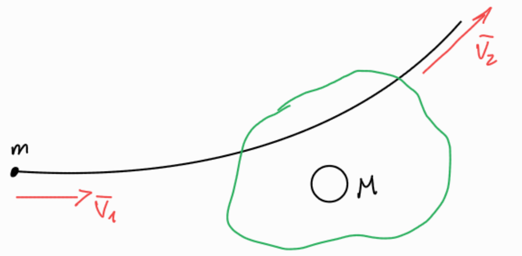
Si verifica per esempio negli urti tra particelle cariche



$M >> m$

In sostanza M genera un campo e.m. fisso (da carica z)

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{ze^2}{4\pi^2 c} \left| \int \frac{d}{dt}\left(\frac{\hat{n}\wedge(\hat{n}\wedge\bar{\beta})}{1-\hat{n}\cdot\bar{\beta}}\right) e^{i\omega\left(t-\frac{\hat{n}\cdot\bar{r}}{c}\right)} dt \right|^2$$

Risolveremo questo in regime di basse frequenze: $\hbar\omega << E$ particella

$$\lim_{\omega\to 0} e^{i\omega\left(t-\frac{\hat{n}\cdot\bar{r}}{c}\right)} = 1 \implies \frac{d^2 I}{d\omega\, d\Omega} = \frac{ze^2}{4\pi^2 c} \left| \int \frac{d}{dt}\left(\frac{\hat{n}\wedge(\hat{n}\wedge\bar{\beta})}{1-\hat{n}\cdot\bar{\beta}}\right) dt \right|^2$$

$$= \frac{ze^2}{4\pi^2 c} \left| \left[\left(\frac{\hat{n}\wedge(\hat{n}\wedge\bar{\beta})}{1-\hat{n}\cdot\bar{\beta}}\right)\right]_{t_1}^{t_2} \right|^2$$

cambia solo $\bar{\beta}$

$$= \frac{ze^2}{4\pi^2 c} \left| \left[\left(\frac{\hat{n}\wedge(\hat{n}\wedge\bar{\beta})}{1-\hat{n}\cdot\bar{\beta}}\right)\right]_{\bar{\beta}_1}^{\bar{\beta}_2} \right|^2$$

Poniamo $\bar{\beta} = \beta\hat{z}$ e $(\hat{n}, \hat{\theta}, \hat{\varphi})$

$$\hat{z} = \cos\theta\,\hat{n} - \sin\theta\,\hat{\theta}$$

$$\hat{n}\wedge\hat{n}\wedge\bar{\beta} = \hat{n}\wedge\left(\underbrace{\hat{n}\wedge(\beta\cos\theta\,\hat{n}}_{\hat{n}\wedge\hat{n}=\bar{0}} - \sin\theta\hat{\theta})\right) = \hat{n}\wedge\left(-\beta\sin\theta\,\overbrace{\hat{n}\wedge\hat{\theta}\big) = \hat{n}\wedge\big(-\beta\sin\theta\,\hat{\varphi}}^{\hat{n}\wedge\hat{\varphi}=-\hat{\theta}}\right) = \beta\sin\theta\,\hat{\theta}$$

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{ze^2}{4\pi^2 c} \left| \left[\left(\frac{\hat{n}\wedge(\hat{n}\wedge\bar{\beta})}{1-\hat{n}\cdot\bar{\beta}}\right)\right]_{\bar{\beta}_1}^{\bar{\beta}_2} \right|^2$$

$$= \frac{ze}{4\pi^2 c} \left| \frac{\beta_2\sin\theta}{1-\hat{n}\cdot\bar{\beta}_2} - \frac{\beta_1\sin\theta}{1-\hat{n}\cdot\bar{\beta}_1} \right|^2$$

$$\propto \frac{\sin^2\theta}{\left(1-\frac{V}{c}\cos\theta\right)^2}$$

Per il massimo
$$\frac{d^3 I}{d\omega\, d\Omega\, d\theta} \propto \frac{d}{d\theta}\left(\frac{\sin^2\theta}{\left(1-\frac{V}{c}\cos\theta\right)^2}\right)$$

$$= \frac{2\sin\theta\cos\theta}{\left(1-\frac{V}{c}\cos\theta\right)^2} - \frac{\sin^2\theta}{\left(1-\frac{V}{c}\cos\theta\right)^3}\, 2\frac{V}{c}\sin\theta$$

$$= \frac{2\sin\theta\cos\theta\left(1-\frac{V}{c}\cos\theta\right) - 2\sin^2\theta\,\frac{V}{c}\sin\theta}{\left(1-\frac{V}{c}\cos\theta\right)^3}$$

$$= \frac{2\sin\theta\left(\cos\theta - \frac{V}{c}\cos^2\theta - \frac{V}{c}\sin^2\theta\right)}{\left(1-\frac{V}{c}\cos\theta\right)^3}$$

$$= \frac{2\sin\theta\left(\cos\theta - \frac{V}{c}\right)}{\left(1-\frac{V}{c}\cos\theta\right)^3} \implies \text{massimi } \theta = \left\{0, \operatorname{acos}\left(\frac{V}{c}\right)\right\}$$

Per $\frac{V}{c} \sim 1$: $\frac{d^2 I}{d\omega\, d\Omega} \propto \frac{\sin^2\theta}{\left(1-\frac{V}{c}\cos\theta\right)^2} \xrightarrow{\frac{V}{c}\to 1} \frac{1}{\left(1-\frac{V}{c}\frac{V}{c}\right)^2} = \gamma^2$ oppure 4?

Ricordando che

$$\bar{E} = -\frac{1}{c}\frac{\partial \bar{A}}{\partial t} \longrightarrow \bar{E}(\bar{r},\omega) = \frac{1}{\sqrt{2\pi}}\int e^{i\omega t} E(\bar{r},t)\, dt$$

$$\lim_{\omega\to 0} \bar{E}(\bar{r},\omega) = \lim_{\omega\to 0} \frac{1}{\sqrt{2\pi}}\int \overbrace{e^{i\omega t}}^{1}\left(-\frac{1}{c}\frac{\partial \bar{A}}{\partial t}\right)dt$$

$$= -\frac{1}{c\sqrt{2\pi}}\left|\bar{A}(t_2) - \bar{A}(t_1)\right|^2$$

$$\bar{A}(\bar{r},t) = \frac{e\bar{\beta}}{1-\bar{\beta}\cdot\hat{n}}$$

$$= -\frac{e}{c\sqrt{2\pi}}\left|\frac{\beta_2}{1-\bar{\beta}_2\cdot\hat{n}} - \frac{\beta_1}{1-\bar{\beta}_1\cdot\hat{n}}\right|^2$$

Questo risultato vale anche quantisticamente se $\mathcal{E} = \hbar\omega \to 0$ ($\omega \to 0$)

Se cercassimo il numero di fotoni emessi:

$$N = \lim_{\hbar\omega\to 0} \frac{d^2 N}{d\Omega_\gamma\, d(\hbar\omega)}$$

Ω_γ fotone

$$= \frac{z^2 e^2}{4\pi^2\hbar\omega c}\left|\frac{\beta_2}{1-\bar{\beta}_2\cdot\hat{n}} - \frac{\beta_1}{1-\bar{\beta}_1\cdot\hat{n}}\right|^2$$

$\frac{e^2}{\hbar c} \doteq \alpha$ cost struttura fine

$$= \frac{z^2\alpha^2}{4\pi\omega}\left|\frac{\beta_2}{1-\bar{\beta}_2\cdot\hat{n}} - \frac{\beta_1}{1-\bar{\beta}_1\cdot\hat{n}}\right|^2$$

Per $\omega \to 0$, $N \to \infty$ $\therefore$ $N \propto \omega^{-1}$

Consideriamo quindi un urto:

sezione d'urto $\frac{d\sigma}{d\Omega_p}$ $\qquad$ Ω_p particella

$$\frac{d^3\sigma}{d\Omega_p \, d\Omega_\gamma \, d(\hbar\omega)} = \left(\frac{d^2 N}{d\Omega_\gamma \, d(\hbar\omega)}\right) \frac{d\sigma}{d\Omega_p}$$

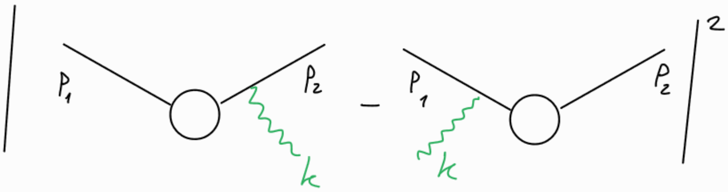
$$= \frac{z^2 \alpha^2}{4\pi\omega} \left| \frac{\beta_2}{1-\bar{\beta}_2\cdot\hat{n}} - \frac{\beta_1}{1-\bar{\beta}_1\cdot\hat{n}} \right|^2 \frac{d\sigma}{d\Omega_p}$$

Questo implica che un urto con campi e.m. non può mai essere totalmente elastico, i.e. c'è sempre radiazione emessa.

4-impulso fotone

$$k^\mu = \left(\hbar\omega, \frac{\hbar\omega}{c}\hat{n}\right) \qquad \text{quindi}$$

$$\left| \frac{\beta_2}{1-\bar{\beta}_2\cdot\hat{n}} - \frac{\beta_1}{1-\bar{\beta}_1\cdot\hat{n}} \right|^2 \propto \left| \frac{p_2^\mu}{k\cdot p_2} - \frac{p_1^\mu}{k\cdot p_2} \right|^2 = $$



Creazione di particelle / Bremsstrahlung interno

decadimento nucleo, Z protoni

$$N(Z) \longrightarrow N'(Z\pm 1) + e^{\mp} + \bar{\nu}/\nu$$

trattiamo questo processo come un bremsstrahlung ma con accelerazione

$$\frac{d^2 I}{d\Omega \, d\omega} = \frac{z^2 e^2}{4\pi^2 c} \left| \frac{\bar{\beta}}{1-\bar{\beta}\cdot\hat{n}} \right|^2$$

l'altro non c'è perché la particella non esisteva prima

$$\tau \sim \frac{1}{\omega} = \frac{\hbar}{\hbar\omega} = \frac{\hbar}{\Delta\tau}$$

tempo di reazione breve $\rightarrow$ energia alta

$$\frac{d^2 I}{d\Omega \, d\omega} = \frac{z^2 e^2}{4\pi^2 c} \left| \frac{\bar{\beta}}{1-\bar{\beta}\cdot\hat{n}} \right|^2 \propto \frac{\sin^2\theta}{(1-\beta\cos\theta)^2}$$

$$\frac{dI}{d\omega} = \int \frac{d^2 I}{d\Omega \, d\omega} \, d\Omega$$

$$= \int \frac{\sin^2\theta}{(1-\beta\cos\theta)^2}\, d\Omega$$

$$= \int \frac{\sin^2\theta}{(1-\beta\cos\theta)^2} \overbrace{\int d\varphi}^{2\pi} \sin\theta\, d\theta$$

$$= -2\pi \int \frac{1-\cos^2\theta}{(1-\beta\cos\theta)^2}\, d\cos\theta \qquad x \doteq \cos\theta$$

$$= -2\pi \int \frac{1-x^2}{(1-\beta x)^2}\, dx$$

$$= \left[\frac{\beta x + \frac{\beta^2-1}{\beta x - 1} + 2\ln(1-\beta x)}{\beta^3} \right]_{-1}^{1} \implies \frac{dI}{d\omega} = \frac{e^2}{\pi c}\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right)$$

$$\mathcal{E}_{TOT} = \int_0^{\mathcal{E}_{max} = \hbar\omega_{max}} \frac{dI}{d\omega}\, d\omega = \frac{e^2}{\pi c}\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right)\omega_{max} = \frac{e^2}{\pi\hbar c}\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right)\mathcal{E}_{max}$$

$$\frac{1+\beta}{1-\beta} = \frac{1+\beta}{1-\beta}\,\frac{1+\beta}{1+\beta} = \frac{(1+\beta)^2}{1-\beta^2} = \gamma^2 \underbrace{(1+\beta)}_{1+\beta<2}{}^2 < 4\gamma^2 \implies \ln\left(\frac{1+\beta}{1-\beta}\right) < \ln(4\gamma^2) = 2\ln(2\gamma)$$

$$\frac{\mathcal{E}_{rad}}{\mathcal{E}_{part}} < \overbrace{\frac{e^2}{\pi\hbar c}}^{\alpha/\pi}\,\frac{2}{\beta}\left(\ln(2\gamma) - 1\right) \implies \mathcal{E}_{rad} \ll \mathcal{E}_{part}$$