10^9 + 7$ .

Example :-

$n = 3,$

$K = 0$

Output = 1

$\{1, 2, 3\}, K=0$

$n = 3, K = 1$

Output = 2

$\{1, 2, 3\},$

$\{1, \boxed{3, 2}\}$   
 $K=1$

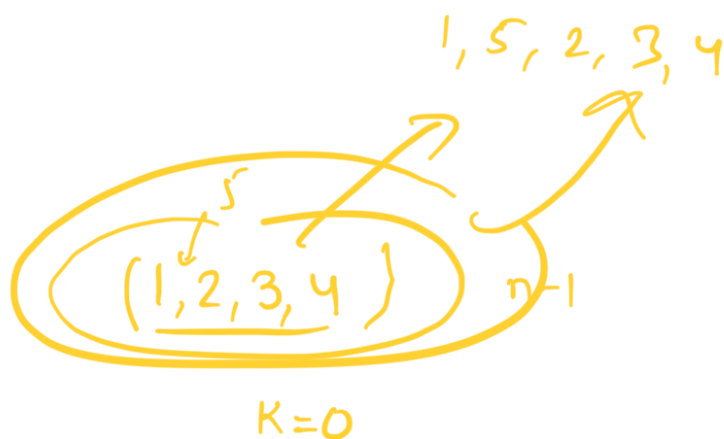
$\{\boxed{2, 1}, 3\}$   
 $K=1$

# Intuition Building

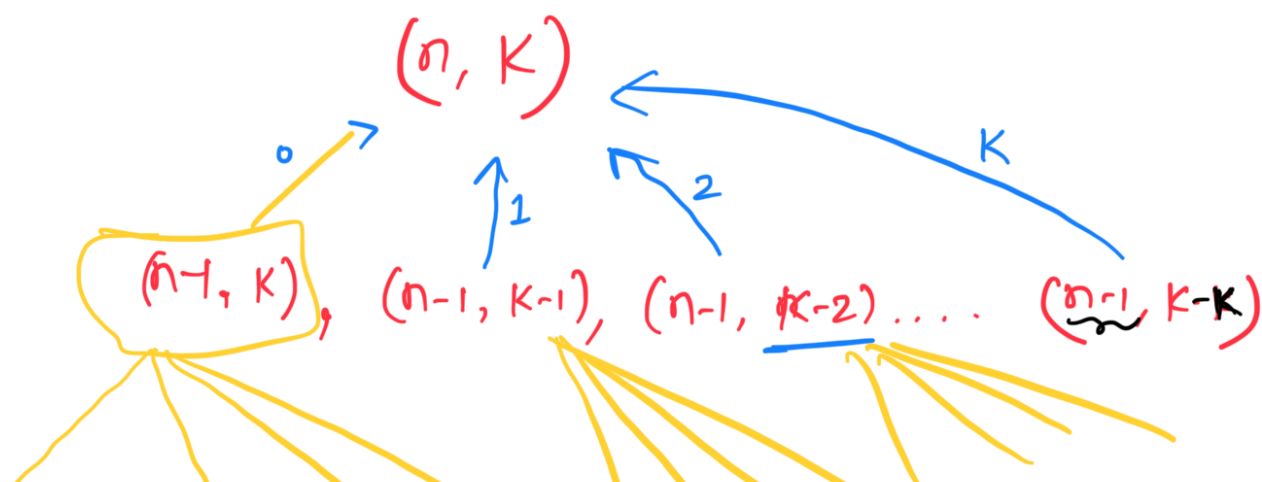
$n=5$   
 $K=3$

$\{1, 2, 3, 4, 5\}$

$K=3$



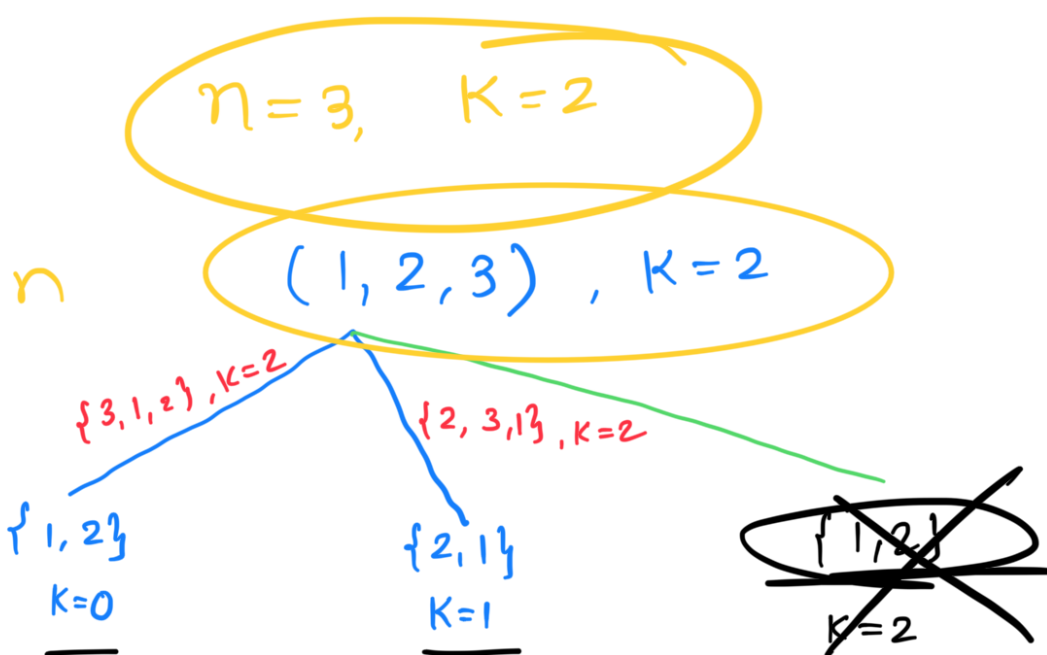
$\{1, 2, \dots, n\}$  ,  $K$



$\{1, 2, \dots, n-2\}$   
K

$(n-2, K-1)$

Example:-



$\text{solve}(n, K) \{$

$\text{if}(n == 0) \{$   
 $\text{return } 0;$

$\}$   
 $\text{if}(K == 0) \text{return } 1;$

$\text{result} = 0;$

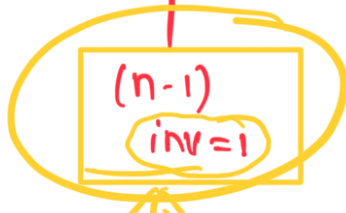
$\text{for} ( \text{inv} = 0; \text{inv} \leq \min(K, n-1); \text{inv}++ ) \{$

result += solve(n-1, K-inv);

3

ret result;

(n, K)



(n-1) inv=2

(K-inv)

Time Complexity:-

① Recursion :-  $O(K^n)$  ~~TLE~~, space = less of 2:-

② Memoization :-  $O(n * k * n)$  ★, space =  $O(n * k)$

$(n * k)$  for inv=0; inv < 2n

# Bottom UP:-

```

int solve(int n, int k) {
    if(n == 0) {
        return 0;
    }

    if(k == 0) {
        return 1; //sorted array
    }

    if(t[n][k] != -1) {
        return t[n][k];
    }

    int result = 0;
    //For an array of length n, you can have at max (n-1) inversions
    for(int inv = 0; inv <= min(k, n-1); inv++) {
        result = (result % MOD + solve(n-1, k-inv) % MOD) % MOD; //I al
        need k-inv inversions
    }

    return t[n][k] = result;
}

```

// State definition

$t[i][j]$  = Total no of arrays of size  $i$

return  $t[n][k]$ ; & exactly  $j$  inversions.

$i$  ↓

|   |   |   |         |
|---|---|---|---------|
|   | 1 | 2 | 3 ← $j$ |
| 0 | 1 | 0 | 0       |
| 1 | ? | ? | ?       |

|   |   |   |   |   |
|---|---|---|---|---|
| 2 | 1 | ? | ? | ? |
| 3 | 1 | ! | . | 1 |
| 4 | 1 |   |   |   |

$(n+1)(k+1)$

for (i = 1 ; i <= n ; i++) {  $\leftarrow n$

for (j = 1 ; j <= k ; j++) {  $\leftarrow k$

{ for (inv = 0 ; inv <= min(j, i-1) ; inv++) {  
 $t[i][j] = (t[i][j] + t[i-1][j-inv]) ;$   
 }  
 }

return t[n][k];

T.C =  $O(n * k * n)$ ;  
 S.C =  $O(n * k)$ .

Optimal

# Bottom Up

$n = 5$

$K = 5$

$j$

|   | 0 | 1 | 2 | 3  | 4  | 5  |
|---|---|---|---|----|----|----|
| 0 | 1 | 0 | 0 | 0  | 0  | 0  |
| 1 | 1 | 0 | 0 | 0  | 0  | 0  |
| 2 | 1 | 1 | 0 | 0  | 0  | 0  |
| 3 | 1 | 2 | 2 | 1  | 0  | 0  |
| 4 | 1 | 3 | 5 | 6  | 5  | 3  |
| 5 | 1 | 4 | 9 | 15 | 20 | 22 |

$i$

$\Rightarrow i = 5$

$j$

$ce = 22$

$i = 5$

$j = 0$

$sum = f(i-1)(j);$   
 $f(i)(j) = sum;$

$j = 5$

$sum += f(i-1)(j);$

$\{ (j \geq i) \{$   
 $\quad sum -= f(i-1)(j-i);$   
 $\}$   
 $f(i)(j) = sum;$

$j-i$   
 $5-5=0$



