H28 長国

[1] (1) $\frac{n}{2n} \times \frac{n-1}{2n-1} \times \frac{n-2}{2n-2} = \frac{n(n-1)(n-2)}{2n(2n-1)(2n-2)}$

$= \frac{n(\cancel{n-1})(n-2)}{4n(2n-1)(\cancel{n-1})} = \frac{n-2}{4(2n-1)}$ //

(2)

$\left(\frac{n}{2n} \times \frac{n-1}{2n-1} \times \frac{n}{2n-2}\right) + \left(\frac{n}{2n} \times \frac{n}{2n-1} \times \frac{n-1}{2n-2}\right)$

$= 2 \times \frac{n(\cancel{n-1})n}{4n(2n-1)(\cancel{n-1})}$

$= \frac{n}{2(2n-1)}$ //

(3)

$E(X) = 2 \cdot \frac{n-2}{4(2n-1)} + 1 \cdot \frac{n}{2(2n-1)}$

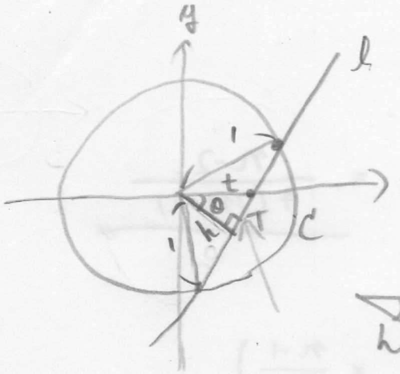
$= \frac{n-2+n}{2(2n-1)}$

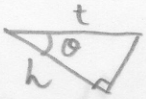
$= \frac{2n-2}{2(2n-1)}$

$= \frac{n-1}{2n-1}$ //

[2]

(1)





$\cos\theta = \dfrac{h}{t}$

$\rightarrow h = t\cos\theta$

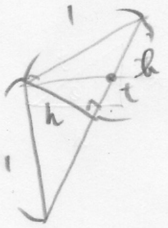
θ は $-90° \sim 90°$

$\rightarrow \cos\theta$ は $0 \sim 1$

$\rightarrow t$ は $0 < t < 1$

$0 < h \leqq t$ //

(2)



$1 = \sqrt{h^2 + b^2}$

$1 = h^2 + b^2$

$b^2 = 1 - h^2 \quad \rightarrow b = \sqrt{1-h^2}$

$\therefore S = \dfrac{1}{2} \ 2\sqrt{1-h^2} \cdot h$

$S = h\sqrt{1-h^2}$ //

(3) h の最大値

$\rightarrow t$ の最大値

$S(h) = h\sqrt{1-h^2}$

$S'(h) = \dfrac{-2h}{2\sqrt{1-h^2}} + \sqrt{1-h^2}\,(-2h) = \dfrac{1-2h^2}{\sqrt{1-h^2}}$

$S'(h) = 0$ となるのは $\dfrac{1}{\sqrt{2}}$

$0 < t < \dfrac{1}{\sqrt{2}}$ のとき

h	0	$\cdots$	t	$\cdots$	$\frac{1}{\sqrt{2}}$
S'	$+$	$+$	$+$	$+$	0
S	↗	↗	↗	↗	

$\dfrac{1}{\sqrt{2}} \leqq t < 1$

h	0	$\cdots$	$\frac{1}{\sqrt{2}}$	$\cdots$	t
S'	$+$	$+$	0	$-$	$-$
S	↗	↗		↘	↘

$\begin{cases} 0 < t < \dfrac{1}{\sqrt{2}} \text{ のとき } S = t\sqrt{1-t^2} \\ \dfrac{1}{\sqrt{2}} \leqq t < 1 \text{ のとき } S = \dfrac{1}{2} \end{cases}$ //

H28

[3] $x^2\dfrac{d^2y}{dx^2}+4x\dfrac{dy}{dx}+2y=0$

$x=e^t$

(1) $\dfrac{dy}{dt}=\dfrac{\frac{dy}{dx}}{\frac{dt}{dx}}=\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$

$x=e^t$ だから

$\dfrac{dx}{dt}=(e^t)'=e^t=x$

$\therefore \dfrac{dy}{dt}=x\dfrac{dy}{dx}$

(2) $\dfrac{d^2y}{dt^2}=\dfrac{d}{dt}\left(\dfrac{dy}{dt}\right)$

$=\dfrac{d}{dt}\left(x\cdot\dfrac{dy}{dx}\right)$

$=\dfrac{dx}{dt}\cdot\dfrac{dy}{dx}+x\cdot\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)$

$=\dfrac{dy}{dt}+x\dfrac{d}{dx}\left(\dfrac{dy}{dt}\right)$

$=x\dfrac{dy}{dx}+x\dfrac{d}{dx}\left(x\dfrac{dy}{dx}\right)$

$=x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$ //

(3) $\dfrac{d^2y}{dt^2}-x\dfrac{dy}{dx}+4\dfrac{dy}{dt}+2y=0$

$\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}+4\dfrac{dy}{dt}+2y=0$

$\dfrac{d^2y}{dt^2}+3\dfrac{dy}{dt}+2y=0$

$\lambda^2+3\lambda+2=0$

$(\lambda+2)(\lambda+1)=0$

$\lambda=-1,-2$

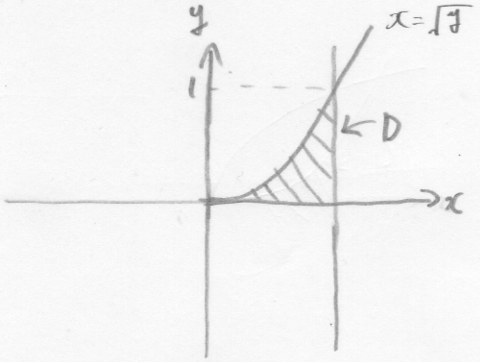
1 X 2 2
1 1 1
3

$x=e^t$

$\therefore y=C_1e^{-t}+C_2e^{-2t}$ $=C_1x^{-1}+C_2x^{-2}$ (C_1, C_2: 定数) //

[4]

(1)



$x = \sqrt{y} \to y = x^2$

(2) $f(x) = x^2$

(3) $V = \int_0^1 \left\{ \int_0^{x^2} \sqrt{1+x^3}\, dy \right\} dx$

$= \int_0^1 \left[y\sqrt{1+x^3} \right]_0^{x^2} dx$

$= \int_0^1 x^2 \sqrt{1+x^3}\, dx$

$1+x^3 = t$ とおいて

$\frac{dt}{dx} = 3x^2 \to dx = \frac{dt}{3x^2}$

x	$0 \to 1$
t	$1 \to 2$

$V = \int_1^2 x^2 \sqrt{t} \cdot \frac{1}{3x^2}\, dt$

$t^{\frac{1}{2}} \to \frac{2}{3} t^{\frac{3}{2}}$

$= \frac{1}{3} \int_1^2 \sqrt{t}\, dt$

$= \frac{1}{3} \cdot \frac{2}{3} \left[t^{\frac{3}{2}} \right]_1^2$

$= \frac{2}{9} \left(\sqrt{2}^3 - 1 \right)$

$= \frac{2}{9} \left(2\sqrt{2} - 1 \right)$