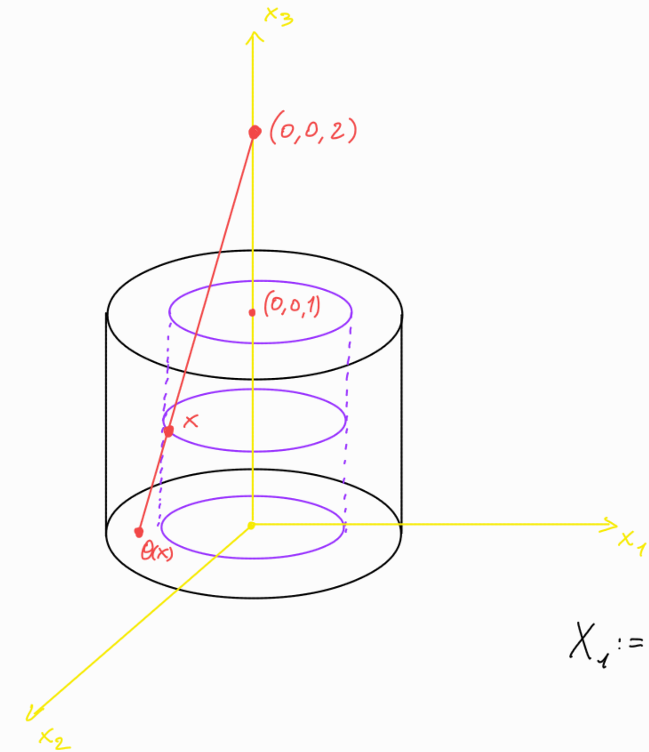
(14) $D^2\times\{0\}\cup S^1\times I$ retratto forte def di $D^2\times I$

$A = D^2\times\{0\}\cup S^1\times I\subset\mathbb{R}^3$

$X = D^2\times I\subseteq\mathbb{R}^3$

$A\subseteq X$ retratto forte deform $\iff \underset{\text{continua}}{\exists F: X\times I\to X} \;\Big|\; \begin{cases} F(x,0)=x \\ F(x,1)\in A, & \forall x\in X \\ F(a,t)=a, & \forall a\in A, \forall t\in I\end{cases}$



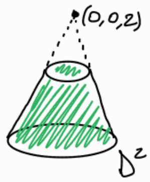
$F: X\times I\to X$

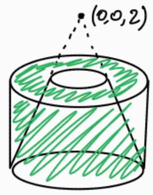
$(x,t)\longmapsto(1-t)x+t\,\theta(x)$

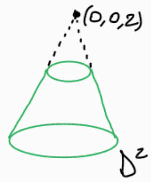
$\theta(x)$: intersezione tra A,

e linea tra $(0,0,2)$ e $x\in X$

$A = D^2\times\{0\}\cup S^1\times I := A_1\cup A_2$

$X_1 := \theta^{-1}(A_1) =$ 

$X_2 := \theta^{-1}(A_2) =$ 

$X_1\cap X_2 = \theta^{-1}(A_1)\cap\theta^{-1}(A_2) =$ 

$\theta|_{X_1}(X_1\cap X_2) = \theta|_{X_2}(X_1\cap X_2) \iff \theta$ continua

$X_1 = \{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2\leq(1-\frac{z}{2})^2,\ z\in[0,1]\}$

$\theta(x,y,z) = (0,0,2)+t\underbrace{((x,y,z)-(0,0,2))}_{\text{vettore direzione retta}} = (tx,\, ty,\, t(z-2)+2)$

dovrebbe essere $2t(z-2)$

$\theta(x,y,z)\in D^2\times\{0\} \Rightarrow \boxed{} \Rightarrow t^2(x^2+y^2)\leq 1 \Rightarrow x^2+y^2\leq\frac{1}{t^2} = (1-\frac{z}{2})^2$

$\Rightarrow t = \frac{2}{z-2}$

$$\theta(x,y,z) = (0,0,2) + \frac{2}{2-z}\Big((x,y,z)-(0,0,2)\Big) = (0,0,2) + \left(\frac{2x}{2-z}, \frac{2y}{2-z}, -2\right) = \left(\frac{2x}{2-z}, \frac{2y}{2-z}, 0\right)$$

$$= \frac{2}{2-z}(x,y,0)$$

$$X_2 = \left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, \left(1-\frac{z}{2}\right)^2 \leq x^2+y^2 \leq 1,\ z \in [0,1]\right\}$$

$$\theta(x,y,z) = (0,0,2) + t\Big((x,y,z)-(0,0,2)\Big) = \Big(tx, ty, 2+t(z-2)\Big)$$

$$\theta(x,y,z) \in S^1 \times I \Rightarrow (tx)^2 + (ty)^2 = 1 \Rightarrow t^2(x^2+y^2) = 1 \Rightarrow t = \frac{1}{\sqrt{x^2+y^2}}$$

$$\theta(x,y,z) = \frac{1}{\sqrt{x^2+y^2}}(x,y,z-2) + (0,0,2)$$

Dunque

$$\theta(x,y,z) = \begin{cases} \frac{2}{2-z}(x,y,0), & x \in X_1 \\[2ex] \frac{1}{\sqrt{x^2+y^2}}(x,y,z-2) + (0,0,2), & x \in X_2 \end{cases}$$

$$X_1 \cap X_2 = \left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, x^2+y^2 = \left(1-\frac{z}{2}\right)^2,\ z \in [0,1]\right\}$$

$$x^2+y^2 = \left(1-\frac{z}{2}\right)^2 \Rightarrow \sqrt{x^2+y^2} = \frac{2-z}{2} \Rightarrow \frac{1}{\sqrt{x^2+y^2}}(x,y,z-2) + (0,0,2) = \frac{2}{2-z}(x,y,0)$$

$$\Rightarrow \quad \theta(x,y,z) \text{ continua} \quad \Rightarrow \quad F(x,t) = (1-t)x + t\,\theta(x) \text{ continua}$$

$$F(x,1) = \theta(x) \in A, \quad \forall x \in X$$

$$F(a,t) = (1-t)a + t\,\theta(a)$$

$$a = (a_1,a_2,a_3) \in D^2 \times \{0\}: \ F(a,t) = (1-t)(a_1,a_2,a_3) + t\,\frac{2}{2-a_3}(a_1,a_2,0)$$

$$\Rightarrow \begin{cases} a_3 = 0 \\ a_1^2 + a_2^2 \leq 1 \end{cases} \qquad = \left((1-t)a_1 + \frac{2ta_1}{2-a_3}, (1-t)a_2 + \frac{2ta_2}{2-a_3}, (1-t)a_3\right)$$

$$= \Big((1-t)a_1 + ta_1, (1-t)a_2 + ta_2, 0\Big)$$

$$= (a_1, a_2, 0)$$

$$= a$$

$$a = (a_1, a_2, a_3) \in S^1 \times I : \quad F(a,t) = (1-t)(a_1, a_2, a_3) + t\left(\cancelto{1}{\frac{1}{\sqrt{a_1^2+a_2^2}}}(a_1, a_2, a_3 - 2) + (0,0,2)\right)$$

$$\Rightarrow \begin{cases} a_1^2 + a_2^2 = 1 \\ a_3 \in [0,1] \end{cases}$$

$$= \left((1-t)\,a_1 + t a_1,\ (1-t)\,a_2 + t a_2,\ (1-t)\,a_3 + t(a_3 - 2 + 2)\right)$$

$$= (a_1, a_2, a_3)$$

$$= a$$