④ X sp top contraibile $\iff$ $id_X \sim \varepsilon$

X contraibile sse $X \sim \{x_0\}$

$$X \sim \{x_0\} \iff \exists\ \pi : X \to \{x_0\},\ i : \{x_0\} \to X \ \middle|\ \pi \circ i \sim id_{\{x_0\}} \wedge i \circ \pi \sim id_X$$

$$\begin{aligned} \pi : X &\to \{x_0\} \\ x &\mapsto x_0 \end{aligned} \qquad \begin{aligned} i : \{x_0\} &\to X \\ x_0 &\mapsto x_0 \end{aligned}$$

$$\begin{tikzcd} X \arrow[r, bend left, "\varepsilon"] & \{x_0\} \arrow[l, bend left, "i"] \end{tikzcd}$$

X contraibile $\Rightarrow X \sim \{x_0\}$

$$\Rightarrow \exists\ \pi : X \to \{x_0\},\ i : \{x_0\} \to X \ \middle|\ \pi \circ i \sim id_{\{x_0\}} \wedge i \circ \pi \sim id_X$$

$$\Rightarrow \exists F : X \times I \to X \ \middle|\ \begin{cases} F(x,0) = id_X(x) \\ F(x,1) = (i \circ \pi)(x) \end{cases} \quad \forall x \in X$$

ma $i \circ \pi = \varepsilon$ costante $\begin{aligned} \varepsilon : X &\to X \\ x &\mapsto x_0 \end{aligned}$

$\Rightarrow id_X \sim \varepsilon$

$$id_X \sim \varepsilon \quad \Rightarrow \quad \exists F : X \times I \to X \ \middle|\ \begin{cases} F(x,0) = id_X(x) = x \\ F(x,1) = \varepsilon(x) = x_0 \end{cases} \quad \forall x \in X$$

$\Rightarrow$ $\varepsilon = i \circ \pi$ definite sopra $\mid\ \pi \circ i \sim id_{\{x_0\}} \wedge i \circ \pi \sim id_X$

$\Rightarrow X \sim \{x_0\}$

$\Rightarrow$ X contraibile