$$\begin{cases}
\rho: \hat{\chi} \to \chi \\
q: \hat{\gamma} \to \gamma
\end{cases}$$
(ivestinent)  $\Rightarrow p \times q: \hat{\chi} \times \hat{\gamma} \to \chi \times \gamma \\
(\hat{\chi}, \hat{\gamma}) \mapsto (p(\hat{\chi}), q(\hat{\gamma}))$ 
(ivestinento

$$P: \widetilde{X} \longrightarrow X$$
 rivest

$$(1)$$
 p suriettiva  $=> p(\tilde{X}) = X$ 

2 
$$\forall x \in X$$
,  $\exists U \subseteq X$  aperto,  $U \ni x \land \exists \{\widehat{U}_j\}_{j \in S} \subseteq \widehat{X}$  aperti

$$\begin{cases} P^{-1}(U) = \bigcup_{j \in S} \widetilde{U}_{j} \\ \widetilde{U}_{j} \cap \widetilde{U}_{k} = \emptyset, \ \forall j, k \in J, \ j \neq k \end{cases}$$

$$\begin{cases} P|_{\widetilde{U}_{j}} : \widetilde{U}_{j} \longrightarrow U \text{ one one of is no } \forall j \in J \end{cases}$$

i.e. 
$$(p \times q)(\widetilde{X} \times \widetilde{Y}) := p(\widetilde{X}) \times q(\widetilde{Y}) = X \times Y$$

$$\forall (x,y) \in X \times Y, \exists U \times V \subseteq X \times Y \text{ aperto, } U \times V \ni (x,y) \land \exists \left\{ \widetilde{U}_{j} \times \widetilde{V}_{j} \right\}_{j \in S} \subseteq \widetilde{X} \times \widetilde{Y} \text{ aperti}$$

$$\left( (p \times q)^{-1} \left( (j \times V) \right) = \bigcup_{j \in J} (\widetilde{U}_j \times \widetilde{V})$$

$$(\widetilde{U}_{j} \times \widetilde{V}_{j}) \wedge (\widetilde{U}_{k} \times \widetilde{V}_{k}) = \emptyset, \forall j, k \in \mathcal{J}, j \neq k$$

$$\left| \left\langle (P \times q)^{-1} (U \times V) = \bigsqcup_{j \in \mathbb{J}} \widetilde{U}_{j} \times \widetilde{V} \right\rangle \right| \left\langle (\widetilde{U}_{j} \times \widetilde{V}_{j}) \wedge (\widetilde{U}_{k} \times \widetilde{V}_{k}) = \emptyset, \ \forall j, k \in \mathbb{J}, \ j \neq k \right|$$

$$\left| P \times q \right|_{\widetilde{U}_{j} \times \widetilde{V}_{j}} = P \left| \sum_{\widetilde{U}_{j} \times q} \times q \right|_{\widetilde{V}_{j}} : \widetilde{U}_{j} \times \widetilde{V}_{j} \longrightarrow U \times V \quad \text{oneomorfism'} \quad \forall j \in \mathbb{J}$$