① Let us take two vector

$\boxed{12, 15, 18, 20}$ not three.

$x(n) = \cancel{y(n-1)+y(n)}\ 3\delta(n-3)$

$z(n) = \cancel{y(n+1)+y(n-1)+y(n)}\ 6+\delta(n-2)$

$x(n) = 3u(n-3)$

$z(n) = 6+u(n-2)$

impulse response

Let $n=-5:5$

$x(-5)=0$; $z(-5)=0$
$x(-4)=0$; $x(-4)=0$
$\vdots$
$x(3)=3$; $x(2)=6$
$x(4)=0$; $x(3)=0$
$x(5)=0$; $x(4)=0$
$x(5)=0$

unit step response

Let $n=-5:5$

$x(-5)=0$; $z(-5)=0$
$x(-4)=0$; $z(-4)=0$
$\vdots$
$x(3)=3$; $z(2)=6$
$x(4)=3$; $z(3)=6$
$x(5)=3$; $z(4)=6$
$z(5)=6$

impulse function

$\delta(n)=\begin{cases}1 & n=0\\ 0, & n\neq 0\end{cases}$

step functi.

$u(n)=\begin{cases}1 & n\geq 0\\ 0, & n<0\end{cases}$

② $x_1(n)=[1,2,3,5,6,-7]$; $x_2(n)=[4,5,6,7,8]$

$N=L+M-1 = 6+5-1$

$\cancel{=4+2-1=5}$

	4	5	6	7	8
1	4	5	6	7	8
2	8	10	12	14	16
3	12	15	18	21	24
~~4~~	~~16~~	~~20~~			
5	20	25	30	35	40
6	24	30	36	42	48
-7	28	35	42	49	56

$y(n)=\{4,13,28,54,89,69,60,40,-1,-56\}$

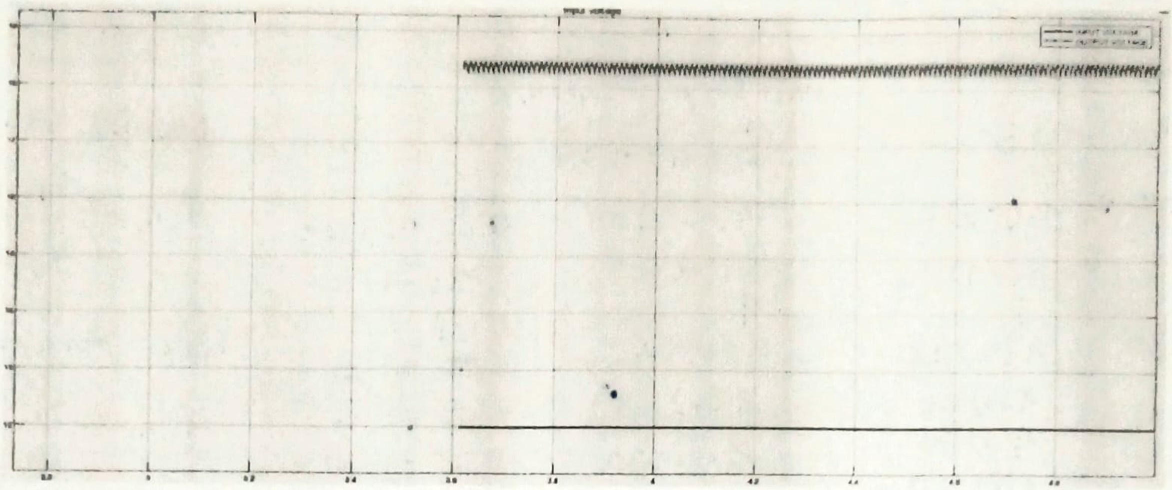
② 14

circular convolution.

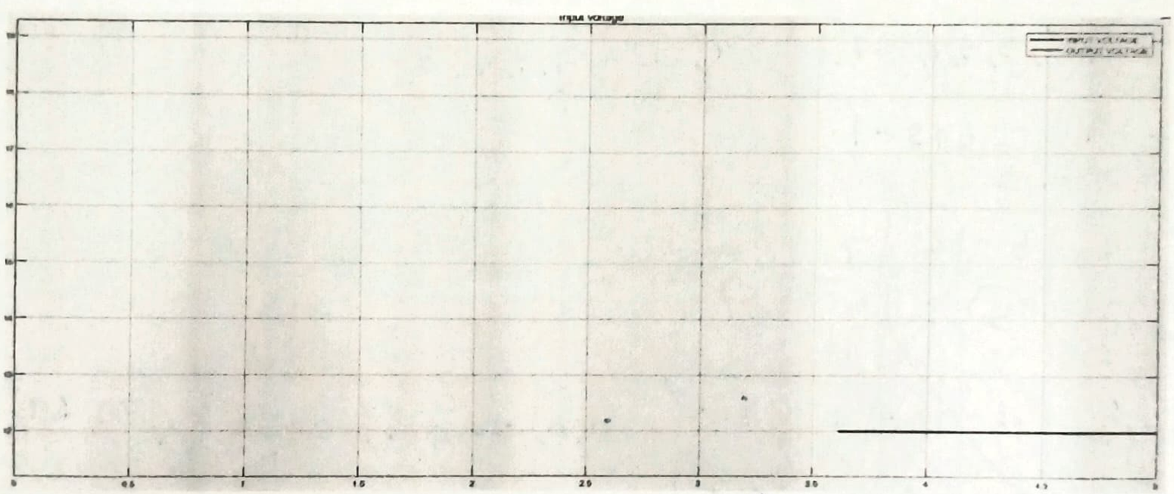
③ $x_1(n)=[0,0,0,2,2,2,2]$
0 1 2 3 4 5 6 7

$x_2(n)=[0,0,1,1,1,1,1,1]$

Duty cycle=30



Duty cycle = 40



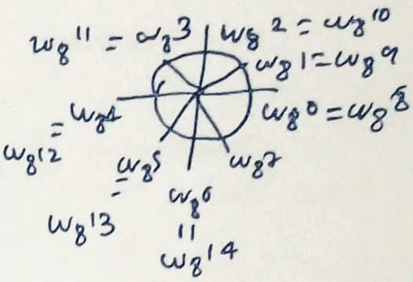
	0	0	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
2	0	0	2	2	2	2	2	2
2	0	0	2	2	2	2	2	2
2	0	0	2	2	2	2	2	2
2	0	0	2	2	2	2	2	2

$y(n) = \{0, 0, 0, 0, 0, 2, 4, 6, 8, 8, 8, 6, 4, 2\}$

④ $x(n) = [2,1,2,1,1,2,1,2]$

$X_N = W_N x_N$

$$W_N = \begin{bmatrix} w_8^0 & w_8^0 & w_8^0 & w_8^0 & w_8^0 & w_8^0 & w_8^0 & w_8^0 \\ w_8^0 & w_8^1 & w_8^2 & w_8^3 & w_8^4 & w_8^5 & w_8^6 & w_8^7 \\ w_8^0 & w_8^2 & w_8^4 & w_8^6 & w_8^8 & w_8^{10} & w_8^{12} & w_8^{14} \\ w_8^0 & w_8^3 & w_8^6 & w_8^9 & w_8^{12} & w_8^{15} & w_8^{18} & w_8^{21} \\ w_8^0 & w_8^4 & w_8^8 & w_8^{12} & w_8^{16} & w_8^{20} & w_8^{24} & w_8^{28} \\ w_8^0 & w_8^5 & w_8^{10} & w_8^{15} & w_8^{20} & w_8^{25} & w_8^{30} & w_8^{35} \\ w_8^0 & w_8^6 & w_8^{12} & w_8^{18} & w_8^{24} & w_8^{30} & w_8^{36} & w_8^{42} \\ w_8^0 & w_8^7 & w_8^{14} & w_8^{21} & w_8^{28} & w_8^{35} & w_8^{42} & w_8^{49} \end{bmatrix}$$



$w_{8N} = e^{\frac{-j2\pi}{N}}$

$w_8^0 = 1$

$w_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$w_8^2 = -j$

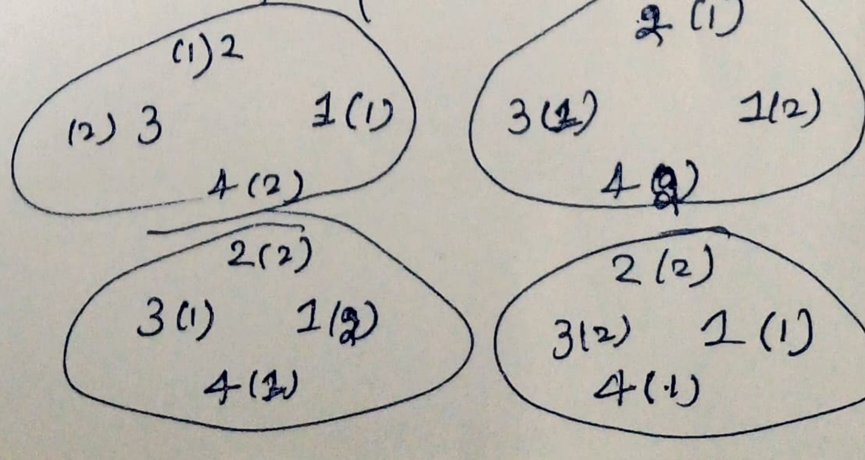
$w_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\quad w_8^4 = -1$ $\quad w_8^5 = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$ $\quad w_8^6 = j$ $\quad w_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$X_N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ 1 & -j & & & & & & \\ 1 & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & & & & & & \\ 1 & -1 & & & & & & \\ 1 & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & & & & & & \\ 1 & j & & & & & & \\ 1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & & & & & & \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

⑤ $x(n) = [1,2,3,4]$

$h(n) = [1,2,2,1]$

$x_3(n) = [17, 15, 13, 15]$



6) $x(N) = \{16, -6-2j, 0, -6+2j\}$

$x(n) = \frac{1}{4} W_N^{q} X(N)$

$$= \frac{1}{4}\begin{bmatrix} W_4^{0\alpha} & W_4^{0\alpha} & W_4^{0\alpha} & W_4^{0\alpha} \\ W_4^{0\alpha} & W_4^{1\alpha} & W_4^{2\alpha} & W_4^{3\alpha} \\ W_4^{0\alpha} & W_4^{2\alpha} & W_4^{4\alpha} & W_4^{6\alpha} \\ W_4^{0\alpha} & W_4^{3\alpha} & W_4^{6\alpha} & W_4^{9\alpha} \end{bmatrix}\begin{bmatrix} 16 \\ -6-2j \\ 0 \\ -6+2j \end{bmatrix}$$

$W_4^{0\alpha} = 1$ $\quad W_4^{5\alpha} = j1$

$W_4^{1\alpha} = j1$ $\quad W_4^{0\alpha} = -1$

$W_4^{2\alpha} = -1$ $\quad W_4^{7\alpha} = -j1$

$W_4^{3\alpha} = -j1$ $\quad W_4^{8\alpha} = 1$

$W_4^{4\alpha} = 1$ $\quad W_4^{9\alpha} = j1$

7) $T = 0.1$

8) $0.6 \le |H(e^{j\omega})| \le 1.0$ for $0 \le \omega \le 0.35\pi$

11) $|H(e^{j\omega})| \le 0.1$; for $0.7\pi \le \omega \le \pi$

$\omega_p = 0.35\pi \qquad A_p = 0.6$

$\omega_s = 0.7\pi \qquad A_s = 0.1$

$$N = \frac{\log \sqrt{\dfrac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

For invarient

9) $\Omega_p = \dfrac{\omega_p}{T}$

11) $\Omega_s = \dfrac{\omega_s}{T}$

For bilinear

$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$

$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$

$$\Omega_c = \frac{1}{2}\left[\frac{\Omega_s}{\left(\frac{1}{A_s^2}-1\right)^{1/2N}} + \frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{1/2N}}\right]$$

$P = \pm \Omega_c e^{j(2K+N+1)\pi/2N}$

invarient

$$\frac{C_K}{S - P_K} = \frac{C_K}{1 - e^{P_K T} z^{-1}}$$

$$H_a(s) = \frac{\Omega_c}{(s-s_1)(s-s_1^*)(s-s_2)(s-s_2^*)}$$

bilinear

7)
8)
10)

$S \rightarrow \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

9)

10) $$N = \frac{\cosh^{-1}\sqrt{\frac{\left(\frac{1}{A_s^2}-1\right)}{\left(\frac{1}{A_p^2}-1\right)}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\therefore p_k = \sigma_k + j\Omega_k$$

$$\sigma_k = a\cos\phi_k$$

$$\Omega_k = b\sin\phi_k$$

$$\phi_k = (2k+N+1)\pi/2N$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\mu = \frac{1+\sqrt{1+\varepsilon^2}}{2}$$

$$\varepsilon = \sqrt{\frac{1}{A_p} - 1}$$

13)