③ $\mathbb{S}^1$ circonferenza ritratto forte di deformazione di N nastro di Möbius

N omotopicamente equivalente al cilindro

C retr forte def di N se $\exists\, r: N \to \mathbb{S}^1 \mid i \circ r \sim_c id_N$

oppure se e solo se $\exists\, F: N \times I \to N \mid F(x,0) = x,\ F(x,1) \in \mathbb{S}^1,\ \forall x \in N \ \wedge\ F(a,t) = a,\ \forall a \in \mathbb{S}^1\ \forall t \in I$

$$N: \begin{cases} x = (R + s\cos(t/2))\cos t \\ y = (R + s\cos(t/2))\sin t \\ z = s \sin t/2 \end{cases} \qquad \begin{cases} s \in [-w, w] \\ t \in [0, 2\pi) \end{cases}$$

$$\mathbb{S}^1 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = R^2 \wedge z = 0\} \subset \mathbb{R}^3$$

$$i: \mathbb{S}^1 \hookrightarrow N$$

$$N = \{(x,y,z) \in \mathbb{R}^3 \mid -R^2 y + x^2 y + y^3 - 2Rxz - 2x^2 z - 2y^2 z + yz^2 = 0\}$$

$$\pi_1: I^2 \to N \qquad (0,t) \rightsquigarrow (1, 1-t)$$

$$\tilde{r}: I^2 \to I \times \{\tfrac{1}{2}\} = I \qquad (s,t) \mapsto (s, \tfrac{1}{2})$$

π_k continue $\because$ proiez quoz

$\Downarrow$?

$id_N \sim_{\mathbb{S}^1} i \circ r$

$$F: I^2 \times I \to I \qquad (s,t,u) \mapsto \left(s, \tfrac{u}{2} + (1-u)t\right)$$

$$id_{I^2} \sim_F j \circ \tilde{r}$$

$$\begin{tikzcd}
I^2 \arrow[r, "\tilde{r}"] \arrow[d, "\pi_1"] & I \arrow[l, bend right, "j"'] \arrow[d, "\pi_2"] \\
N \arrow[r, "r"] & \mathbb{S}^1 \arrow[l, bend left, "i"]
\end{tikzcd}$$

$$\begin{cases} F(s,t,0) = (s,t), & \forall (s,t) \in N \\ F(s,t,1) = (s, \tfrac{1}{2}) \in I, & \forall (s,t) \in N \\ F(s, \tfrac{1}{2}, u) = (s, \tfrac{1}{2}) \in I, & \forall s,u \in I \end{cases}$$