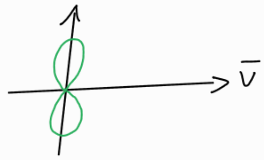
È interessante analizzare la distribuzione angolare della radiazione emessa da una particella accelerata

non relativistica: $$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}|\dot{\bar{v}}|^2 \sin^2\theta$$




Ponendo $\bar{\beta} = \frac{\bar{v}}{c}$ e $\hat{n} = \frac{\bar{R}}{R}$ :

campi ritardati
$$\begin{cases} \bar{E} = e\left(\frac{\hat{n}-\bar{\beta}}{\gamma^2(1-\bar{\beta}\cdot\hat{n})^3}\right)\frac{1}{R^2} + \underbrace{\frac{e}{c}\left(\frac{\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}}{(1-\bar{\beta}\cdot\hat{n})^3}\right)\frac{1}{R}} \\ \bar{B} = \hat{n}\wedge\bar{E} \end{cases}$$

solo questo $\because$ più importante per $R \to +\infty$

$$\bar{S} = \frac{c}{4\pi}\bar{E}\wedge\bar{B} = \frac{c}{4\pi}\bar{E}\wedge(\hat{n}\wedge\bar{E}) = \frac{cE^2}{4\pi}\hat{n} \simeq \frac{e}{4\pi c}\frac{|\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}|^2}{(1-\bar{\beta}\cdot\hat{n})^6}\Bigg|_{rit.}\frac{\hat{n}}{R^2}$$

$\bar{S}$ al tempo ritardato

Per calcolare l'energia usiamo il tempo ritardato:

$t' = t - \frac{R}{c}$ $$\begin{cases} t_1 = T_1 + \frac{R}{c} \\ t_2 = T_2 + \frac{R}{c} \end{cases}$$

$t_i$ : tempi di osservazione

$T_i$ : tempi di emissione

$$\mathcal{E} = \int_{t_1}^{t_2}\underbrace{\bar{S}(t')\cdot\hat{n}}_{\text{flusso}}\,dt = \int_{t_1}^{t_2}\bar{S}\cdot\hat{n}\underbrace{\frac{dt}{dt'}}_{1-\bar{\beta}\cdot\hat{n}}dt'$$

$1-\bar{\beta}\cdot\hat{n}$ ricavato in qualche pagina precedente

$$dP = (\bar{S}\cdot\hat{n})(1-\bar{\beta}\cdot\hat{n})\,R^2 d\Omega$$

Potenza istantanea

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\frac{|\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}|^2}{(1-\bar{\beta}\cdot\hat{n})^6}\frac{\cancel{R^2}}{\cancel{R^2}}\underbrace{\hat{n}\cdot\hat{n}}_{1}(1-\bar{\beta}\cdot\hat{n}) \Rightarrow \boxed{\frac{dP}{d\Omega} = \frac{e}{4\pi c}\frac{|\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}|^2}{(1-\bar{\beta}\cdot\hat{n})^5}}$$

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## Accelerazione lineare

$\bar{\beta} \parallel \dot{\bar{\beta}} \Rightarrow \bar{\beta}\cdot\dot{\bar{\beta}} = 0$ $\qquad \theta = \widehat{\hat{n}\,\dot{\bar{\beta}}}$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\frac{|\hat n \wedge (\hat n - \bar\beta)\wedge \dot{\bar\beta}|^2}{(1-\bar\beta\cdot\hat n)^5} = \frac{e}{4\pi c}\frac{|\hat n \wedge(\hat n \wedge \dot{\bar\beta})|^2}{(1-\bar\beta\cdot\hat n)^5} = \frac{e}{4\pi c}\frac{\dot\beta^2 \sin^2\theta}{(1-\beta\cos\theta)^5}$$

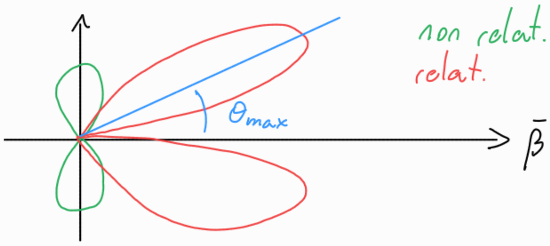
Cerchiamo il massimo angolare di $\frac{dP}{d\Omega}$:

$$\frac{\sin^2\theta}{(1-\beta\cos\theta)^5} \overset{x \doteq \cos\theta}{=} \frac{1-x^2}{(1-\beta x)^5} \longrightarrow \frac{d}{dx}\frac{1-x^2}{(1-\beta x)^5} = -\frac{2x}{(1-\beta x)^5} + \frac{5\beta(1-x^2)}{(1-\beta x)^6} = \frac{-2x(1-\beta x)+5\beta(1-x^2)}{(1-\beta x)^6} = 0$$

$$\Rightarrow \quad 2\beta x^2 - 5\beta x^2 - 2x + 5\beta = 0 \quad \Rightarrow \quad 3\beta x^2 + 2x - 5\beta = 0 \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{1+15\beta^2}}{3\beta}$$

$$\Rightarrow \quad \theta_{max} = \cos^{-1}\left(\frac{-1\pm\sqrt{1+15\beta^2}}{3\beta}\right) \simeq \frac{1}{2\gamma}$$

$$\gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \beta \simeq 1 - \frac{1}{2\gamma^2} \quad \text{per } \gamma \to +\infty$$




per $\beta$ maggiori, $\theta_{max}$ diminuisce

questo spostamento è dato dal termine $(1-\hat n\cdot\bar\beta)^{-5}$

Caso ultrarelativistico:

$$\frac{dP}{d\Omega} \simeq \frac{e}{4\pi c}\dot\beta^2 \frac{\theta^2}{\left(1-\beta+\frac{\beta\theta^2}{2}\right)^5} \overset{\theta \propto \gamma^{-1}}{\simeq} \frac{8e}{\pi c}\dot\beta^2\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^5}$$