1. Verifica la topologia dei compatto-aperti

$$C(X,Y) = \{ f : X \to Y \text{ continua} \}$$

$$S(K,W) = \{ f \in C(X,Y) \mid f(K) \subseteq W \} \subset C(X,Y) \quad \text{aperto}$$

$$\mathcal{S} = \{ S(K,W) \subset C(X,Y) \mid K \subset X \text{ compatto} \wedge W \subseteq Y \text{ aperto} \} \cup C(X,Y) \cup \emptyset$$

$$U \subseteq C(X,Y) \text{ aperto} \iff \begin{cases} U = C(X,Y) \\ U = \emptyset \\ \forall f \in U,\ \exists S_1, \dots, S_k \in \mathcal{S} \mid f \in \bigcap_{j=1}^{k} S_j \subseteq U \end{cases}$$

Intersezione di due aperti come aperto:

$U, V \subset C(X,Y)$, $U \cap V$ aperto

$$\forall f \in U, \forall g \in V,\ \exists S_1, \dots, S_p, S_{p+1}, \dots, S_q \in \mathcal{S} \ \Big| \ f \in \bigcap_{j=1}^{p} S_j \subseteq U \wedge g \in \bigcap_{j=p+1}^{q} S_j \subseteq V$$

$$\Rightarrow \forall h \in U \cap V,\ \exists S_m, \dots, S_n \in \{ S_1, \dots, S_p, S_{p+1}, \dots, S_q \} \subseteq \mathcal{S} \ \Big| \ h \in \bigcap_{j=m}^{n} S_j \subseteq U \cap V$$

$$\Rightarrow U \cap V \in \tau(C(X,Y))$$

Unione di aperti come aperto:

$$\forall h \in \bigcup_{j \in J} U_j,\ \exists S_{j,1}, \dots, S_{j,p} \in \mathcal{S} \ \Big| \ h \in \bigcup_{j \in J} \left(\bigcap_{q=1}^{p} S_{j,q} \right) \subseteq \bigcap_{q=1}^{p} \underbrace{\left(\bigcup_{j \in J} S_{j,q} \right)}_{:= \tilde{S}_q \in \mathcal{S} \text{ aperto} \because \text{unione di aperti}}$$

$$\Rightarrow U \cup V \in \tau(C(X,Y))$$