$\bar{A}$ indipendente dal tempo $\Rightarrow \begin{cases} \bar{E} = -\bar{\nabla}\varphi \\ \bar{B} = \bar{\nabla}\wedge\bar{A} \end{cases}$

$\bar{E}$ uniforme $\Rightarrow \varphi = \bar{E}\cdot\bar{r}$

$\bar{B}$ uniforme $\Rightarrow \bar{A} = \frac{1}{2}\bar{B}\wedge\bar{r}$

$$\bar{\nabla}\wedge(\bar{B}\wedge\bar{r}) = (\bar{\nabla}\cdot\bar{r})\bar{B} - \underbrace{(\bar{\nabla}\cdot\bar{B})}_{0}\bar{r} + \underbrace{(\bar{r}\cdot\bar{\nabla})\bar{B}}_{0} - (\bar{B}\cdot\bar{\nabla})\bar{r}$$

$\bar{B}$ uniforme

$$= (\bar{\nabla}\cdot\bar{r})\bar{B} - (\bar{B}\cdot\bar{\nabla})\bar{r} = 2\bar{B}$$

$$\underbrace{(\bar{\nabla}\cdot\bar{r})}_{3}B_x - \left(B_x\frac{\partial}{\partial x} + \cancel{B_y\frac{\partial}{\partial y}} + \cancel{B_z\frac{\partial}{\partial z}}\right)x = 3B_x - B_x = 2B_x$$

Campo elettrico costante e uniforme

$\bar{E} = (E,0,0) \qquad \bar{F} = \frac{d\bar{p}}{dt} = e\bar{E} = (eE,0,0) \qquad$ poniamo $\bar{p}(t=0) = (0,p_0,0)$

$$\begin{cases} \dot{p}_x = eE \\ \dot{p}_y = 0 \\ \dot{p}_z = 0 \end{cases} \Rightarrow \bar{p} = (eEt, p_0, 0)$$

energia cinetica

$$T = m\gamma c^2 = \sqrt{m^2c^4 + p^2c^2} = \sqrt{m^2c^4 + c^2(eEt)^2 + p_0^2c^2} = \sqrt{(\mathcal{E}_0)^2 + (ceEt)^2}$$

$\mathcal{E}_0$ energia iniziale

$$\begin{cases} \bar{p} = m\gamma\bar{v} \\ E = m\gamma c^2 \end{cases} \Rightarrow \bar{v} = \frac{\bar{p}c^2}{E} \rightarrow \begin{cases} v_x = \dfrac{eEtc^2}{\sqrt{(\mathcal{E}_0)^2 + (ceEt)^2}} \\ v_y = \dfrac{p_0c^2}{\sqrt{(\mathcal{E}_0)^2 + (ceEt)^2}} \\ v_z = 0 \end{cases}$$

aumenta

diminuisce

la condizione $v^2 = v_x^2 + v_y^2 + v_z^2 < c^2$ accoppia le velocità fra loro (in meccanica classica, le componenti sono linearmente indipendenti)

fa eccezione l'attrito viscoso $a = \frac{F}{m} \propto |\bar{v}|$

$$\lim_{t\to+\infty} v_x(t) = \lim_{t\to+\infty} \frac{eEtc^2}{\sqrt{(\mathcal{E}_0^2) + (ceEt)^2}} = \frac{\cancel{eEt}c^2}{\cancel{eEt}c\sqrt{1 + \underbrace{\left(\frac{\mathcal{E}_0}{ceEt}\right)^2}_{\to 0}}} = c$$

$$\lim_{t\to+\infty} v_y(t) = \lim_{t\to+\infty} \frac{p_0c^2}{\sqrt{(\mathcal{E}_0)^2 + (ceEt)^2}} = 0$$

$$\sqrt{(\mathcal{E}_0)^2+\underbrace{(ceEt)^2}_{\to+\infty}}$$

<u>leggi orarie:</u>

$$\frac{\mathcal{E}_0}{ceEt}=\cos\theta$$

$$\begin{cases} x(t)=\int v_x\,dt=\int \dfrac{eEt\,c^2}{\sqrt{(\mathcal{E}_0)^2+(ceEt)^2}}\,dt=\int \dfrac{c}{\sqrt{1+\left(\dfrac{\mathcal{E}_0}{ceEt}\right)^2}}\,dt=\dfrac{1}{eE}\sqrt{(\mathcal{E}_0)^2+(ceEt)^2} \\[2ex] y(t)=\int v_y\,dt=\int \dfrac{p_0c^2}{\sqrt{(\mathcal{E}_0)^2+(ceEt)^2}}\,dt=\dfrac{p_0c}{eE}\,\mathrm{asinh}\left(\dfrac{ceE}{\mathcal{E}_0}t\right) \end{cases}$$

$$\int\frac{d\xi}{\sqrt{1+\xi^2}}=\mathrm{asinh}(\xi)=\ln\left(\xi+\sqrt{1+\xi^2}\right)$$

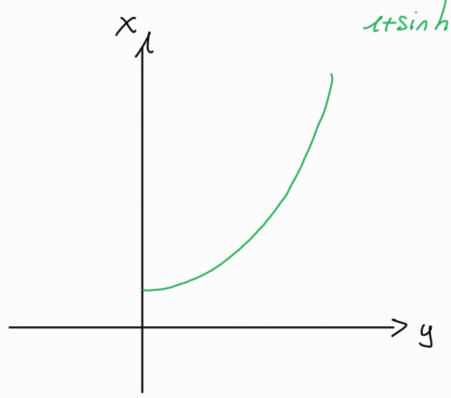
<u>traiettorie:</u>

$$y=\frac{p_0c}{eE}\,\mathrm{asinh}\left(\frac{ceE}{\mathcal{E}_0}t\right)\to\frac{eE}{p_0c}y=\mathrm{asinh}\left(\frac{ceE}{\mathcal{E}_0}t\right)\to\frac{ceE}{\mathcal{E}_0}t=\sinh\left(\frac{eE}{p_0c}y\right)$$

$$x=\frac{1}{eE}\sqrt{(\mathcal{E}_0)^2+(ceEt)^2}=\frac{\mathcal{E}_0}{eE}\sqrt{1+\left(\frac{ceE}{\mathcal{E}_0}t\right)^2}=\frac{\mathcal{E}_0}{eE}\underbrace{\sqrt{1+\sinh^2\left(\frac{eE}{p_0c}y\right)}}_{1+\sinh^2(\xi)=\cosh^2(\xi)}$$

$$x=\frac{\mathcal{E}_0}{eE}\cosh\left(\frac{2E}{p_0c}y\right)$$

catenaria



diventa una parabola (moto del proiettile) per $c\to+\infty$

Campo magnetico uniforme e costante

$\bar{B}=(0,0,B)$ $\quad\frac{d\bar{p}}{dt}=\frac{e}{c}\bar{v}\wedge\bar{B}$ $\quad\bar{p}=m\gamma\bar{v}=\frac{\mathcal{E}}{c^2}\bar{v}$

L'energia sarà costante perché solo il campo magnetico la cambia dunque

$$\bar{\omega}\doteq\frac{ec\bar{B}}{\mathcal{E}}$$

$$\frac{d\bar{p}}{dt}=\frac{d}{dt}\left(\frac{\mathcal{E}}{c^2}\bar{v}\right)=\frac{\mathcal{E}}{c^2}\frac{d\bar{v}}{dt}=\frac{e}{c}\bar{v}\wedge\bar{B}\implies\dot{\bar{v}}=\frac{ec}{\mathcal{E}}\bar{v}\wedge\bar{B}=\bar{v}\wedge\bar{\omega}$$

$$\bar{\omega}=(0,0,\omega)$$

$$\begin{cases}\dot{v}_x=\omega_z v_y-\cancel{\omega_y}v_z=\omega v_y\end{cases}$$

accorpiamo ...

accoppiamo le velocità come numeri complessi

$$\begin{cases} \dot v_y = \cancel{\omega_x} v_z - \omega_z v_x = -\omega v_x \\ \dot v_z = \cancel{\omega_x} v_y - \cancel{\omega_y} v_z = 0 \end{cases} \Longrightarrow \quad v_z = cost.$$

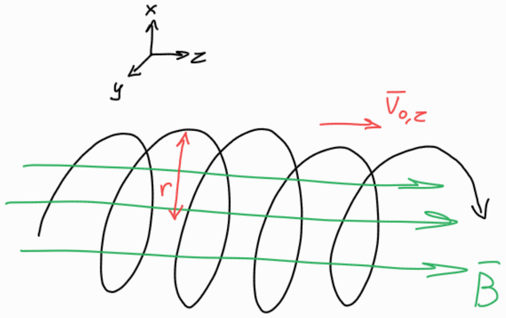
$$\frac{d}{dt}\underbrace{(v_x + i v_y)}_{u} = -i\omega \underbrace{(v_x + i v_y)} \quad \Longrightarrow \quad \dot u = -i\omega \dot u \quad \Longrightarrow \quad u \propto e^{-i\omega u}$$

$$v_x + i v_y = a\, e^{-i\omega t} \qquad a = v_0^{\perp} e^{-i\alpha}$$

perpendicolare a z

2 costanti per 2 eqq differenziali

$$v_x + i v_y = v_0^{\perp} e^{-i(\omega t + \alpha)} \Longrightarrow \begin{cases} v_x = v_0^{\perp} \cos(\omega t + \alpha) \\ v_y = -v_0^{\perp} \sin(\omega t + \alpha) \\ v_z = v_z(t=0) \doteq v_{0,z} \end{cases}$$



$$\boxed{\begin{cases} x(t) = x_0 + r \sin(\omega t + \alpha) \\ y(t) = y_0 + r\cos(\omega t + \alpha) \\ z(t) = z_0 + v_{0,z} t \end{cases}}$$

$$r = \frac{v_0^{\perp}}{\omega} = \frac{v_0^{\perp} \mathcal{E}}{ecB} \quad \text{raggio elica}$$

Nel caso di basse velocità $\tilde{\mathcal{E}} = mc^2 \Longrightarrow$ $\tilde\omega = \frac{ecB}{\tilde{\mathcal{E}}} = \frac{eB}{mc}$ pulsazione di ciclotrone

$$\tilde r = \frac{v_0^{\perp}}{\omega} = \frac{mc\, v_0^{\perp}}{eB} \Longrightarrow \frac{v_\perp c}{rB} = \frac{e}{m}$$

Nel caso relativistico $\frac{v_\perp c}{rB} = \frac{e}{m\gamma} = \frac{e}{m}\sqrt{1 - \frac{v^2}{c^2}}$