$$\bar\nabla \wedge \bar E = -\frac{1}{c}\frac{\partial \bar B}{\partial t} \qquad \bar\nabla\cdot\bar E = 0$$

$$\bar\nabla \wedge \bar B = -\frac{1}{c}\frac{\partial \bar E}{\partial t} \qquad \bar\nabla\cdot\bar B = 0$$

$$\longrightarrow \quad \partial_\mu F^{\mu\nu} = 0 \qquad \text{no sorgenti}$$

gauge: $\varphi = 0 \ \wedge \ \bar\nabla\cdot\bar A = 0$ $\qquad \partial_\mu A^\mu = 0$

$$\bar E = -\frac{1}{c}\frac{\partial \bar A}{\partial t} \quad \wedge \quad \bar B = \bar\nabla\wedge\bar A$$

$$\bar\nabla\wedge\bar B = -\frac{1}{c}\frac{\partial \bar E}{\partial t}$$

$$\bar\nabla\wedge\bar\nabla\wedge\bar A = \bar\nabla(\cancelto{0}{\bar\nabla\cdot\bar A}) - \nabla^2 A = -\frac{1}{c}\frac{\partial^2 \bar A}{\partial t^2} \implies \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar A = \square \bar A = 0$$

<u>4D:</u>

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\partial_\mu F^{\mu\nu} = \partial_\mu\partial^\mu A^\nu - \partial^\nu \overbrace{\cancel{\partial_\mu A^\mu}}^{\text{gauge di Lorentz}} = \partial_\mu\partial^\mu A^\nu = \square A^\nu = 0$$

4Dg: $\partial_\mu A^\mu = 0$

3Dg: $\varphi = 0 \wedge \bar\nabla\cdot\bar A = 0$

3Dg $\subset$ 4Dg $\wedge$ 4Dg covariante

$\square \bar A = 0$ si può usare come soluzione del tipo:
(ansatz)

$$\bar A(x,t) = \frac{1}{(2\pi)^{3/2}}\int d^3\bar k \, \bar{\mathcal{A}}(k,t)\, e^{i\bar k\cdot\bar x} \qquad \text{trasf. di Fourier}$$

$$\nabla^2\left(e^{i\bar k\cdot\bar x}\right) = \bar\nabla\cdot\left(i\bar k e^{i\bar k\cdot\bar x}\right) = -k^2 e^{i\bar k\cdot\bar x}$$

$$\square \bar A = \frac{1}{(2\pi)^{3/2}}\int\left(-k^2\bar{\mathcal{A}} - \frac{1}{c^2}\frac{\partial^2\bar{\mathcal{A}}}{\partial t^2}\right)e^{i\bar k\cdot\bar x} = 0 \qquad \text{vera } \forall \bar k, \bar x$$

$$\implies k^2\bar{\mathcal{A}} + \frac{1}{c^2}\frac{\partial^2\bar{\mathcal{A}}}{\partial t^2} = 0 \implies \left(\frac{\partial^2}{\partial t^2} + \omega^2\right)\bar{\mathcal{A}}(\bar k,t) = 0$$

$\omega \doteq ck$

eq. diff. oscillatore armonico

$$\bar{\mathcal{A}}(\bar k, t) = \bar a(\bar k)\, e^{-i\omega t} + \bar a^*(\bar k)\, e^{i\omega t}$$

$A(k,t) = a(k)\, e \quad + a(k)\, e$

$$\bar A(\bar x,t) = \frac{1}{(2\pi)^{3/2}} \int d^3\bar k \left(\bar a(\bar k)\, \underbrace{e^{-i\omega t + i\bar k\cdot\bar x} + \bar a'(\bar k)\, e^{i\omega t + i\bar k\cdot \bar x}}_{\text{onde piane}} \right)$$

solitamente $\tilde{\bar a}(\bar k) \doteq \bar a'(-\bar k)$

in più $\tilde{\bar a} = (\bar a)^*$

$$\bar A(\bar x,t) = \frac{1}{(2\pi)^{3/2}} \int d^3\bar k \left(\bar a(\bar k)\, e^{i(\bar k\cdot\bar x - \omega t)} + \bar a^*(\bar k)\, e^{-i(\bar k\cdot\bar x - \omega t)} \right)$$

$$\bar\nabla\cdot\bar A = \frac{1}{(2\pi)^{3/2}} \int d^3\bar k \left(i\bar k\cdot\bar a(\bar k)\, e^{i(\bar k\cdot\bar x - \omega t)} - i\bar k\cdot\bar a^*(\bar k)\, e^{-i(\bar k\cdot\bar x - \omega t)} \right) = 0$$

$\Longrightarrow \quad \bar a\cdot\bar k = \bar a^*\cdot\bar k = 0$ le ampiezze devono essere perpendicolari alla direzione di propagazione

equivalenti $\because \bar k \in \mathbb{R}^n$

$$\begin{cases} \bar E = -\frac{1}{c}\frac{\partial \bar A}{\partial t} \\ \bar B = \bar\nabla\wedge\bar A \end{cases}$$

Anche i campi rispettano l'eq. di d'Alembert perché $-\frac{1}{c}\frac{\partial}{\partial t}$ e $\bar\nabla\wedge$ commutano con $\Box$

$$\bar E(\bar x,t) = \int \frac{d^3\bar k}{(2\pi)^{3/2}} \left(\bar{\mathcal E}(\bar k,t)\, e^{-i(\bar k\cdot\bar x - \omega t)} + \bar{\mathcal E}^*(\bar k,t)\, e^{i(\bar k\cdot\bar x - \omega t)} \right)$$

$$\bar B(\bar x,t) = \int \frac{d^3\bar k}{(2\pi)^{3/2}} \left(\bar{\mathcal B}(\bar k,t)\, e^{-i(\bar k\cdot\bar x - \omega t)} + \bar{\mathcal B}^*(\bar k,t)\, e^{i(\bar k\cdot\bar x - \omega t)} \right)$$

Siccome $\begin{cases} \bar\nabla\cdot\bar E = 0 \\ \bar\nabla\cdot\bar B = 0 \end{cases} \Longrightarrow \bar{\mathcal E}\cdot\bar k = \bar{\mathcal B}\cdot\bar k = 0$

Sostituendo $\bar E$ e $\bar B$ trovati in $\begin{cases} \bar\nabla\wedge\bar E = -\frac{1}{c}\frac{\partial\bar B}{\partial t} \\ \bar\nabla\wedge\bar B = -\frac{1}{c}\frac{\partial\bar E}{\partial t} \end{cases} \Longrightarrow \bar B = \hat k\wedge\bar E$

$\{\bar B, \bar E, \bar k\}$ terna ortogonale

$$\bar B = \hat k\wedge\bar E \Longrightarrow B = |\hat k\wedge\bar E| = E$$

Nello spazio di Minkowski:

$$\partial_\mu F^{\mu\nu} = 0 \quad \wedge \quad \partial_\mu F_{\rho\sigma} + \partial_\rho F_{\sigma\mu} + \partial_\sigma F_{\mu\rho} = 0$$

$$\underbrace{\partial^\mu \partial_\mu}_{\Box} F_{\rho\sigma} + \partial_\rho \overbrace{\partial^\mu F_{\sigma\mu}}^{0} + \partial_\sigma \overbrace{\partial^\mu F_{\mu\rho}}^{0} = 0 \quad \Rightarrow \quad \Box F^{\mu\nu} = 0$$

Con il gauge $\partial_\mu A^\mu = 0$, $\quad \Box F^{\mu\nu} = 0 \Rightarrow \Box A^\mu = 0$

da cui la soluzione $\quad A^\mu = a^\mu(k)\, e^{ik\cdot x} \quad k\cdot x = k_\nu x^\nu$

con $k^\mu = (|\bar{k}|, \bar{k})$, $k^2 = 0$ (tipo luce)

$\Box A^\mu = 0$

$$\partial_\nu \partial^\nu A^\mu = \partial_\nu \partial^\nu \left(a^\mu e^{ik\cdot x}\right) = \partial_\nu \left(i k^\nu a^\mu e^{ik\cdot x}\right) = -k^\nu k_\nu\, a^\mu e^{ik\cdot x} = 0 \quad \Rightarrow \quad k^\nu k_\nu = k^2 = 0$$

Polarizzazione

$\partial_\mu A^\mu = 0 \quad$ su $\quad A^\mu = a^\mu e^{ik\cdot x}$

$$\partial_\mu \left(a^\mu e^{ik\cdot x}\right) = -i k_\mu a^\mu e^{ik\cdot x} = 0 \quad \Rightarrow \quad k_\mu a^\mu = 0$$

$\begin{cases} k^2 = 0 \\ k_\mu a^\mu = 0 \end{cases}$ queste permettono di imporre condizioni sull'ampiezza a^μ

Poniamo $k^\mu = (k, 0, 0, k) \quad \Rightarrow \quad a^0 = a^3$

$\partial_\mu A^\mu = 0$ lascia ancora la libertà di gauge $\quad A^\mu \to A^\mu + \partial^\mu \chi$

$$\cancel{\partial_\mu A^\mu} + \partial_\mu \partial^\mu \chi = 0 \iff \Box \chi = 0 \quad \Rightarrow \quad \chi = \tilde{\chi}(k)\, e^{i\bar{k}\cdot\bar{x}}$$

$$A^\mu \to A^\mu + \partial^\mu \chi \quad \Rightarrow \quad a^\mu \to a^\mu + i k^\mu \chi$$

A questo punto a^μ ha solo 2 componenti libere: le 2 polarizzazioni (fisiche)

Tensore $F^{\mu\nu}$

$A^\mu = a^\mu e^{ik\cdot x}$

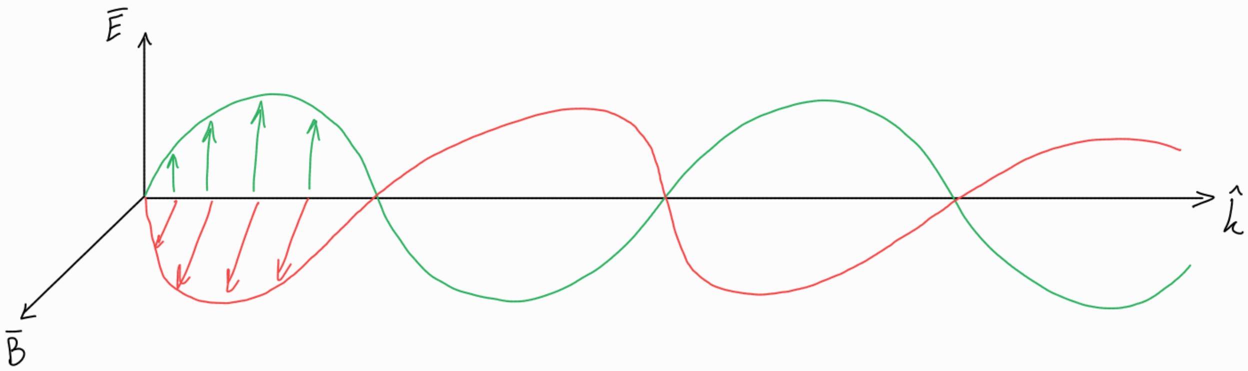
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \left(-i k^\mu a^\nu - i k^\nu a^\mu\right) e^{ik\cdot x}$$

$$F^{\mu\nu} F_{\mu\nu} = \left(-i k^\mu a^\nu - i k^\nu a^\mu\right)\left(-i k_\mu a_\nu - i k_\nu a_\mu\right) e^{i2k\cdot x}$$

$$= -\left(\underbrace{k^\mu k_\mu} a^\nu a_\nu + \underbrace{k^\nu a_\nu}\, \underbrace{k_\mu a^\mu} + \underbrace{k^\mu a_\mu}\, \underbrace{k_\nu a^\nu} + \underbrace{k^\nu k_\nu}_{k^2=0} a^\mu a_\mu\right) e^{i2k\cdot x}$$

$k^2 = 0$ $\qquad\qquad$ $k_\mu a^\mu = 0$

$$\Longrightarrow \quad F^{\mu\nu}F_{\mu\nu} = -4\left(E^2 - B^2\right) = 0 \quad \Longrightarrow \quad |E| = |B| \quad \text{in tutti i SR}$$



$$\tilde{F}^{\mu\nu}F_{\mu\nu} \propto \bar{E}\cdot\bar{B} = 0 \quad \Longrightarrow \quad \bar{E} \perp \bar{B} \quad \text{in tutti i SR}$$