H30 長岡

[1] $A = \begin{bmatrix} a & 1-a \\ 1-a & a \end{bmatrix}$

(1) $\begin{vmatrix} a-\lambda & 1-a \\ 1-a & a-\lambda \end{vmatrix} = (a-\lambda)^2 - (1-a)^2$

$= ((a-\lambda)+(1-a))((a-\lambda)-(1-a))$

$= (1-\lambda)(2a-1-\lambda)$

$\therefore \lambda = 1,\ 2a-1$ //

○ $\lambda = 1$ のとき

$\begin{bmatrix} a-1 & 1-a \\ 1-a & a-1 \end{bmatrix} \longrightarrow \begin{bmatrix} a-1 & 1-a \\ 0 & 0 \end{bmatrix}$

$(a-1)x + (1-a)y = 0$

$(a-1)x = -(1-a)y$

$(a-1)x = (a-1)y$

$\therefore x_1 = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (c_1 は 0 でない任意の定数) //

○ $\lambda = 2a-1$ のとき

$\begin{bmatrix} a-2a+1 & 1-a \\ 1-a & a-2a+1 \end{bmatrix} \to \begin{bmatrix} 1-a & 1-a \\ 1-a & 1-a \end{bmatrix} \to \begin{bmatrix} 1-a & 1-a \\ 0 & 0 \end{bmatrix}$

$(1-a)x + (1-a)y = 0$

$(1-a)x = -(1-a)y$

$\therefore x_2 = c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (c_2 は 0 でない任意の定数) //

(2) 直交行列 $\to {}^tAA = E$

$k\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} k\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = k^2\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2k^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2k^2E$

$2k^2E = E \to 2k^2 = 1 \to k^2 = \frac{1}{2} \to k = \pm\frac{1}{\sqrt{2}}$

1つあげればよいので

$P = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ // , $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 2a-1 \end{bmatrix}$ //

[2] $xy'' + 2y' + 4xy = 0$

(1) $z = xy$ とおく

$$y = \frac{z}{x} = z \cdot x^{-1}$$

zはxの関数であることに注意して

$$\frac{dy}{dx} = \frac{dz}{dx}x^{-1} + z(-x^{-2}) = \frac{dz}{dx}x^{-1} - zx^{-2} = z'x^{-1} - zx^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{d^2z}{dx^2}x^{-1} + \frac{dz}{dx}(-x^{-2}) - \left(\frac{dz}{dx}x^{-2} + z(-2x^{-3})\right)$$

$$= z''x^{-1} - z'x^{-2} - z'x^{-2} + 2zx^{-3} = x^{-1}z'' - 2x^{-2}z' + 2x^{-3}z$$

与式に代入

$$x(x^{-1}z'' - 2x^{-2}z' + 2x^{-3}z) + 2(x^{-1}z' - x^{-2}z) + 4z = 0$$

$$z'' - \cancel{2x^{-1}z'} + \cancel{2x^{-2}z} + \cancel{2x^{-1}z'} - \cancel{2x^{-2}z} + 4z = 0$$

$$z'' + 4z = 0$$

(2) $\lambda^2 + 4 = 0$

$$\lambda = \frac{-0 \pm \sqrt{0^2 - 16}}{2} = \pm\frac{\sqrt{-16}}{2} = \pm i\frac{4}{2} = \pm 2i$$

$$\therefore z = C_1 \cos 2x + C_2 \sin 2x$$

$$\therefore y = \frac{C_1 \cos 2x + C_2 \sin 2x}{x} \quad (C_1, C_2 : 定数)$$

[3] (1) $P(X=1)=\frac{1}{2}$, $P(X=2)=\frac{1}{2}$ //

$$\left.\begin{array}{lll} 1 & 1 & \to 1 \\ 1 & 2 & \to 2 \\ 2 & 1 & \to 2 \\ 2 & 2 & \to 2 \end{array}\right\} \to P(Y=1)=\frac{1}{4},\quad P(Y=2)=\frac{3}{4}$$ //

(2) $P(X>Y)=\frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$ //

$P(X<Y)=\frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$ //

$$\begin{array}{r} 75 \\ 4\overline{)300} \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

(3) $E_A = \frac{1}{8}\cdot 300 - \frac{3}{8}n = \frac{300-3n}{8}$ //

$E_B = \frac{3}{8}\cdot n - \frac{1}{8}\cdot 300 = \frac{3n-300}{8}$ //

(4)

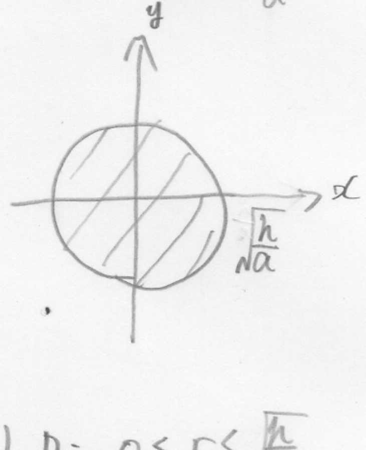
$300-3n = 3n-300$

$6n = 600$

$n = 100$ //

[4] (1) $h - a(x^2+y^2) \geqq 0$

$(x^2+y^2) \leqq \frac{h}{a}$



$S = \pi\left(\sqrt{\frac{h}{a}}\right)^2$

$= \frac{\pi h}{a}$ //

(2) $D = 0 \leqq r \leqq \sqrt{\frac{h}{a}}$, $0 \leqq \theta \leqq 2\pi$

$x^2+y^2=r^2$ より

$V = \int_0^{2\pi}\left\{\int_0^{\sqrt{\frac{h}{a}}} (h-ar^2)\,r\,dr\right\}d\theta$

$= \int_0^{2\pi}\left[\frac{1}{2}hr^2 - \frac{1}{4}ar^4\right]_0^{\sqrt{\frac{h}{a}}} d\theta$

$= 2\pi\left[\frac{1}{2}hr^2 - \frac{1}{4}ar^4\right]_0^{\sqrt{\frac{h}{a}}}$

$\to V = 2\pi\left(\frac{1}{2}h\cdot\frac{h}{a} - \frac{1}{4}a\cdot\frac{h^2}{a^2}\right)$

$= 2\pi\left(\frac{h^2}{2a} - \frac{h^2}{4a}\right) = 2\pi\left(\frac{h^2}{4a}\right)$

$= \frac{1}{2}\cdot\frac{\pi h^2}{a}$

$= \frac{1}{2}Sh$ //