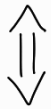


$$\textcircled{8} \left. \begin{array}{l} p: \tilde{X} \rightarrow X \text{ rivest} \\ Y \subseteq \tilde{X} \text{ comp conn} \\ X \text{ conn} + \text{loc conn} \end{array} \right\} \Rightarrow p|_Y: Y \rightarrow X \text{ rivest}$$

$$X \text{ loc conn} \stackrel{\text{def}}{\iff} \forall x \in X, \forall U \ni x \text{ aperto}, \exists V \text{ connesso} \mid x \in V \subset U$$

$$p: \tilde{X} \rightarrow X \text{ rivest}$$

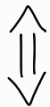


$$\textcircled{1} p \text{ suriettiva} \Rightarrow p(\tilde{X}) = X$$

$$\textcircled{2} \forall x \in X, \exists U \subseteq X \text{ aperto}, U \ni x \wedge \exists \{\tilde{U}_j\}_{j \in J} \subseteq \tilde{X} \text{ aperti} \mid$$

$$\left\{ \begin{array}{l} p^{-1}(U) = \bigsqcup_{j \in J} \tilde{U}_j \\ \tilde{U}_j \cap \tilde{U}_k = \emptyset, \forall j, k \in J, j \neq k \\ p|_{\tilde{U}_j}: \tilde{U}_j \rightarrow U \text{ omeomorfismo } \forall j \in J \end{array} \right.$$

$$p|_Y: Y \rightarrow X \text{ rivest}$$



$$\textcircled{1} p|_Y \text{ suriettiva} \Rightarrow p(Y) = X$$

$$\textcircled{2} \forall x \in X, \exists U \subseteq X \text{ aperto}, U \ni x \wedge \exists \{\tilde{U}_j\}_{j \in J} \subseteq Y \text{ aperti} \mid$$

$$\left\{ \begin{array}{l} p^{-1}(U) = \bigsqcup_{j \in J} \tilde{U}_j \\ \tilde{U}_j \cap \tilde{U}_k = \emptyset, \forall j, k \in J, j \neq k \\ p|_{\tilde{U}_j}: \tilde{U}_j \rightarrow U \text{ omeomorfismo } \forall j \in J \end{array} \right.$$

X loc conn $\Rightarrow \exists U \subset X$ aperto + conn $\mid x \in U \subseteq V, \forall x \in X, \forall V \subset X$ aperto

Sia $\forall x \in U \subseteq p(Y) \Rightarrow p^{-1}(x) \cap Y \neq \emptyset \Rightarrow p^{-1}(U) \cap Y \neq \emptyset$

connesso $\because (U \text{ connesso} \wedge p \text{ cont})$

p rivest $\Rightarrow \begin{cases} p^{-1}(U) = \bigsqcup_{j \in \mathcal{J}} \tilde{U}_j \\ U \cong^p \tilde{U}_j, \forall j \in \mathcal{J} \end{cases} \longrightarrow \tilde{U}_j \cap Y \neq \emptyset, j \in \mathcal{J}' \subseteq \mathcal{J}$

consideriamo solo quelli che intersecano Y

$\left. \begin{array}{l} Y \subseteq \tilde{X} \text{ comp conn} \\ \tilde{U}_j \text{ connessi} \end{array} \right\} \Rightarrow (\tilde{U}_j \subseteq Y \vee \tilde{U}_j \not\subseteq Y)$

$\left. \begin{array}{l} \tilde{U}_j \cap Y \neq \emptyset \\ (\tilde{U}_j \subseteq Y \vee \tilde{U}_j \not\subseteq Y) \end{array} \right\} \Rightarrow \tilde{U}_j \subseteq Y$

questi fungono da preimmagine per $p|_Y$ quindi la ② della definizione di rivestimento è verificata

$\tilde{U}_j \subseteq Y \Rightarrow p(\tilde{U}_j) = U \subset p(Y), \forall x \in p(Y) \Rightarrow p(Y)$ aperto

Invece sia $x \notin p(Y), x \in U \subset X \setminus p(Y)$

se $\tilde{U}_i \cap Y \neq \emptyset$ per qualche $i \in I \Rightarrow \tilde{U}_i \subset Y$

$\Rightarrow x \in U = p(\bigsqcup_{i \in I} \tilde{U}_i) \subset p(Y)$

contraddizione

Dunque $\tilde{U}_i \cap Y = \emptyset \Rightarrow X \setminus p(Y) = \bigcup_{k \in K} U_k$ aperti $\Rightarrow X \setminus p(Y)$ aperto $\Rightarrow p(Y)$ chiuso

$p(Y)$ aperto+chiuso in X connesso $\Rightarrow p(Y)=X \Rightarrow p$ suriettiva ① ✓

