8 $\left.\begin{array}{l} p:\tilde{X}\to X \text{ rivest} \\ Y\subseteq\tilde{X} \text{ comp conn} \\ X \text{ conn + loc conn}\end{array}\right\} \Rightarrow p|_Y : Y\to X \text{ rivest}$

X loc conn $\overset{\text{def}}{\iff} \forall x\in X,\ \forall U\ni x$ aperto, $\exists V$ connesso $\mid x\in V\subset U$

$p:\tilde{X}\to X$ rivest

$\Updownarrow$

1. p suriettiva $\Rightarrow p(\tilde{X}) = X$
2. $\forall x\in X,\ \exists U\subseteq X$ aperto, $U\ni x \wedge \exists\{\tilde{U}_j\}_{j\in J}\subseteq\tilde{X}$ aperti $\Bigg|$

$$\left|\ \begin{cases} p^{-1}(U) = \bigsqcup_{j\in J}\tilde{U}_j \\ \tilde{U}_j\cap\tilde{U}_k = \emptyset, & \forall j,k\in J,\ j\neq k \\ p|_{\tilde{U}_j} : \tilde{U}_j\to U \text{ omeomorfismo} & \forall j\in J\end{cases}\right.$$

$p|_Y : Y\to X$ rivest

$\Updownarrow$

1. $p|_Y$ suriettiva $\Rightarrow p(Y) = X$
2. $\forall x\in X,\ \exists U\subseteq X$ aperto, $U\ni x \wedge \exists\{\tilde{U}_j\}_{j\in J}\subseteq Y$ aperti $\Bigg|$

$$\left|\ \begin{cases} p^{-1}(U) = \bigsqcup_{j\in J}\tilde{U}_j \\ \tilde{U}_j\cap\tilde{U}_k = \emptyset, & \forall j,k\in J,\ j\neq k \\ p|_{\tilde{U}_j} : \tilde{U}_j\to U \text{ omeomorfismo} & \forall j\in J\end{cases}\right.$$

$(p|_{\tilde{U}_j}: \tilde{U}_j \to U$ omeomorfismo $\forall j \in J$

X loc conn $\Rightarrow \exists U \subset X$ aperto + conn $\mid x \in U \subseteq V,\ \forall x \in X,\ \forall V \subset X$ aperto

Sia $\forall x \in U \subseteq p(Y) \Rightarrow p^{-1}(x) \cap Y \neq \emptyset \Rightarrow p^{-1}(U) \cap Y \neq \emptyset$

connesso :: (U connesso $\wedge$ p cont)

$$p \text{ rivest} \Rightarrow \begin{cases} p^{-1}(U) = \bigsqcup_{j \in J} \tilde{U}_j \longrightarrow \tilde{U}_j \cap Y \neq \emptyset,\ j \in J' \subseteq J \\ U \overset{p}{\cong} \tilde{U}_j,\ \forall j \in J \end{cases}$$

consideriamo solo quelli che intersecano Y

$$\left.\begin{array}{l} Y \subseteq \tilde{X} \text{ comp conn} \\ \tilde{U}_j \text{ connessi} \end{array}\right\} \Rightarrow \left(\tilde{U}_j \subseteq Y \ \vee\ \tilde{U}_j \not\subset Y\right)$$

$$\left.\begin{array}{l} \tilde{U}_j \cap Y \neq \emptyset \\ \left(\tilde{U}_j \subseteq Y \ \vee\ \tilde{U}_j \not\subset Y\right) \end{array}\right\} \Rightarrow \tilde{U}_j \subseteq Y$$

questi fungono da preimmagine per $p|_Y$ quindi la ② della definizione di rivestimento è verificata

$\tilde{U}_j \subseteq Y \Rightarrow p(\tilde{U}_j) = U \subset p(Y),\ \forall x \in p(Y) \Rightarrow p(Y)$ aperto

contraddizione

Invece sia $x \notin p(Y)$, $x \in U \subseteq X \setminus p(Y)$

se $\tilde{U}_i \cap Y \neq \emptyset$ per qualche $i \in I \Rightarrow \tilde{U}_i \subset Y$

$\Rightarrow x \in U = p\left(\bigsqcup_{i \in I} \tilde{U}_i\right) \subset p(Y)$

Dunque $\tilde{U}_i \cap Y = \emptyset \Rightarrow X \setminus p(Y) = \bigcup_{k \in K} U_k$ aperti $\Rightarrow X \setminus p(Y)$ aperto

$\Rightarrow p(Y)$ chiuso

$p(Y)$ aperto + chiuso in X connesso $\Longrightarrow p(Y) = X \Longrightarrow p$ suriettiva (1) ✓