② $X \sim Y$, $f: X \longrightarrow Y$ omotopia, $g: Y \longrightarrow X$, $f \circ g \sim id_Y \wedge g \circ f \sim id_X$ inversa omotopica. Dimostra g unica a meno di omotopie

Suppongo $h: Y \longrightarrow X \mid f \circ h \sim id_Y \wedge h \circ f \sim id_X$

$$g \circ f \sim id_X$$
$$g \circ f \circ h \sim id_X \circ h$$
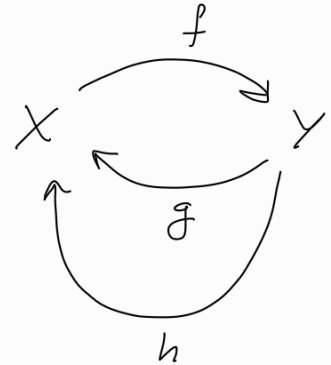$$g \circ (f \circ h) \sim h$$
$$g \sim h$$

$$F: X \times I \longrightarrow X$$
$$\begin{cases} F(x,0) = (g \circ f)(x) \\ F(x,1) = id_X(x) = x \end{cases}$$

$$(g \circ f \circ h)(y)$$

$$(g \circ f)(x) = \tilde{x}$$
$$\begin{cases} h(\tilde{y}) = \tilde{x} \\ (g \circ f \circ h)(y) = x \end{cases}$$

$g \circ f \sim id_X \wedge h \circ f \sim id_X \implies$ $g \circ f \circ h \circ f \sim \overbrace{id_X \circ id_X}^{\sim id_Y}$

$$g \circ id_Y \circ f \sim id_X$$
$$g \circ f \sim id_X$$

$$g \circ f \sim id_X$$
$$g \circ f \sim h \circ f$$