6a) $f, g : I \to X$, $f(0) = g(0) = x$, $f(1) = g(1) = y$, $u_{f,g} : \pi_1(X,x) \to \pi_1(X,y)$, $u_f \equiv u_g$

$$\Updownarrow$$

$$[g \cdot i(f)] \in Z(\pi_1(X,x))$$

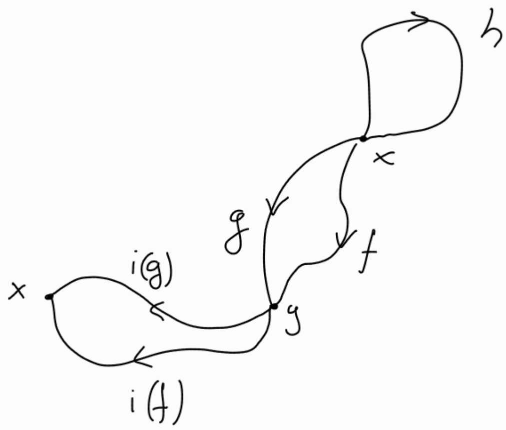
G gruppo, $Z(G) = \{ z \in G \mid \forall g \in G, zg = gz \}$

$$u_f : \pi_1(X,x) \to \pi_1(X,y) \qquad [h] \mapsto [i(f) \cdot h \cdot f]$$

$$u_g : \pi_1(X,x) \to \pi_1(X,y) \qquad [h] \mapsto [i(g) \cdot h \cdot g]$$

$\Leftarrow$

$$
\begin{aligned}
u_g([h]) &= [i(g) \cdot h \cdot g] \\
&= [i(g) \cdot \overbrace{f \cdot i(f)}^{\varepsilon_x} \cdot h \cdot \overbrace{f \cdot i(f)}^{\varepsilon_x} \cdot g] \\
&= [i(g) \cdot f] \cdot [i(f) \cdot h \cdot f] \cdot [i(f) \cdot g] \\
&= [i(g) \cdot f] \cdot [i(f) \cdot g] \cdot [i(f) \cdot h \cdot f] \\
&= [i(g) \cdot f \cdot i(f) \cdot g] \cdot [i(f) \cdot h \cdot f] \\
&= \overbrace{[i(g) \cdot g]}^{[\varepsilon_y]} \cdot [i(f) \cdot h \cdot f] \\
&= [i(f) \cdot h \cdot f] \\
&= u_g([h])
\end{aligned}
$$



$\Rightarrow$

$$u_g([h]) = u_f([h])$$

$$[i(g) \cdot h \cdot g] = [i(f) \cdot h \cdot f]$$

$$[\overbrace{g \cdot i(g)}^{\varepsilon_x} \cdot h \cdot g \cdot i(f)] = [g \cdot i(f) \cdot h \cdot \overbrace{f \cdot i(f)}^{\varepsilon_x}]$$

$$[h \cdot g \cdot i(f)] = [g \cdot i(f) \cdot h], \ \forall h \in C^1(I,X) \Rightarrow [g \cdot i(f)] \in Z(\pi_1(X,x))$$

$$[f_0] = [f_1] \wedge [g_0] = [g_1]$$

$$\Downarrow$$

$$[f_0 \cdot g_0] = [f_1 \cdot g_1]$$

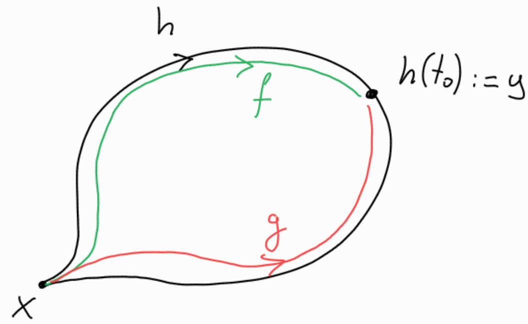
$⑥b$ $u_f : \pi_1(X,x) \to \pi_1(X,y)$ indipendente da $f \iff \pi_1(X,x)$ abeliano

$$\pi_1(X,x) \in Ab \iff [f]\cdot[g] = [g]\cdot[f], \quad \forall [f],[g] \in \pi_1(X,x)$$

$$h : I \to X \qquad t_0 \in I,\ h(t_0) = y \in X$$

$$\exists\, f,g : I \to X,\ f,g \in C^1(I,X),\ f(0) = g(0) = h(0) = h(1) := x,\ f(1) = g(1) = h(t_0) := y$$

$$h = f \cdot i(g), \quad \forall h \in [h] \in \pi_1(X,x)$$



$$\Rightarrow \quad \pi_1(X,x) \subseteq Z(\pi_1(X,x))$$

siccome $Z(\pi_1(X,x)) \subseteq \pi_1(X,x)$

$$\Rightarrow \pi_1(X,x) = Z(\pi_1(X,x)) \Rightarrow \pi_1(X,x) \in Ab$$