1a $f, g: I \to X,\ f(1) = g(0),\ k \in C(X,Y) \implies k \circ (f \cdot g) = (k \circ f) \cdot (k \circ g)$

$$(f \cdot g)(t) := \begin{cases} f(2t), & t \in [0, \frac{1}{2}] \\ g(2t-1), & t \in [\frac{1}{2}, 1] \end{cases}$$

$$\big(k \circ (f \cdot g)\big)(t) = \begin{cases} k\big(f(2t)\big), & t \in [0, \frac{1}{2}] \\ k\big(g(2t-1)\big), & t \in [\frac{1}{2}, 1] \end{cases}$$

$$= \begin{cases} (k \circ f)(2t), & t \in [0, \frac{1}{2}] \\ (k \circ g)(2t-1), & t \in [\frac{1}{2}, 1] \end{cases}$$

$$= \big((k \circ f) \cdot (k \circ g)\big)(t), \qquad \forall t \in I$$

1a $f: I \to X,\ k \in C(X,Y) \implies k \circ i(f) = i(k \circ f)$

$$i(k \circ f)(t) = (k \circ f)(1-t)$$
$$= k\big(f(1-t)\big)$$
$$= k\big(i(f)(t)\big)$$
$$= \big(k \circ i(f)\big)(t), \qquad \forall t \in I$$

1a $f, g: I \to X,\ f \sim_{\{0,1\},F} g,\ k \in C(X,Y) \implies k \circ f \sim_{\{0,1\},k \circ F} k \circ g$

$$f \sim_{\{0,1\},F} g \iff \exists F: I \times I \to X \ \Big| \ \begin{cases} F(t,0) = f(t) \\ F(t,1) = g(t) \\ F(0,s) = f(0), & \forall s \in I \\ F(1,s) = f(1), & \forall s \in I \end{cases} \implies \begin{cases} f(0) = g(0) \\ f(1) = g(1) \end{cases}$$

$$\begin{cases} F(t,0) = f(t) \\ F(t,1) = g(t) \end{cases} \implies \begin{cases} k\big(F(t,0)\big) = k\big(f(t)\big) \\ k\big(F(t,1)\big) = k\big(g(t)\big) \end{cases}$$

$$\begin{cases} F(0,s) = f(0), & \forall s \in I \\ F(1,s) = f(1), & \forall s \in I \end{cases} \implies \begin{cases} k(F(0,s)) = k(f(0)), & \forall s \in I \\ k(F(1,s)) = k(f(1)), & \forall s \in I \end{cases}$$

$$\implies \begin{cases} (k \circ F)(t,0) = (k \circ f)(t) \\ (k \circ F)(t,1) = (k \circ g)(t) \\ (k \circ F)(0,s) = (k \circ f)(0), & \forall s \in I \\ (k \circ F)(1,s) = (k \circ f)(1), & \forall s \in I \end{cases} \implies \begin{cases} (k \circ f)(0) = (k \circ g)(0) \\ (k \circ f)(1) = (k \circ g)(1) \end{cases}$$

Quindi consideriamo l'omotopia $k \circ F : I \times I \to Y$ da cui $k \circ f \sim_{\{0,1\},\, k \circ F} k \circ g$