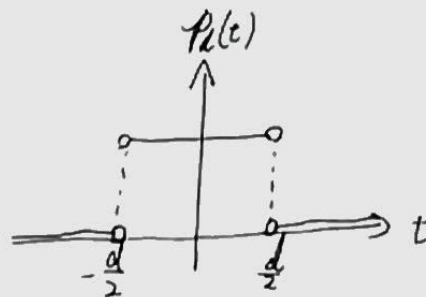


問題 4.10 (p.94)

$$p_d(t) = \begin{cases} 1 & |t| < \frac{1}{2}d \\ 0 & |t| > \frac{1}{2}d \end{cases}$$



$$F(\omega) = F[p_d(t)] = \int_{-\infty}^{\infty} p_d(t) e^{-j\omega t} dt$$

$$= \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{-j\omega t} dt$$

$$= -\frac{1}{j\omega} \left[e^{-j\omega t} \right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$= -\frac{1}{j\omega} (e^{-j\omega \frac{d}{2}} - e^{j\omega \frac{d}{2}})$$

$$= \frac{1}{j\omega} (e^{+j\omega \frac{d}{2}} - e^{-j\omega \frac{d}{2}})$$

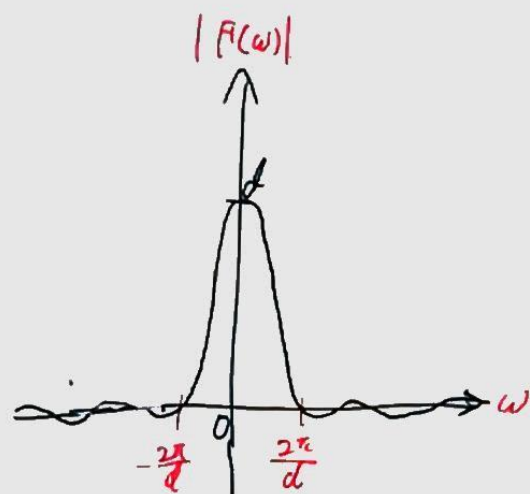
$$= \frac{1}{j\omega} \cdot 2j \sin\left(\frac{\omega d}{2}\right)$$

$$= \frac{2}{\omega} \sin\left(\frac{\omega d}{2}\right)$$

$$= \frac{1}{\frac{\omega}{2}} \sin\left(\frac{\omega d}{2}\right)$$

$$= \frac{d}{\frac{\omega d}{2}} \sin\left(\frac{\omega d}{2}\right)$$

$$= d \operatorname{sinc}\left(\frac{\omega d}{2}\right)$$



问题 5.7 (p.28)

$$\mathcal{F}[1] = 2\pi \delta(\omega), \quad \mathcal{F}[f(t) e^{j\omega_0 t}] = \mathcal{F}(\omega - \omega_0)$$

$$\mathcal{F}[e^{j\omega_0 t}] = 2\pi \delta(\omega - \omega_0)$$

问题 5.8 (p.28)

$$\cos \omega_0 t = \frac{1}{2} (e^{j\omega_0 t} + e^{-j\omega_0 t})$$

$$\begin{aligned} \mathcal{F}[\cos \omega_0 t] &= \mathcal{F}\left[\frac{1}{2} (e^{j\omega_0 t} + e^{-j\omega_0 t})\right] \\ &= \frac{1}{2} \mathcal{F}[e^{j\omega_0 t}] + \frac{1}{2} \mathcal{F}[e^{-j\omega_0 t}] \\ &= \frac{1}{2} \cdot 2\pi \delta(\omega - \omega_0) + \frac{1}{2} \cdot 2\pi \delta(\omega + \omega_0) \\ &= \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) \end{aligned}$$

$$\sin \omega_0 t = \frac{1}{2j} (e^{j\omega_0 t} - e^{-j\omega_0 t})$$

$$\begin{aligned} \mathcal{F}[\sin \omega_0 t] &= \mathcal{F}\left[\frac{1}{2j} (e^{j\omega_0 t} - e^{-j\omega_0 t})\right] \\ &= \frac{1}{2j} \mathcal{F}[e^{j\omega_0 t}] - \frac{1}{2j} \mathcal{F}[e^{-j\omega_0 t}] \\ &= \frac{1}{2j} 2\pi \delta(\omega - \omega_0) - \frac{1}{2j} 2\pi \delta(\omega + \omega_0) \\ &= -j\pi (\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) \\ &= j\pi (\delta(\omega + \omega_0) - \delta(\omega - \omega_0)) \end{aligned}$$
