

# 公式 三角関数

## 相互関係

$$\begin{aligned} \textcircled{1} \sin^2 \theta + \cos^2 \theta &= 1 \\ \textcircled{2} \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \textcircled{3} 1 + \tan^2 \theta &= \frac{1}{\cos^2 \theta} \\ \textcircled{4} 1 + \frac{1}{\tan^2 \theta} &= \frac{1}{\sin^2 \theta} \end{aligned}$$

## ③

$$\textcircled{1} \in \cos^2 \theta \text{ 区間}$$

## ④

$$\textcircled{1} \in \sin^2 \theta \text{ 区間}$$

## 加法定理

$$\begin{aligned} \textcircled{1} \sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\ \textcircled{2} \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \textcircled{3} \tan(a \pm b) &= \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \end{aligned}$$

## 2倍角

$$\begin{aligned} \textcircled{1} \sin 2\theta &= 2 \sin \theta \cos \theta \\ \textcircled{2} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ \textcircled{3} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \end{aligned}$$

加法定理の  $a, b \in \mathbb{R}$  区間

$$\begin{aligned} \textcircled{2} \cos^2 \theta - \sin^2 \theta &= (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2 \sin^2 \theta \\ &= \cos^2 \theta - (1 - \cos^2 \theta) = 2 \cos^2 \theta - 1 \end{aligned}$$

## 半角

$$\begin{aligned} \textcircled{1} \sin^2 \theta &= \frac{1}{2} (1 - \cos 2\theta) \\ \textcircled{2} \cos^2 \theta &= \frac{1}{2} (1 + \cos 2\theta) \end{aligned}$$

2倍角の②区間

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$2 \sin^2 \theta = 1 - \cos 2\theta$$

$$\therefore \sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$2 \cos^2 \theta = 1 + \cos 2\theta$$

$$\therefore \cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

積和

$$\begin{aligned} \textcircled{1} \sin a \cos b &= +\frac{1}{2}(\sin(a+b) + \sin(a-b)) \\ \textcircled{2} \cos a \sin b &= +\frac{1}{2}(\sin(a+b) - \sin(a-b)) \\ \textcircled{3} \cos a \cos b &= +\frac{1}{2}(\cos(a+b) + \cos(a-b)) \\ \textcircled{4} \sin a \sin b &= -\frac{1}{2}(\cos(a+b) - \cos(a-b)) \end{aligned}$$

ex)  $\sin a \sin b$

加法定理

$$\begin{aligned} \textcircled{1} \sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\ \textcircled{2} \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \end{aligned}$$

$\sin a \sin b$  を求むには ③と④の差をとる。

$$\textcircled{4} \cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\rightarrow \textcircled{3} \cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) - \cos(a+b) = 0 + 2 \sin a \sin b$$

$$\rightarrow \sin a \cos b = -\frac{1}{2}(\cos(a+b) - \cos(a-b)) //$$

和積

$$\begin{aligned} \textcircled{1} \sin A + \sin B &= +2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \textcircled{2} \sin A - \sin B &= +2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \textcircled{3} \cos A + \cos B &= +2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \textcircled{4} \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned}$$

ex) ①  $\sin A + \sin B$

積和の①を  $a+b=A$ ,  $a-b=B$  とおきかえ

$$\begin{cases} a+b=A \\ a-b=B \end{cases} \Rightarrow \begin{cases} 2a=A+B \\ 2b=A-B \end{cases} \Rightarrow \begin{cases} a=\frac{A+B}{2} \\ b=\frac{A-B}{2} \end{cases}$$

$$\sin a \cos b = \frac{1}{2}(\sin A + \sin B)$$

$$\sin A + \sin B = 2 \sin a \cos b$$

$$= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} //$$