H29 長園

[1] (1) $\begin{vmatrix} 3-\lambda & -2 \\ 1 & -\lambda \end{vmatrix} = -3\lambda+\lambda^2+2 = \lambda^2-3\lambda+2 = (\lambda-2)(\lambda-1)$

$\therefore \lambda = 1, 2$ //

○ $\lambda = 1$

$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \to x-y=0 \to x=y$

$\therefore x_1 = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (c_1：定数) //

○ $\lambda = 2$

$\begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \to x = 2y$ 　$y = c_2$ とおいて

$\therefore x_2 = c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (c_2：定数) //

(2) $P = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ //

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$P^{-1} = \frac{1}{1-2} \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = -\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ //

(3) $D = P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ とおいて

$A^n = (PDP^{-1})^n$

$= (PD\underbrace{P^{-1})(P}_{消える}D\underbrace{P^{-1})(P}_{消える}DP^{-1}) \cdots (PDP^{-1})$

$= PD^nP^{-1}$

$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1^n & 0 \\ 0 & 2^n \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2\cdot 2^n \\ 1 & 2^n \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1+2\cdot 2^n & 2-2\cdot 2^n \\ -1+2^n & 2-2^n \end{bmatrix}$

$= \begin{bmatrix} 2^{n+1}-1 & 2-2^{n+1} \\ 2^n-1 & 2-2^n \end{bmatrix}$ //

[2] $y'' - y = e^{x}\sin x$

(1) $\lambda^2 - 1 = 0$

$\lambda^2 = 1 \rightarrow \lambda = \pm 1$

$\underline{y = C_1 e^{-x} + C_2 e^{x} \quad (C_1,\ C_2:\text{定数})}$ //

(2) $y = a e^{x}\cos x + b e^{x}\sin x$

$\dfrac{dy}{dx} = a(e^{x}\cos x - e^{x}\sin x) + b(e^{x}\sin x + e^{x}\cos x)$

$\dfrac{d^2y}{dx^2} = a(\cancel{e^{x}\cos x} - e^{x}\sin x - e^{x}\sin x - \cancel{e^{x}\cos x}) + b(\cancel{e^{x}\sin x} + e^{x}\cos x + e^{x}\cos x - \cancel{e^{x}\sin x})$

$= -2a e^{x}\sin x + 2b e^{x}\cos x$

与式に代入

$2b e^{x}\cos x - 2a e^{x}\sin x - a e^{x}\cos x - b e^{x}\sin x = e^{x}\sin x$

$(2b - a) e^{x}\cos x + (-2a - b) e^{x}\sin x = e^{x}\sin x$

$\begin{cases} 2b - a = 0 \\ -2a - b = 1 \end{cases} \Rightarrow \begin{matrix} a = 2b \\ -4b - b = 1 \end{matrix} \quad \begin{cases} a = -\frac{2}{5} \\ b = -\frac{1}{5} \end{cases}$ //

(3) $\underline{y = C_1 e^{-x} + C_2 e^{x} - \frac{2}{5} e^{x}\cos x - \frac{1}{5} e^{x}\sin x}$ //

[3] (1) $N = {}_5C_3 = \dfrac{5!}{(5-3)!} \cdot \dfrac{1}{3!} = \dfrac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{2}} = 10$

$\underline{\therefore p_1 = \frac{1}{10}}$ //

(2) $\underline{\therefore p_2 = \frac{1}{10}}$ //

(3) $X = \{1, 2, 3\}$ のとき $Y = \{k, 4, 5\}$

$\dfrac{3 \cdot {}_2C_2}{{}_5C_3} = \underline{\dfrac{3}{10}}$ //

[4] (1) $f_x = 2x$, $f_y = 2y$

$z - a^2 - b^2 = 2a(x-a) + 2b(y-b)$

$z = a^2 + b^2 + 2ax - 2a^2 + 2by - 2b^2$

$z = 2ax + 2by - (a^2 + b^2)$ //

(2) $-1 = -(a^2 + b^2)$

$1 = a^2 + b^2$

このときの接点のz座標は

$z = x^2 + y^2 = a^2 + b^2 = 1$

つまり点$(a, b, f(a,b))$の軌跡は

平面$z=1$と

$z = f(x,y)$の交わる円 $\{(x,y,z) \mid x^2+y^2=1,\ z=1\}$ である

よってSは

$z=1$ //

(3) $D: x^2+y^2 \leqq 1$

Sと$z=f(x,y)$の共通部分は x^2+y^2

$V = \iint_D (1-(x^2+y^2))\,dx\,dy$

$= \int_0^{2\pi}\left\{\int_0^1 r(1-r^2)\,dr\right\}d\theta$

$= \int_0^{2\pi}\left[\frac{1}{2}r^2 - \frac{1}{4}r^4\right]_0^1 d\theta$

$= 2\pi\left(\frac{1}{2} - \frac{1}{4}\right)$

$= 2\pi\left(\frac{2}{4} - \frac{1}{4}\right)$

$= \frac{1}{2}\pi$ //