

$A^\mu = a^\mu e^{ik \cdot x}$ onda di una sorgente a velocità $\vec{\beta} = (\beta, 0, 0)$

$$k \cdot x = k^\mu x_\mu = k t - \vec{k} \cdot \vec{x} = \frac{\omega}{c} t - \vec{k} \cdot \vec{x} \quad \text{con} \quad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$k^\mu = \Lambda^\mu{}_\nu k'^\nu \rightarrow \begin{aligned} k^0 &= \gamma(k'^0 - \beta k'^1) & k^2 &= k'^2 \\ k^1 &= \gamma(k'^1 - \beta k'^0) & k^3 &= k'^3 \end{aligned}$$

$$\begin{aligned} k^0 = k &= \frac{\omega}{c} = \gamma \left(\frac{\omega'^0}{c} - \beta k'^1 \right) & \begin{cases} k^1 = k \cos \alpha = \frac{\omega}{c} \cos \alpha \\ \alpha = \widehat{\vec{k} \vec{\beta}} \end{cases} \\ &= \gamma \left(\frac{\omega'^0}{c} - \beta \frac{\omega}{c} \cos \alpha \right) \end{aligned}$$

$$\Rightarrow \boxed{\omega = \gamma \omega' (1 - \beta \cos \alpha)} \quad \text{Effetto Doppler}$$