$\textcircled{10}$ $\mathbb{T}^2 \setminus \{p\} \sim \infty \quad \wedge \quad \mathbb{T}^2 \setminus \{p\} \sim \mathbb{S}^2 \setminus \{x_1, x_2, x_3\}$

$X := \mathbb{T}^2 \setminus \{p\}, \quad p \in \mathbb{T}^2 \quad , \quad Y := \mathbb{S}^2 \setminus \{x_1, x_2, x_3\}, \quad x_1, x_2, x_3 \in \mathbb{S}^2$

$\infty := C_1 \cup C_2$

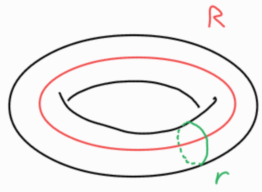
$C_1 := \{(x,y) \in \mathbb{R}^2 \mid (x-1)^2 + y^2 = 1\} = \{ (\cos(\pi(s+1)) + 1, \sin(\pi(s+1))) \in \mathbb{R}^2 \mid s \in [-1,1] \}$

$C_2 := \{(x,y) \in \mathbb{R}^2 \mid (x+1)^2 + y^2 = 1\} = \{ (\cos(\pi s) - 1, \sin(\pi s)) \in \mathbb{R}^2 \mid s \in [-1,1] \}$

$X \sim \infty \implies \exists\, f : X \to \infty,\ g : \infty \to X$ continue $\mid g \circ f \sim id_X \wedge f \circ g \sim id_\infty$

$\mathbb{T}^2 := \{(x,y,z) \in \mathbb{R}^3 \mid (x^2+y^2+z^2+R^2-r^2)^2 = 4R^2(x^2+y^2)\}$

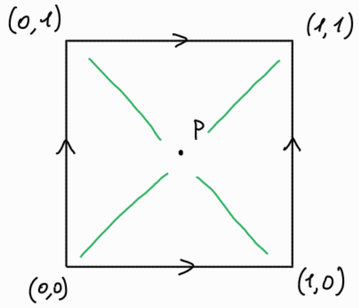
$= \{((R + r\cos\theta)\cos\varphi, (R+r\cos\theta)\sin\varphi, r\sin\theta) \in \mathbb{R}^3 \mid \theta, \varphi \in [0, 2\pi)\}$



$$\begin{array}{c} I^2 \\ \big\downarrow \pi \\ \mathbb{T}^2 \end{array}$$

$\mathbb{T}^2 = I^2/_\sim$ con $x \sim y \iff x = y \vee x_1 = y_1 + 1 \vee x_2 = y_2 + 1$

$X = (I^2/_\sim) \setminus \{p\}$



$r : I^2 \to \partial I^2 \quad \left(p = \left(\tfrac{1}{2}, \tfrac{1}{2}\right) \subset I^2 \setminus \partial I^2\right)$

$\tilde{r} : X \to \partial I^2/_\sim$

$r : I^2 \setminus \{p\} \to \partial I^2$

$\{x = y > 0\} \mapsto (1,1)$

$\{x = y < 0\} \mapsto (0,0)$

$\left\{\begin{array}{l} x = -y \\ x > 0 \end{array}\right\} \mapsto (1,0)$

$\left\{\begin{array}{l} x = -y \\ x < 0 \end{array}\right\} \mapsto (0,1)$

$\left\{\begin{array}{l} x > y \\ x > 1-y \end{array}\right\} \mapsto (1,y)$

$\left\{\begin{array}{l} x > y \\ x < 1-y \end{array}\right\} \mapsto (x,0)$

$\left\{\begin{array}{l} x < y \\ x > 1-y \end{array}\right\} \mapsto (x,1)$

$\left\{\begin{array}{l} x < y \\ x < 1-y \end{array}\right\} \mapsto (0,y)$

```tikzcd
I^2 \setminus \{p\} \arrow[r, "r"] \arrow[d, "\pi"] & \partial I^2 \arrow[l, bend right, "i"'] \arrow[d, "\tilde{\pi}"] \\
I^2 \setminus \{p\}/_\sim \arrow[r, "\tilde{r}"] & \partial I^2/_\sim \arrow[l, bend left, "j"]
```

$\partial I^2 \subseteq I^2 \setminus \{p\}$ retratto $\because$ $r \circ i = id_{\partial I^2}$

$A \subseteq X$ retratto di X se $\exists\, r : X \to A$ continua $\mid r \circ i = id_A$, $i : A \hookrightarrow X$ inclusione

$\partial I^2/_\sim \subseteq I^2 \setminus \{p\}/_\sim$ retratto se $\exists\, \tilde{r} : I^2\setminus\{p\}/_\sim \longrightarrow \partial I^2/_\sim$ continua |

$\tilde{r} \circ j = id_{\partial I^2/_\sim}$, $j : \partial I^2/_\sim \hookrightarrow I^2\setminus\{p\}/_\sim$ inclusione

$\tilde{r} : I^2\setminus\{p\}/_\sim \longrightarrow \partial I^2/_\sim$

$[x] \longmapsto [r(x)]$

$\tilde{r} \circ \pi = \tilde{\pi} \circ r \quad \wedge \quad \pi = j \circ \tilde{\pi} \circ r$

$\tilde{r} \circ \pi \circ i = \tilde{\pi} \circ \underbrace{r \circ i}$

$\tilde{r} \circ \overbrace{j \circ \tilde{\pi} \circ r}^{} \circ\, i$

$\tilde{r} \circ j \circ \tilde{\pi} \circ \underbrace{r \circ i}_{id_{\partial I^2}} = \tilde{\pi} \circ id_{\partial I^2}$

$\tilde{r} \circ j \circ \tilde{\pi} = \tilde{\pi} \implies \tilde{r} \circ j = id_{\partial I^2/_\sim}$

$X := \dfrac{I^2\setminus\{p\}}{\sim} \sim \dfrac{\partial I^2}{\sim} \equiv S^1 \vee S^1 \cong \infty$ ✓

$S^n \setminus \{N\} \cong \mathbb{R}^n \implies S^2 \setminus\{N\} \cong \mathbb{R}^2$

$\Downarrow$

$\infty \sim \mathbb{R}^2 \setminus \{a,b\} \implies S^2 \setminus \{N, p, q\} \cong \mathbb{R}^2 \setminus \{a,b\} \sim \infty$ ✓