particella - nucleo = poca energia persa + grosso cambiamento di direzione

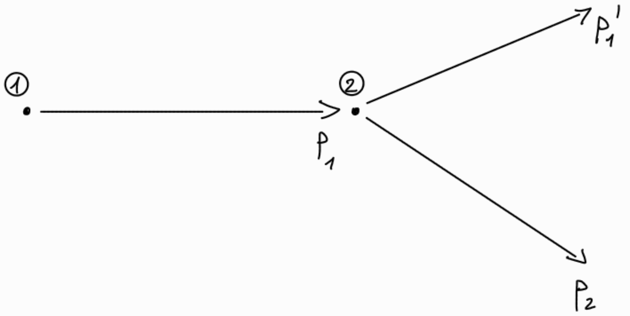
particella - particella = viceversa

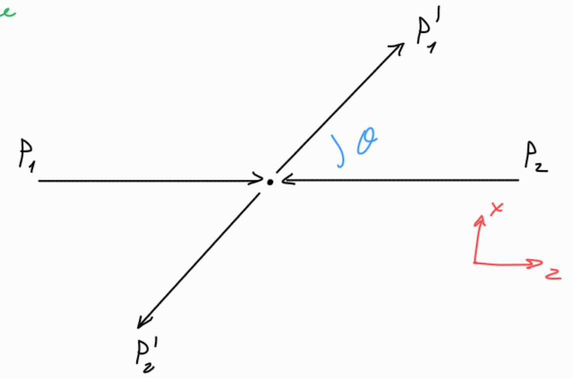
LAB

$P_1 + P_2 = P_1' + P_2'$

conservazione 4-impulso

CM ← centro di massa





$$P_1 = (\mathcal{E}_1, 0, 0, P_1) = (\mathcal{E}_1, \bar{P}_1)$$

$$P_2 = (mc^2, 0, 0, 0) = (mc^2, \bar{0})$$

$$P_1 = (\mathcal{E}_1, 0, 0, p)$$

$$P_2 = (\mathcal{E}_2, 0, 0, -p)$$

$$P_1' = (\mathcal{E}_1', p\sin\theta, 0, p\cos\theta)$$

$$P_2' = (\mathcal{E}_2', -p\sin\theta, 0, -p\cos\theta)$$

impulso trasferito

$$Q^2 = -(P_1' - P_1)^2$$

$$\overset{CM}{=} -\left((\mathcal{E}_1', p\sin\theta, 0, p\cos\theta) - (\mathcal{E}_1, 0, 0, p)\right)^2$$

$$= -(\cancel{\mathcal{E}_1' - \mathcal{E}_1}, p\sin\theta, 0, p\cos\theta - p)^2$$

$$= -\left(0 - p^2\sin^2\theta - p^2(\cos\theta - 1)^2\right)$$

$$= p^2\sin^2\theta + p^2(\cos^2\theta - 2\cos\theta + 1)$$

$$= \underbrace{p^2\sin^2\theta + p^2\cos^2\theta}_{p^2} - 2p^2\cos\theta + p^2$$

$$= 2p^2(1 - \cos\theta)$$

$$= 4p^2\sin^2\left(\frac{\theta}{2}\right)$$

$\mathcal{E}_1' \overset{CM}{=} \mathcal{E}_1 \quad \because \; m_1' = m_1$

$$P^2 = (E, \bar{P})^2$$
$$= E^2 - \bar{P}\cdot\bar{P}$$
$$= m^2 + |\bar{p}|^2 - |\bar{p}|^2$$
$$= m^2$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$$

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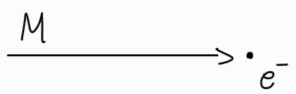
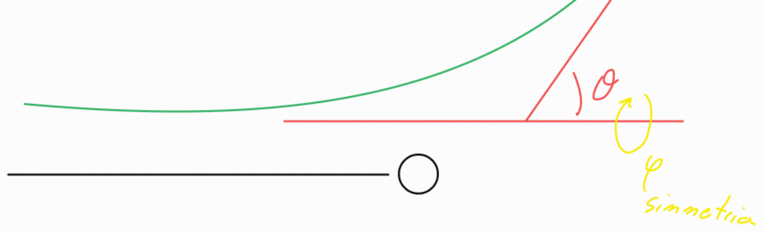
particella grande + e^- $\Rightarrow$ $M \gg m_e$

LAB

CM



$$\frac{d\sigma}{d\Omega} = \left(\frac{ze^2}{2pv}\right)^2 \sin^{-4}\left(\frac{\theta}{2}\right)$$ Rutherford



$$d\Omega = \overbrace{2\pi}^{\int d\varphi} \sin\theta \, d\theta$$

$$Q^2 = 4p^2 \sin^2\frac{\theta}{2}$$

$$d\left(\sin^2\frac{\theta}{2}\right) = \underbrace{\cancel{2}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}_{\sin\theta}\frac{1}{\cancel{2}}\,d\theta = \frac{1}{2}\sin\theta\, d\theta \implies d\Omega = 2\pi\, 2\, d\left(\sin^2\frac{\theta}{2}\right) = 4\pi\, d\left(\frac{Q^2}{4p^2}\right) = \frac{4\pi}{4p^2}\, d(Q^2)$$

p^2 e' dato

$$d\Omega = \pi p^{-2}\, d(Q^2)$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d(Q^2)}\frac{p^2}{\pi} = \left(\frac{ze^2}{2pv}\right)^2 \sin^{-4}\left(\frac{\theta}{2}\right) = \left(\frac{ze^2}{2pv}\right)^2\left(\frac{4p^2}{Q^2}\right)^2 = \left(\frac{ze^2\,\cancel{4}^{2}p\cancel{^2}}{\cancel{2}pv\,Q^2}\right)^2 = \left(\frac{ze^2}{\beta c Q^2}\right)^2$$

$$\implies \frac{d\sigma}{d(Q^2)} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2$$

RICONTROLLA

$\begin{cases} p = (m, 0, 0, 0) & \text{iniziale elettrone} \\ p' = (\varepsilon, 0, 0, p) & \text{finale elettrone} \end{cases}$

$$\begin{aligned} Q^2 &= -(p'-p)^2 \\ &= (\varepsilon - m, 0, 0, p)^2 \\ &= -\left((\varepsilon-m)^2 - p^2\right) \\ &= -\left(\varepsilon^2 + m^2 - 2\varepsilon m - p^2\right) \\ &= -\left(\overbrace{m^2+p^2}^{\varepsilon^2} - 2\overbrace{(T+m)}^{\varepsilon}\, m - p^2\right) \\ &= 2Tm \end{aligned}$$

energia cinetica $T = \varepsilon - m$ $\frac{\text{persa dalla particella}}{\text{acquisita dall'elettrone}}$

$$\frac{d\sigma}{d(Q^2)} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2 \quad \text{con} \quad d(Q^2) = 2m\, dT$$

$$\frac{1}{2m}\frac{d\sigma}{dT} = 4\pi\left(\frac{ze^2}{\beta c\, 2mT}\right)^2 \implies \frac{d\sigma}{dT} = 2\pi\,\frac{1}{m}\left(\frac{ze^2}{\beta c T}\right)^2 = \frac{2\pi z^2 e^4}{mc^2\beta^2 T^2}$$

$T_{min} \sim \overbrace{\hbar\omega}^{\text{energia legame}}$ quindi integriamo $\frac{d\sigma}{dT}$

riflessione totale

$T_{max} \rightarrow E' = \gamma m c^2, \quad p' = \gamma m \beta c$ trasf. Lorentz

$$T_{max} = E - mc^2 = \gamma(E' + \beta c p') - mc^2$$

$$= \gamma^2 mc^2 + \gamma^2 m\beta^2 c^2 - mc^2$$

$$= \gamma^2 mc^2 \left(1 + \beta^2 - (1-\beta^2)\right) \qquad \gamma^{-2} = 1-\beta^2$$

$$= \gamma^2 mc^2 \; 1 + \beta^2 - 1 + \beta^2$$

$$= 2\gamma^2 mc^2 \beta^2$$

Queste formule non considerano gli effetti quantistici: usando $\left.\frac{d\sigma}{d\Omega}\right|_{Mott}$ si ottengono delle correzioni per spin $\propto \left(1-\beta^2 \sin^2\frac{\theta}{2}\right)$ per la sezione d'urto

$$Q^2 = 2mT = 4p^2 \sin^2\frac{\theta}{2}$$

$$2m T_{max} = 4p^2 \sin^2\left(\frac{\pi}{2}\right) = 4p^2 \implies 1-\beta^2\sin^2\frac{\theta}{2} = 1-\beta^2\frac{T}{T_{max}}$$

A questo punto integriamo considerando la particella che entra in un materiale

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2\beta^2 T^2}\left(1-\beta^2\frac{T}{T_{max}}\right) \qquad N \; \frac{\text{atoni}}{\text{volume}} \qquad Z \text{ elettroni}$$

stopping power: energia persa per unità di lunghezza

media pesata in base all'energia di ogni urto

$$\frac{d\mathcal{E}}{dx} = NZ \int_{T_{min}}^{T_{max}} T \frac{d\sigma}{dT} dT$$

$$\propto \int_{T_{min}}^{T_{max}} \frac{T}{T^2}\left(1-\beta^2\frac{T}{T_{max}}\right) dT$$

$$= \left[\ln(T) - \beta^2 \frac{T}{T_{max}}\right]_{T_{min}}^{T_{max}}$$

$$= \ln\left(\frac{T_{max}}{T_{min}}\right) - \beta^2\left(1 - \underbrace{\frac{T_{min}}{T_{max}}}_{\ll 1}\right)$$

$$\simeq \ln\left(\frac{T_{max}}{T_{min}}\right) - \beta^2 \xRightarrow{\text{con costanti}}$$

Formula di Bethe-Bloch

$$\boxed{\frac{d\mathcal{E}}{dx} = NZ \frac{2\pi z^2 e^4}{mc^2\beta^2}\left(\ln\left(\frac{2\gamma^2 m\beta^2 c^2}{\hbar\langle\omega\rangle}\right) - \beta^2\right)}$$

condizioni
$$\begin{cases} M \gg m_e \\ T_{min} > \hbar\omega \\ \text{piccolo } [illegible] \end{cases}$$

Z nuclei materiale

z particella incidente

è piccolo bremmstrahlung

Muone in rame