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$$\left.\begin{array}{l} G := (G, \cdot, *) \\ a \cdot \varepsilon = \varepsilon \cdot a = a * \varepsilon = \varepsilon * a, \quad \forall a \in G \\ (a \cdot b) * (c \cdot d) = (a * c) \cdot (b * d), \quad \forall a, b, c, d \in G \end{array}\right\} \Longrightarrow \begin{cases} \cdot \equiv * \\ a \cdot b = b \cdot a, \quad \forall a, b \in G \end{cases} \Longrightarrow G \in Ab$$

(Eckmann-Hilton theorem)

$\forall a, b \in G$

$$\begin{aligned} a \cdot b &= (\varepsilon * a) \cdot (b * \varepsilon) \\ &= (\varepsilon \cdot b) * (a \cdot \varepsilon) \\ &= b * a \\ &= (b \cdot \varepsilon) * (\varepsilon \cdot a) \\ &= (b * \varepsilon) \cdot (\varepsilon * a) \\ &= b \cdot a \end{aligned}$$

$\longrightarrow \cdot \equiv *$

$\longrightarrow G \in Ab$