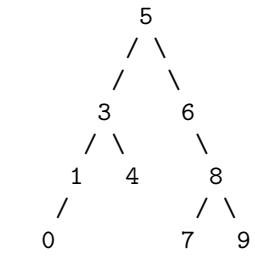
Tree-based Hegselmann-Krause update

The algorithm

For a Hegselmann-Krause update of agent i one has to find all agents j with an opinion in the range $[x_i - \varepsilon_i, x_i + \varepsilon_i]$.

The main idea is to maintain a Search Tree of all n agents' opinions. Every time an agent changes its opinion, the tree needs to be updated. However, in the tree, one can find efficiently the entry $x_i - \varepsilon_i$ in $O(\log(n))$. As soon, as this node is reached, the tree can be traversed in order until a node with a key larger than the upper bound $x_i + \varepsilon_i$ is encountered.

In a search tree each node has a `key` and at most two children, such that its `left` child's key is always smaller than its own key and the key of its `right` child is always larger than its own key, e.g.:



Using a python-esque, recursive pseudocode, this traversal could look like the following. Note that the comparisons in front of the recursive calls cut all subtrees from the traversal, which contain only nodes which are outside of the range, i.e., it uses the order of the tree to find the first element of the range and traverses only until the last. Therefore, this algorithm has (for a balanced tree) a complexity of $O(\log(n) + m)$, where m is the number of elements in the range.

```
def traverse_range(node, queue, lower, upper):
    if not node:
        return

    if node.key > lower:
        traverse_range(node.left, lower, upper)

    if lower <= node.key <= upper:
        queue.push(node.key)

    if node.key < upper:
        traverse_range(node.right, lower, upper)

queue = []
```

```
traverse_range(root, queue, x-eps, x+eps)
average = sum(queue) / len(queue)
```

Additional to this simple traversal, we have to consider that opinions may occur multiple times, but keys in a search tree need to be unique. This can be solved simply by storing, additionally to the children, also a counter, which keeps track of the number of agents, which hold this opinion. This does, especially in the case, where agents are converged into tight clusters, reduce the computation time even more, since fewer nodes have to be traversed.

Since, we have to perform the simulation until all agents reach their final state, this contributes most to the speedup of this algorithm.

Technical details

To ensure that the `finding` phase is fast, the tree needs to be balanced, i.e., the number of nodes on each level should increase exponentially to ensure that finding the lower bound can be done in $O(\log n)$. This could be achieved by using AVL- or red-black trees. But generally binary trees are not very efficient for in-order traversal. We decided to use a B-tree instead, which stores multiple values in each node, such that in-order traversal benefits from the caches of modern processors.

Example implementation

In the `src` directory there is an example implementation for the tree-based Hegselmann-Krause update. It is written in the rust programming language for its high performance and high-level abstractions. (For installation instructions, see rustup.rs).

This example can be compiled with `cargo build` and the included tests and benchmarks can be executed with `cargo test` and `cargo bench`. When run as a program, it takes some parameters to perform simulations and saves the cluster configuration into a file (more info with `cargo run -- -h`).

The most interesting file is probably `src/hegselmannkrause.rs`, which includes an implementation of the algorithm.