④ Esempio X connesso con $\pi_1(X,x)$ dipendente da $x \in X$

$$S = \mathbb{S}^1_{(-1,1)} = \{(x,y) \in \mathbb{R}^2 \mid (x+1)^2 + (y-1)^2 = 1\}$$

$$C = \left\{\left(\frac{1}{n}, y\right) \in \mathbb{R}^2 \mid y \in [0,1],\ n \in \mathbb{N} \setminus \{0\}\right\}$$

$$I_x = \{(x,0) \in \mathbb{R}^2 \mid x \in (0,1]\}$$

$$\left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \longrightarrow \begin{cases} \tilde{X} = C \cup I_x \\ X = S \cup \tilde{X} \end{cases}$$

X connesso $\because$ S e $\tilde{X}$ connessi

$X \neg$ cpa $\because$ $\nexists\, f : I \to X,\ f \in C^1(I,X) \mid f(0) \in S \wedge f(1) \in \tilde{X}$

$x \in S,\ y \in \tilde{X} \Rightarrow \pi_1(X,x) = \mathbb{Z} \neq \{e\} = \pi_1(X,y)$