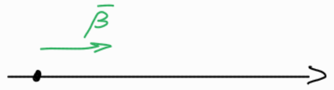
Acceleratori lineari

$$P = \frac{2e^2\gamma^2}{3m^2c^3}\left(\left(\frac{dp}{d\tau}\right)^2 - \frac{1}{c^2}\left(\frac{d\mathcal{E}}{d\tau}\right)^2\right) = \frac{2e^2\gamma^6}{3c}\left(\dot{\beta}^2 - (\bar{\beta}\wedge\dot{\bar{\beta}})^2\right)$$



$$\bar{\beta} \parallel \dot{\bar{\beta}} \Rightarrow \bar{\beta}\wedge\dot{\bar{\beta}} = \bar{0} \Rightarrow P = \frac{2e^2\gamma^6}{3c}\dot{\beta}^2 = \frac{2e^2}{3m^2c^3}\left(\frac{dp}{dt}\right)^2$$

$$\hookrightarrow \frac{d\bar{p}}{dt} = |\bar{p}|\cancelto{0}{\frac{d\hat{p}}{dt}} + \frac{d\bar{p}}{dt}\cdot\bar{p}$$

$$\left(\frac{dp}{d\tau}\right)^2 - \frac{1}{c^2}\left(\frac{d\mathcal{E}}{d\tau}\right)^2 = \left(\frac{dp}{d\tau}\right)^2 - \frac{\beta^2}{c^2}\left(\frac{dp}{d\tau}\right)^2 = \left(\frac{dp}{d\tau}\right)^2(1-\beta^2) = \frac{1}{\gamma^2}\left(\frac{dp}{d\tau}\right)^2 = \left(\frac{dp}{dt}\right)^2$$

Un parametro importante è il campo elettrico per unità di lunghezza $\frac{dE}{dx}$

$$\frac{dp}{dt} = F = |\bar{\nabla}V| = \frac{d\mathcal{E}}{dx} \qquad \bar{F} = e\bar{E}$$

$$P = \frac{2e^2}{3m^2c^3}\left(\frac{dp}{dt}\right)^2 = \frac{2e^2}{3m^2c^3}\left(\frac{d\mathcal{E}}{dx}\right)^2$$

diventa rilevante quando $\frac{d\mathcal{E}}{dx} \sim mc^2$

$$P \sim \frac{2e^2}{3mc}$$

__Esempio con elettrone:__

lunghezza fondamentale: $\frac{e^2}{m^2c^3}$

$$\begin{cases} m_e = 0.511\ \text{MeV} & (c=1) \\ \alpha = \frac{e^2}{4\pi} = \frac{1}{137} & \left(\alpha = \frac{e^2}{4\pi\hbar c}\right) \\ \hbar c \simeq 197\ \frac{\text{MeV}}{\text{fm}} & \Rightarrow 1\,\text{fm} = \frac{1}{197}\ \text{MeV}^{-1} \quad (\hbar = 1) \end{cases}$$

$$\Rightarrow \frac{e^2}{mc^2} = \frac{4\pi}{137}\,\frac{1}{0.511\,\text{MeV}} = \frac{4\pi}{137\cdot 0.511}\,197\,\text{fm} \simeq 0.03\,\text{fm}$$

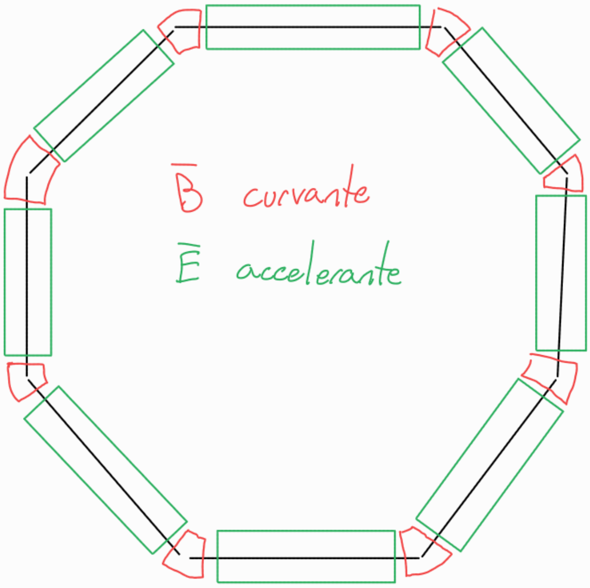
Finché $\frac{d\mathcal{E}}{dx} \sim \frac{0.511\ \text{MeV}}{0.03\ \text{fm}}$ (numero minuscolo)

LINAC (CERN) : $\frac{d\mathcal{E}}{dx} = \frac{160\,\text{MeV}}{80\,\text{m}}$

$\Rightarrow$ P di radiazione e' trascurabile

Acceleratori circolari

$$P = \frac{2e^2\gamma^2}{3m^2c^3}\left(\left(\frac{dp}{d\tau}\right)^2 - \frac{1}{c^2}\left(\frac{d\mathcal{E}}{d\tau}\right)^2\right) = \frac{2e^2\gamma^6}{3c}\left(\dot{\beta}^2 - (\bar{\beta}\wedge\dot{\bar{\beta}})^2\right)$$



$\bar{E}$ accelera poco ma molte volte

$\bar{B}$ fa perdere energia per radiazione

$$\frac{d\bar{p}}{d\tau} \gg \frac{d\mathcal{E}}{d\tau}$$

$$\frac{d\bar{p}}{d\tau} = \cancel{\frac{dp}{d\tau}\hat{p}} + \frac{d\hat{p}}{d\tau}p = p\frac{d\hat{p}}{d\tau} = p\gamma\frac{d\hat{p}}{dt}$$

$$\omega \doteq \frac{c\beta}{\rho} \implies P = \frac{2e^2}{3m^2c^3}\gamma^2\omega^2|\bar{p}|^2 = \frac{2e^2}{3m^2c^3}\gamma^2\frac{c^2\beta^2}{\rho^2}(\gamma m\beta c)^2$$

$$\implies P = \frac{2e^2c}{3\rho^2}\beta^4\gamma^4$$

energia persa accelerando

Per ogni giro: $\Delta\mathcal{E} = \frac{2\pi\rho}{c\beta}P = \frac{4\pi e^2}{3\rho}\beta^3\gamma^4$

$\gamma = \frac{E}{mc^2}$ ∴ più importante per particelle leggere

$$\Delta\mathcal{E} = 8.85\cdot 10^{-2}\frac{(\mathcal{E}\,[GeV])^2}{\rho\,[m]}$$

energia particella

CERN

$$\begin{cases}\mathcal{E} = 14\cdot 10^3\,GeV \\ \rho = 9\cdot 10^3\,m\end{cases}$$

Queste perdite di energia sono molto importanti