⑧ Retratto di contraibile $\Rightarrow$ contraibile

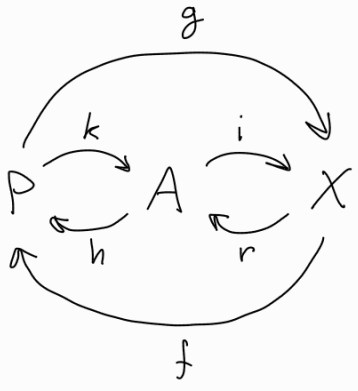
$A \subseteq X$ retratto di X se $\exists\, r: X \to A$ continua $\mid r \circ i = id_A$, $i: A \hookrightarrow X$ inclusione

X contraibile $\iff X \sim P = \{x_0\},\ x_0 \in X \iff \exists\, f: X \to P,\ g: P \to X \mid \begin{matrix} f \circ g \sim id_P \\ g \circ f \sim id_X \end{matrix}$

$r \circ i = id_A \Rightarrow r$ suriettiva $\Rightarrow r(X) = A$

Siccome $P = \{x_0\}$ ha solo un punto $\quad f \circ g \sim id_P \longrightarrow f \circ g = id_P$

A contraibile $\iff A \sim P,\ x_0 \in A \subseteq X \iff \exists\, h: A \to P,\ k: P \to A \mid \begin{matrix} h \circ k = id_P \\ k \circ h \sim id_A \end{matrix}$



$$\begin{cases} k = r \circ g \\ h = f \circ i \end{cases}$$

$$\begin{cases} h \circ k = \overbrace{h \circ r}^{f} \circ g = f \circ g = id_P \\ k \circ h = (r \circ g) \circ (f \circ i) = r \circ (g \circ f) \circ i \sim r \circ id_X \circ i = r \circ i = id_A \end{cases}$$