(12) $SL(n,\mathbb{R})$ retratto forte deform di $GL^+(n,\mathbb{R})$

$A \subseteq X$ retr forte def di X se $\exists\, r : X \to A \mid i \circ r \sim_A id_X$

oppure se e solo se $\exists\, F : X \times I \to X \mid F(x,0) = x,\ F(x,1) \in A,\ \forall x \in X \ \wedge\ F(a,t) = a,\ \forall a \in A,\ \forall t \in I$

$$GL(n,\mathbb{R}) = \{ M \in M(n,\mathbb{R}) \mid \det M \neq 0 \}$$

$$SL(n,\mathbb{R}) = \{ M \in GL(n,\mathbb{R}) \mid \det M = 1 \}$$

$$GL^+(n,\mathbb{R}) = \{ M \in GL(n,\mathbb{R}) \mid \det M > 0 \}$$

$$r : GL^+(n,\mathbb{R}) \longrightarrow SL(n,\mathbb{R})$$
$$M \longmapsto (\det M)^{-1/n} M$$

$$\det r(M) = \det\left((\det M)^{-1/n} M \right)$$
$$= \left((\det M)^{-1/n} \right)^n \det M$$
$$= (\det M)^{-1} \det M$$
$$= 1$$

$$i : SL(n,\mathbb{R}) \hookrightarrow GL^+(n,\mathbb{R})$$

$$(i \circ r)(M) = i\left((\det M)^{-1/n} M \right) = (\det M)^{-1/n} M \in GL^+(n,\mathbb{R})$$

$$F : GL^+(n,\mathbb{R}) \times I \longrightarrow GL^+(n,\mathbb{R})$$
$$(M,t) \longmapsto (\det M)^{-t/n} M$$

$$\det F(M,t) = \det\left((\det M)^{-t/n} M \right)$$
$$= \underbrace{(\det M)^{-t}}_{>0,\ \forall t \in I} \overbrace{\det M}^{>0}$$
$$\Longrightarrow (\det M)^{-t/n} M \in GL^+(n,\mathbb{R})$$

$$F(M,0) = M = id_{GL(n,\mathbb{R})}(M)$$

$$F(M,1) = (\det M)^{-1/n} M = r(M) \in SL(n,\mathbb{R}),\quad \forall M \in GL(n,\mathbb{R})$$

Sia $N \in SL(n,\mathbb{R}) \Rightarrow \det N = 1$

$$F(N,t) = \underline{(\det N)}^{-t/n} N = N,\quad \forall N \in SL(n,\mathbb{R}),\ \forall t \in I$$

$= 1$