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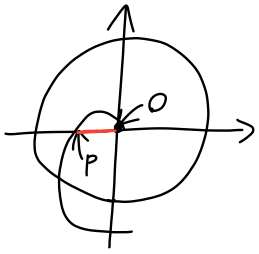
制御理論　演習問題２



$\dfrac{60}{s(s+2)(s+6)}$　のゲイン余有を求めよ

$g_m = 20\log\dfrac{1}{OP} = -20\log OP$

留数

$\dfrac{1}{s} \to \dfrac{60}{2\times 6} = 5$　，　$\dfrac{1}{s+2} \to \dfrac{60}{-2\times 4} = -7.5$　，　$\dfrac{1}{s+6} \to \dfrac{60}{-6\times(-4)} = 2.5$

与式の伝達関数を$G(j\omega)$とおく

$$G(j\omega) = \frac{60}{s(s+2)(s+6)} = \frac{60}{j\omega(j\omega+2)(j\omega+6)} = \frac{60}{j\omega(-\omega^2+8j\omega+12)} = \frac{60}{-j\omega^3-8\omega^2+j12\omega}$$

$$= \frac{60}{-8\omega^2+j(12\omega-\omega^3)} \times \frac{-8\omega^2-j(12\omega-\omega^3)}{-8\omega^2-j(12\omega-\omega^3)}$$

$$= \frac{-480\omega^2-j60(12\omega-\omega^3)}{64\omega^4+(12\omega-\omega^3)^2}$$

このときのRe軸上の点を知りたいので，$\mathrm{Im}\{G(j\omega)\}$のときの$\omega$をもとめる．

$60(12\omega-\omega^3)=0 \to 12\omega-\omega^3=0 \to 12-\omega^2=0 \to \omega^2=12 \to \omega=2\sqrt{3}$

よって

$$OP = \left|Re\{G(j2\sqrt{3})\}\right| = \left|\frac{-480\cdot(2\sqrt{3})^2}{64(2\sqrt{3})^4+(12\cdot 2\sqrt{3}-(2\sqrt{3})^3)^2}\right| = \left|\frac{-5760}{9216+0}\right| = 0.625$$

$\therefore g_m = -20\log OP = -20\log_{10}0.625 = 4.08240\ldots$

$g_m \fallingdotseq 4.08\,dB$