① Esempio 1.1.2 → $f_0 \not\simeq f_1$

$$\mathbb{S}^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 = 1\} \qquad Y = \{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2+y^2 \le 4\}$$

$$f_0 : \mathbb{S}^1 \to Y$$
$$(\cos t, \sin t) \mapsto \tfrac{3}{2}(\cos t, \sin t)$$

↳ circonferenza centro $(0,0)$ raggio $\frac{3}{2}$

$$f_1 : \mathbb{S}^1 \to Y$$
$$(\cos t, \sin t) \mapsto \tfrac{1}{5}(\cos t + \tfrac{35}{4}, \sin t)$$

sbagliato nel testo

↳ circonferenza centro $(\frac{7}{4}, 0)$ raggio $\frac{1}{5}$

$\pi_1(\mathbb{S}^1) = \pi_1(Y) = \mathbb{Z}$ ma $\pi_1(f_0(\mathbb{S}^1)) = \mathbb{Z} \neq \{e\} = \pi_1(f_1(\mathbb{S}^1))$

sufficiente?

$F : \mathbb{S}^1 \times I \to Y$ continua

$$\begin{cases} F((x,y),0) = f_0 \\ F((x,y),1) = f_1 \end{cases}$$

$f_0(\mathbb{S}^1) \sim f$ con $f : I \to ?$

$f_1(\mathbb{S}^1) \sim \varepsilon_Y$

Esempio 1.2.2 → $f_0 \not\simeq_A f_1$

$$Y = \{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2+y^2 \le 4\}$$

$$A = \{0,1\} \subset I$$

$$f_0 : I \to Y$$
$$t \mapsto \tfrac{3}{2}(\cos(\pi t), \sin(\pi t))$$

senso antiorario

$$f_1 : I \to Y$$
$$t \mapsto \tfrac{3}{2}(\cos(\pi t), -\sin(\pi t))$$

senso orario

circonferenze centro $(0,0)$ raggio $\frac{3}{2}$

$$\left((1-s)f_0(t) + s f_1(t)\right)$$

$F : I^2 \longrightarrow Y$

$$\frac{3}{2}\left((1-s)\cos(\pi t) + s\cos(\pi t),\ (1-s)\sin\pi t - s\sin(\pi t)\right)$$

$$\begin{cases} F(0,t) = f_0(t) \\ F(1,t) = f_1(t) \end{cases}$$

$$\frac{3}{2}\left(\cos(\pi t),\ (1-2s)\sin(\pi t)\right)$$

$s = \frac{1}{2}$

$F(s,0) = F(s,1) = \left(\frac{3}{2}, 0\right)$

$$\left\{\left(\frac{3}{2}\cos(\pi t), 0\right) \in \mathbb{R}^2 \;\middle|\; t \in I\right\} \not\subset Y$$