

⑤ Calcolare π_1 di:

$$A := \{x \in \mathbb{R}^2 \mid \|x\| > 1\}$$

$$B := \{x \in \mathbb{R}^2 \mid \|x\| < 1\}$$

$$C := \{x \in \mathbb{R}^2 \mid \|x\| \geq 1\}$$

$$D := \{(x, y) \in \mathbb{R}^2 \mid y \geq x^2\}$$

$$E := \{(x, y) \in \mathbb{R}^2 \mid y \leq x^2\}$$

$$F := \mathbb{S}^1 \cup (\mathbb{R}_+ \times \mathbb{R})$$

$$G := \mathbb{R}^2 \setminus (\mathbb{R}_+ \times \{0\})$$

$$\mathbb{R}_+ := \{x \in \mathbb{R} \mid x \geq 0\}$$

$$A, C \simeq \mathbb{S}^1 \Rightarrow \pi_1(A) = \pi_1(C) = \mathbb{Z}$$

$$B, D, E, G \text{ convessi} \Rightarrow \text{contraibili}$$

$$\Rightarrow \text{semp/ conn}$$

$$\Rightarrow \pi_1(B) = \pi_1(D) = \pi_1(E) = \pi_1(G) = \{e\}$$

$$\pi_1(F, x) = \begin{cases} \mathbb{Z}, & x \in \mathbb{S}^1 \\ \{e\}, & x \in (\mathbb{R}_+ \times \mathbb{R}) \end{cases}$$

