電気電子情報数学及び演習Ⅱ－演習問題（１）

<u>問題 1.1 (p.8)</u>

$z_1 = 4 + 3i$、$z_2 = 2 - 5i$ とする。つぎの演算で得られる複素数を $x + iy$ の形で求めよ。

４．$(3z_1 - z_2)^2$　　　１０．$\dfrac{1}{z_1^2}$、$\dfrac{1}{\bar{z}_1^2}$　　　１２．　$z_2\bar{z}_2/(z_1\bar{z}_1)$

$z = x + iy$とする。計算の詳細を示してつぎを計算せよ。

１６．$\mathrm{Re}(z/\bar{z})$

<u>問題 1.2 (pp.13-14)</u>

つぎの複素数を極形式で表し、複素平面上に図示せよ。

２．　$z = -2 + 2i$　　　４．　$z = -10$　　　８．　$z = \dfrac{i}{3 + 3i}$

すべてのべき根を求めて図示せよ。

２２．　$w = \sqrt[3]{8i}$　　　２３．　$w = \sqrt[3]{216}$

<u>問題 1.3 (p.21)</u>

４．つぎの不等式で定義される集合を複素平面上に図示せよ。

$$0 < |\ z - 1 - i\ | < \sqrt{2}$$

１２．　$f(z) = (z - 2)/(z + 2)$において、$z = 4i$ での$\mathrm{Re}\{f(z)\}$、$\mathrm{Im}\{f(z)\}$を求めよ。

$f(0) = 0$で$z \neq 0$に対して関数 f が以下に等しいとき、$f(z)$が$z = 0$で連続であるかどうか調べよ。

１４．　$f(z) = \dfrac{(\mathrm{Re}\, z^2)}{|z|}$

つぎの導関数の値を求めよ。

１７．　i　における$f(z) = \dfrac{z - i}{z + i}$

1.1 (p 8)

4. $\left(3(4+3i)-(2-5i)\right)^2$

$= (10+14i)^2$

$= 100 + 280i - 196$

$= -96 + 280i$ //

10. (1) $\dfrac{1}{(4+3i)^2} = \dfrac{1}{16+24i-9}$

$= \dfrac{1}{(7+24i)} \times \dfrac{(7-24i)}{(7-24i)}$

$= \dfrac{7-24i}{49+576}$

$= \dfrac{7}{625} - \dfrac{24}{625}i$ //

10(2) $\dfrac{1}{(4-3i)^2} = \dfrac{1}{16-9-24i}$

$= \dfrac{1}{7-24i} \times \dfrac{7+24i}{7+24i}$

$= \dfrac{7+24i}{49+576}$

$= \dfrac{7}{625} + \dfrac{24}{625}i$ //

12. $\dfrac{(2-5i)\ (2+5i)}{(4+3i)(4-3i)} = \dfrac{4+25}{16+9}$

$= \dfrac{29}{25}$ //

16. $\dfrac{x+iy}{x-iy} = \dfrac{(x+iy)^2}{(x-iy)(x+iy)} = \dfrac{x^2-2ixy-y^2}{x^2+y^2} = \dfrac{x^2-y^2}{x^2+y^2} - \dfrac{2ixy}{x^2+y^2}$

$\therefore Re\left(\dfrac{z}{\bar{z}}\right) = \dfrac{x^2-y^2}{x^2+y^2}$ //

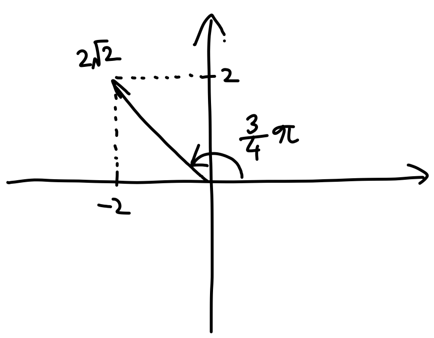
1.2 (pp. 13-14)

2. $z = -2+2i$

$|z| = \sqrt{4+4} = 2\sqrt{2}$

$\theta = \tan^{-1}\left(\dfrac{2}{-2}\right) = \dfrac{3}{4}\pi$

$\therefore z = 2\sqrt{2}\, e^{j\frac{3}{4}\pi}$ //

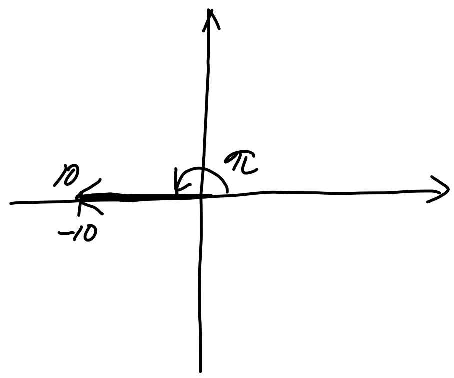


4. $z = -10$

$|z| = 10$

$\theta = \pi$

$\therefore z = 10e^{j\pi}$



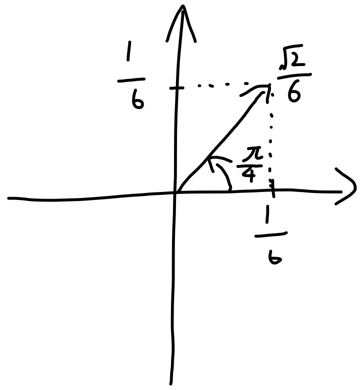
1.2 (pp. 13-14)

8. $z = \dfrac{i}{3+3i} = \dfrac{i(3-3i)}{9+9} = \dfrac{3+3i}{18} = \dfrac{1}{6} + \dfrac{1}{6}i$

$|z| = \sqrt{\dfrac{1}{36} + \dfrac{1}{36}} = \sqrt{\dfrac{2}{36}} = \dfrac{\sqrt{2}}{6}$

$\theta = \tan^{-1}\left(\dfrac{\frac{1}{6}}{\frac{1}{6}}\right) = \dfrac{\pi}{4}$

$\therefore z = \dfrac{\sqrt{2}}{6} e^{i\frac{\pi}{4}}$ //



22. $W = \sqrt[3]{8i}$

$z = 8i$ として極形式にすると.

$z = 8i = 8e^{i(\theta + 2n\pi)}$

$\theta = \tan^{-1}\dfrac{1}{0} = \dfrac{\pi}{2}$

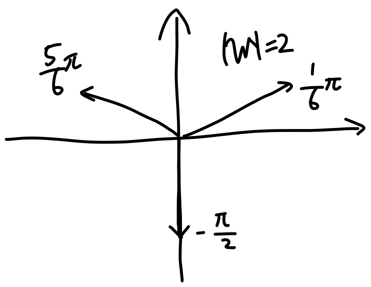
$\therefore z = 8e^{i(\frac{\pi}{2} + 2n\pi)}$

$\therefore W = z^{\frac{1}{3}}$

$= 2e^{\frac{1}{3}(\frac{\pi}{2} + 2n\pi)i}$

$= 2e^{i\frac{\pi + 4n\pi}{6}}$ $\qquad \dfrac{9}{6}$

$\therefore W_1 = 2e^{i\frac{\pi}{6}}, W_2 = 2e^{i\frac{5}{6}\pi}, W_3 = 2e^{i\frac{\pi}{2}}$ //



23. $W = \sqrt[3]{216}$

$z = 216$ と置くと

$z = 216 e^{i0}$

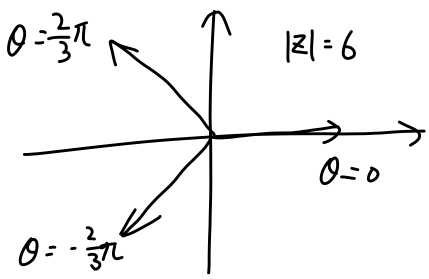
$= 216 e^{i(0 + 2n\pi)}$

$\therefore W = z^{\frac{1}{3}}$

$= \left(216 e^{i2n\pi}\right)^{\frac{1}{3}}$

$= 6 e^{i\frac{2n\pi}{3}}$

$\therefore W_1 = 6, \quad W_2 = 6e^{i\frac{2}{3}\pi}, \quad W_3 = 6e^{-i\frac{2}{3}\pi}$ //

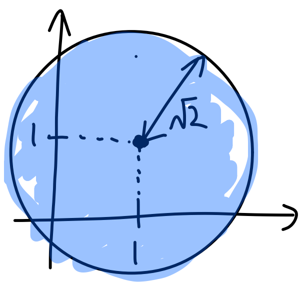


1.3 (p. 21)

4. $0 < |z - 1 - i| < \sqrt{2}$

中心が $(1, 1)$

半径が $0 < r < \sqrt{2}$



境界は含まない

$(0 < r < \sqrt{2})$

1.3 (p.21)

12. $f(z)=\dfrac{z-2}{z+2}$ $\quad z=4i$

$$f(4i)=\frac{4i}{4i+2}$$
$$=\frac{16+16i-4}{20}$$
$$=\frac{12}{20}+\frac{16}{20}i$$
$$=\frac{3}{5}+\frac{4}{5}i$$

$$\therefore\begin{cases}\mathrm{Re}\{f(z)\}=\dfrac{3}{5}\\ \mathrm{Im}\{f(z)\}=\dfrac{4}{5}\end{cases}$$ //

14. $f(z)=\dfrac{\mathrm{Re}\,z^2}{|z|}$

$z=x+iy$ として

$$f(z)=\frac{\mathrm{Re}(z^2)}{|z|}=\frac{\mathrm{Re}(x^2+2ixy-y^2)}{\sqrt{x^2+y^2}}$$
$$=\frac{x^2-y^2}{\sqrt{x^2+y^2}}$$

$$\lim_{z\to0}f(z)=\lim_{y\to0}\left[\lim_{x\to0}f(z)\right]=\lim_{y=0}\frac{-y^2}{\sqrt{y^2}}=0$$

$$\lim_{z\to0}f(z)=\lim_{x\to0}\left[\lim_{y\to0}f(z)\right]=\lim_{x\to0}\frac{x^2}{\sqrt{x^2}}=0$$

$f(0)=0$ であるから, $f(z)=0$ で連続. //

17. i における $f(z)=\dfrac{z-i}{z+i}$

$$f'(z)\Big|_{z=i}=\lim_{z\to i}\frac{f(z)-f(i)}{z-i}$$
$$=\lim_{z\to i}\frac{\dfrac{z-i}{z+i}-\dfrac{i-i}{i+i}}{z-i}$$
$$=\lim_{z\to i}\frac{z-i}{(z-i)(z+i)}$$
$$=\lim_{z\to i}\frac{1}{z+i}$$
$$=\frac{1}{2i}$$
$$=-\frac{1}{2}i$$ //