$$S=\int\left(-mc\,ds-\frac{e}{c}A_\mu dx^\mu\right),\qquad \delta S=0\qquad ds=\sqrt{dx_\mu dx^\mu}\qquad \delta(ds)=\frac{dx_\mu}{ds}\delta(dx^\mu)$$

$$\delta S=-\int\left(mc\,\frac{dx_\mu}{ds}\delta(dx^\mu)+\frac{e}{c}A_\mu\delta(dx^\mu)+\frac{e}{c}\delta A_\mu dx^\mu\right)$$

$d\,\delta(x^\mu)=\delta(dx^\mu)$

$$=-\int\left(mc\,\frac{dx_\mu}{ds}d(\delta x^\mu)+\frac{e}{c}A_\mu d(\delta x^\mu)+\frac{e}{c}\delta A_\mu dx^\mu\right)=0$$

$$\int mc\,\underbrace{\frac{dx_\mu}{ds}}_{u_\mu}d(\delta x^\mu)=mc\frac{dx_\mu}{ds}\delta x^\mu-\int mc\,\frac{d^2x_\mu}{ds^2}\delta x^\mu ds$$

$$=mc\,u_\mu\delta x^\mu-\int mc\,\frac{d}{ds}\left(\frac{dx_\mu}{ds}\right)\delta x^\mu ds$$

$$=mc\,u_\mu\delta x^\mu-\int mc\,du_\mu\delta x^\mu$$

$$\int\frac{e}{c}A_\mu d(\delta x^\mu)=\frac{e}{c}A_\mu\delta x^\mu-\int\frac{e}{c}dA_\mu\delta x^\mu$$

non variamo le condizioni al contorno

$$\delta S=\int_\Omega\left(mc\,du_\mu\delta x^\mu+\frac{e}{c}dA_\mu\delta x^\mu-\frac{e}{c}\delta A_\mu dx^\mu\right)+\cancel{\left(mc\,u_\mu+\frac{e}{c}A_\mu\right)\delta x^\mu\Big|_{\partial\Omega}}=0$$

$$dA_\mu=\frac{\partial A_\mu}{\partial x^\nu}dx^\nu\qquad\wedge\qquad\delta A_\mu=\frac{\partial A_\mu}{\partial x^\nu}\delta x^\nu$$

indici muti $\mu \leftrightarrow \nu$

$$\delta S=\int\left(mc\,du_\mu\delta x^\mu+\frac{e}{c}\frac{\partial A_\mu}{\partial x^\nu}dx^\nu\delta x^\mu-\frac{e}{c}\frac{\partial A_\mu}{\partial x^\nu}\delta x^\nu dx^\mu\right)$$

$$=\int\left(mc\,du_\mu-\frac{e}{c}\left(\frac{\partial A_\mu}{\partial x^\nu}-\frac{\partial A_\nu}{\partial x^\mu}\right)dx^\nu\right)\delta x^\mu$$

$$=\int\left(mc\,\frac{du_\mu}{ds}-\frac{e}{c}\underbrace{\left(\frac{\partial A_\mu}{\partial x^\nu}-\frac{\partial A_\nu}{\partial x^\mu}\right)}_{F_{\mu\nu}}\overbrace{\frac{dx^\nu}{ds}}^{u^\nu}\right)ds\,\overbrace{\delta x^\mu}^{\text{arbitrario}}=0$$

tensore di Lorentz

$$F_{\mu\nu}\doteq\partial_\nu A_\mu-\partial_\mu A_\nu$$

$$\Rightarrow\quad\boxed{mc\,\frac{du_\mu}{ds}=\frac{e}{c}F_{\mu\nu}u^\nu}$$

equazione di Minkowski-Lorentz

$$\frac{d^2x_\mu}{ds^2}=\frac{e}{mc^2}F_{\mu\nu}u^\nu$$

$F_{\mu\nu} = -F_{\nu\mu}$ antisimmetrico $\Longrightarrow$ $F_{\mu\mu} = 0$ $\Longrightarrow$ 6 entrate indipendenti $(\bar{E} \text{ e } \bar{B})$

$$A^\mu = (\varphi, \bar{A})$$

$$\begin{cases} \bar{E} = -\bar{\nabla}\varphi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t} \\ \bar{B} = \bar{\nabla} \wedge \bar{A} \end{cases}$$

$$F^{\mu 0} = \partial^\mu A^0 - \partial^0 A^\mu = \partial^i \varphi - \partial^0 A^i = (-\bar{\nabla}\varphi)^i - \frac{1}{c}\left(\frac{\partial \bar{A}}{\partial t}\right)^i = E^i$$

$$F^{00} = 0$$

$$F^{12} = \partial^1 A^2 - \partial^2 A^1 = \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x} = (\bar{\nabla} \wedge \bar{A})_z = B_z$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

Invarianza di gauge

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad (A')^\mu = A^\mu - \partial^\mu \chi$$

$$(F')^{\mu\nu} = \partial^\mu (A^\nu - \partial^\nu \chi) - \partial^\nu (A^\mu - \partial^\mu \chi)$$

$$= \partial^\mu A^\mu - \cancel{\partial^\mu \partial^\nu} \chi - \partial^\nu A + \cancel{\partial^\nu \partial^\mu} \chi$$

$$= F^{\mu\nu}$$

$$mc \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu \Longrightarrow \begin{cases} \frac{d\bar{p}}{dt} = e\bar{E} + \frac{e}{c}\bar{v} \wedge \bar{B} \\ \frac{d\mathcal{E}}{dt} = e\bar{E} \cdot \bar{v} \end{cases}$$

solo 3 di queste 4 sono indipendenti