6 $\left.\begin{array}{l} p:\tilde{X}\to X \quad \text{omeo locale + propria} \\ \tilde{X}, X \quad \text{var top connesse}\end{array}\right\} \Rightarrow p$ rivest grado finito

$p:\tilde{X}\to X$ propria $\overset{def}{\Longleftrightarrow}$ $\left(K\subset X \text{ compatto} \Rightarrow p^{-1}(K)\subset\tilde{X} \text{ compatto}\right)$

$\tilde{X}, X$ var top $\Longrightarrow$ $\tilde{X}, X \cong \mathbb{R}^n$ $\overset{\text{Heine Borel}}{\Longrightarrow}$ $\tilde{X}, X$ localmente compatti

Lemma (online "When is the image of a proper map closed?"):

$$\left.\begin{array}{l} p:\tilde{X}\to X \quad \text{propria} \\ \tilde{X}, X \quad \text{local. compatti}\end{array}\right\} \Rightarrow p \text{ chiusa}$$

p omeomorfismo locale $\Rightarrow$ p continua + aperta $\Rightarrow$ $p(\tilde{X})\subseteq X$ aperto

p aperta + chiusa $\Rightarrow$ $\left.\begin{array}{r} p(\tilde{X})\subseteq X \text{ aperto + chiuso} \\ X \text{ connesso}\end{array}\right\} \Rightarrow p(\tilde{X}) = X \Rightarrow p$ suriettiva

p omeo locale $\overset{\text{Prop 3.2.1}}{\Longrightarrow}$ $p^{-1}(x)$ topologia discreta

top discreta + fibra finita ??

Sia $x\in X$, $\{\tilde{x}_1,\dots,\tilde{x}_n\} = p^{-1}(x)$

$\tilde{X}$ var top $\Rightarrow$ X Hausdorff

$\Rightarrow \exists\, \tilde{U}_1,\dots,\tilde{U}_n$ aperti $\mid \tilde{U}_i \ni \tilde{x}_i \wedge \tilde{U}_i\cap\tilde{U}_j = \emptyset, \quad i\neq j$

p omeo loc

restringiamo $\tilde{U}_i$ sino a che $p(\tilde{U}_i)\cong U_i \ni x$

$\bigcup_{i=1}^n \tilde{U}_i$ aperto $\Rightarrow$ $\left.\begin{array}{r} C := \tilde{X}\setminus \bigcup_{i=1}^n \tilde{U}_i \text{ chiuso} \\ p \text{ chiusa}\end{array}\right\} \Rightarrow p(C)\subseteq X$ chiuso

$U := \left(\bigcap_{i}^{n} U_i\right)\setminus p(C) \subseteq X$ aperto

$\underbrace{\left(\bigcup_{i=1} \dots \right)}_{\text{aperto}} \quad \underbrace{\dots}_{\text{chiuso}}$

$$p^{-1}(x) \in \bigcup_{i=1}^{n} \tilde{U}_i \implies p^{-1}(x) \cap C = \emptyset$$

$$\implies x \notin p(C)$$

$$\implies x \in U \quad \text{intorno aperto di } x, \ \forall x \in X$$

Generalizzando $p^{-1}(x) \longrightarrow p^{-1}(V) = \bigsqcup_{i=1}^{n} \tilde{U}_i \,, \quad \tilde{U}_i \overset{p}{\cong} V \,, \quad \forall i \in \{1, \dots, n\}$

siccome p suriettiva $\implies$ p rivestimento grado finito