② $I/_{\{0,1\}} \cong \mathbb{S}^1 \Rightarrow \begin{cases} \pi_1(X,x) = \{[f] \mid f: \mathbb{S}^1 \to X,\ f \in C^0(\mathbb{S}^1),\ f(1,0) = x\} \\ f, g \in [f] \iff f \sim g \text{ rel } \{(1,0) \in \mathbb{S}^1\} \end{cases}$

$$\pi_1(X,x) := \{[\gamma] \mid \gamma: I \to X,\ \gamma(0) = \gamma(1) = x\}$$

$f$ equiv $g \iff f \sim g$ rel $\{0,1\} \subset I$, $\forall f, g \in [f]$

$$\Downarrow$$

$\gamma$ equiv $\delta \iff \gamma \sim \delta$ rel $\{(0,1) \in \mathbb{S}^1\}$, $\forall \gamma, \delta \in [\gamma]$

poiché $I/_{\{0,1\}} \cong \mathbb{S}^1$

Prop universale quozienti:

$$\left(0 \sim 1 \Rightarrow \gamma(0) = \gamma(1)\right) \Rightarrow \left(\exists f: \mathbb{S}^1 \to X \mid \gamma = f \circ \pi\right)$$

i.e. il diagramma seguente commuta:

$$\begin{tikzcd} I \arrow[r, "\gamma"] \arrow[d, "\pi"'] & X \\ I/_{\{0,1\}} \cong \mathbb{S}^1 \arrow[ur, "f"'] & \end{tikzcd}$$