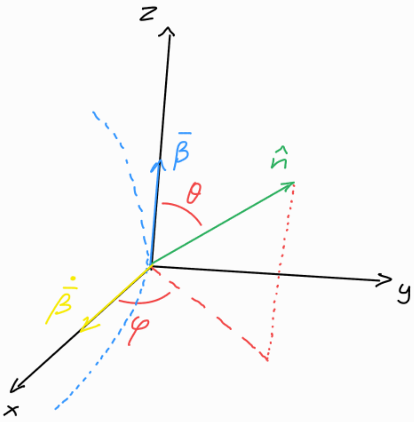
$$dP = \bar S\cdot\hat n\, R^2\, d\Omega \longrightarrow \frac{dP}{d\Omega} = R^2\,\bar S\cdot\hat n$$ $S \propto E^2$

Usando i campi ritardati: $$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\,\frac{\left(\hat n\wedge(\hat n-\bar\beta)\wedge\dot{\bar\beta}\right)^2}{(1-\hat n\cdot\bar\beta)^5}$$

$\hat n$: punta dalla particella all'osservatore



Poniamo ora: $\bar\beta = \beta\hat z$, $\dot{\bar\beta} = \dot\beta\hat x$

Usiamo le coordinate sferiche: $(\hat n, \hat\theta, \hat\varphi)$

$$\begin{cases}\hat n\wedge\hat\theta = \hat\varphi\\ \hat\theta\wedge\hat\varphi = \hat n\\ \hat\varphi\wedge\hat n = \hat\theta\end{cases} \qquad \begin{cases}\hat x = \sin\theta\cos\varphi\,\hat n + \cos\theta\cos\varphi\,\hat\theta - \sin\varphi\,\hat\varphi\\ \hat z = \cos\theta\,\hat n - \sin\theta\,\hat\theta\end{cases}$$

non usiamo $\hat y$

$$(\hat n-\bar\beta)\wedge\dot{\bar\beta} = (\hat n - \beta\hat z)\wedge\dot\beta\hat x$$
$$= \dot\beta\left(\hat n - \beta\left(\cos\theta\,\hat n - \sin\theta\,\hat\theta\right)\right)\wedge\left(\sin\theta\cos\varphi\,\hat n + \cos\theta\cos\varphi\,\hat\theta - \sin\varphi\,\hat\varphi\right)$$
$$= \dot\beta\Big(\xi_1\underbrace{\hat n\wedge\hat n}_{\bar 0} + \cos\theta\cos\varphi\underbrace{\hat n\wedge\hat\theta}_{\hat\varphi} - \sin\varphi\underbrace{\hat n\wedge\hat\varphi}_{-\hat\theta} + \xi_2\underbrace{\hat n\wedge\hat n}_{\bar 0} +$$
$$-\beta\cos^2\theta\cos\varphi\underbrace{\hat n\wedge\hat\theta}_{\hat\varphi} + \beta\cos\theta\sin\varphi\underbrace{\hat n\wedge\hat\varphi}_{-\hat\theta} + \beta\sin^2\theta\cos\varphi\underbrace{\hat\theta\wedge\hat n}_{-\hat\varphi} + \xi_3\underbrace{\hat\theta\wedge\hat\theta}_{\bar 0} - \beta\sin\theta\sin\varphi\underbrace{\hat\theta\wedge\hat\varphi}_{\hat n}\Big)$$
$$= \dot\beta\Big(-\beta\sin\theta\sin\varphi\,\hat n + \sin\varphi\,(1-\beta\cos\theta)\,\hat\theta + \big(\cos\theta\cos\varphi\underbrace{-\beta\cos^2\theta\cos\varphi - \beta\sin^2\theta\cos\varphi}_{-\beta\cos\varphi}\big)\,\hat\varphi$$
$$= \dot\beta\left(-\beta\sin\theta\sin\varphi\,\hat n + \sin\varphi(1-\beta\cos\theta)\,\hat\theta + \cos\varphi\,(\cos\theta-\beta)\,\hat\varphi\right)$$

$$\hat n\wedge(\hat n-\bar\beta)\wedge\dot{\bar\beta} = \dot\beta\Big(-\beta\sin\theta\sin\varphi\underbrace{\hat n\wedge\hat n}_{\bar 0} + \sin\varphi(1-\beta\cos\theta)\underbrace{\hat n\wedge\hat\theta}_{\hat\varphi} + \cos\varphi(\cos\theta-\beta)\underbrace{\hat n\wedge\hat\varphi}_{-\hat\theta}\Big)$$
$$= \dot\beta\left(\cos\varphi(\beta-\cos\theta)\,\hat\theta + \sin\varphi(1-\beta\cos\theta)\,\hat\varphi\right)$$

$$\left|\hat n\wedge(\hat n-\bar\beta)\wedge\dot{\bar\beta}\right|^2 = \dot\beta^2\left|\cos\varphi(\beta-\cos\theta)\,\hat\theta + \sin\varphi(1-\beta\cos\theta)\,\hat\varphi\right|^2$$
$$= \dot\beta^2\left(\cos^2\varphi\,(\beta-\cos\theta)^2 + \sin^2\varphi\,(1-\beta\cos\theta)^2\right)$$
$$= \dot\beta^2\Big(\cos^2\varphi\big(\beta^2+\cos^2\theta - 2\beta\cos\theta\big) + \dots$$

$$\dot\beta^2\left(\cos^2\varphi\left(\beta^2+\cos^2\theta-2\beta\cos\theta\right)+\sin^2\varphi\left(1-2\beta\cos\theta+\beta^2\cos^2\theta\right)\right)$$

$$=\dot\beta^2$$

$$=\dot\beta^2\left(1+\beta^2\cos^2\theta+2\beta\cos\theta\sin(2\varphi)\right)$$

DOVREBBE DARE QUESTO

$$\dot\beta^2\left(\cos^2\varphi\left(\beta^2+\cos^2\theta-1-\beta^2\cos^2\theta\right)+\left(1-\beta\cos\theta\right)^2\right)$$

$$=\dot\beta^2\left(\cos^2\varphi\sin^2\theta\left(1-\beta^2\right)+\left(1-\beta\cos\theta\right)^2\right)$$

$$\hat n\cdot\bar\beta=\beta\,\hat n\cdot\hat z=\beta\cos\theta$$

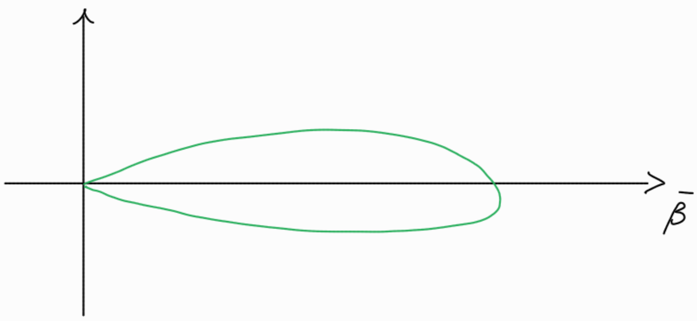
$$\frac{dP}{d\Omega}=\frac{e^2}{4\pi c}\,\frac{\dot\beta^2\left(\cos^2\varphi\sin^2\theta\overbrace{\left(1-\beta^2\right)}^{\gamma^{-2}}+\left(1-\beta\cos\theta\right)^2\right)}{\left(1-\beta\cos\theta\right)^5}$$

controlla per il segno meno

$$\boxed{\frac{dP}{d\Omega}=\frac{e}{4\pi c}\,\frac{\dot\beta^2}{\left(1-\beta\cos\theta\right)^3}\left(1-\frac{\cos^2\varphi\sin^2\theta}{\gamma^2\left(1-\beta\cos\theta\right)^2}\right)}$$

Radiazione di sincrotrone

$$\left.\frac{dP}{d\Omega}\right|_{\theta=0}\neq0$$



Limite ultrarelativistico:

$$\frac{dP}{d\Omega}\simeq\frac{2e^2}{\pi c}\,\frac{\gamma^6\dot\beta^2}{\left(1+\gamma^2\theta^2\right)^3}\left(1-\frac{4\gamma^2\theta^2\cos^2\varphi}{\left(1+\gamma^2\theta^2\right)^2}\right)$$

Lineare	Circolare
$P\propto e^2\dot\beta^2\gamma^6$	$P\propto e^2\dot\beta^2\gamma^4$
$F=\dot p=m\gamma^3\dot v$	$F=\dot p=m\gamma\dot v$
$P\propto\frac{e^2}{m^2c^3}\,\dot p^2$	$P\propto\frac{e^2\gamma^2}{m^2c^3}\,\dot p^2$

Anche se la potenza circolare è γ^2 volte più piccola, vista in funzione delle forze è γ^2 volte più grande

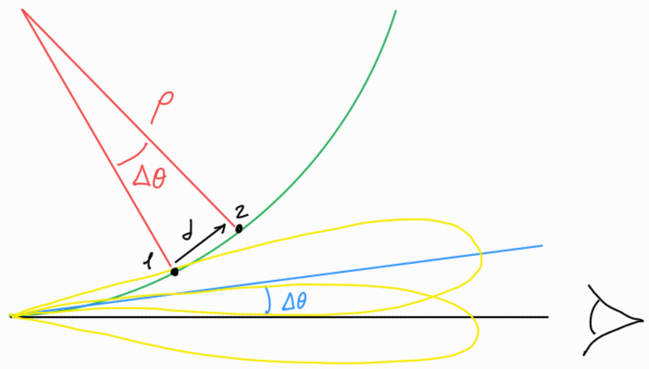
A parità di $\dot\beta$, $P_{lin}>P_{circ}$ ma è necessaria una forza $F_{lin}>F_{circ}$ per ottenere la stessa β

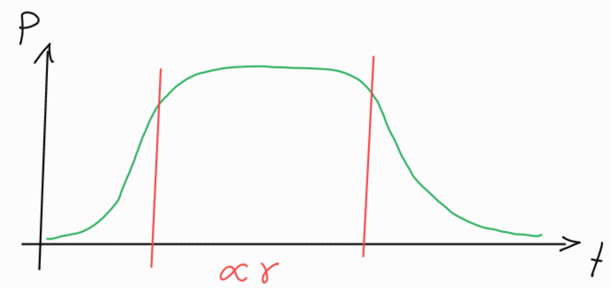
Sincrotrone

Di fatto, la radiazione in moto circolare e' (quasi) sempre piu' importante di quella da moto lineare

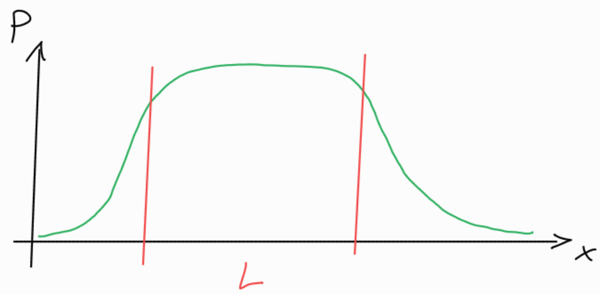
$$\rho = \frac{v^2}{a} = \frac{v^2}{\dot{v}_\perp} \quad \text{raggio}$$

La radiazione si vede solo per un certo intervallo di tempo.





$$\langle\theta\rangle \sim \frac{1}{\gamma} \implies d = \rho\Delta\theta = \frac{\rho}{\gamma}$$



R_i = distanza particella-osservatore

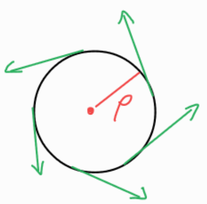
$$\Delta t = t_2 - t_2 = \frac{L}{c} = \left(t_2' - \frac{R_2}{c}\right) - \left(t_1' - \frac{R_1}{c}\right)$$

$$= \Delta t' - \frac{\Delta R}{c} = \Delta t' - \beta\Delta t'$$

$$= (1-\beta)\Delta t' \simeq \frac{2}{\gamma^2}\Delta t' \propto \frac{\rho}{v 2\gamma^3} \implies \Delta t \sim \frac{\rho}{\gamma^3}$$

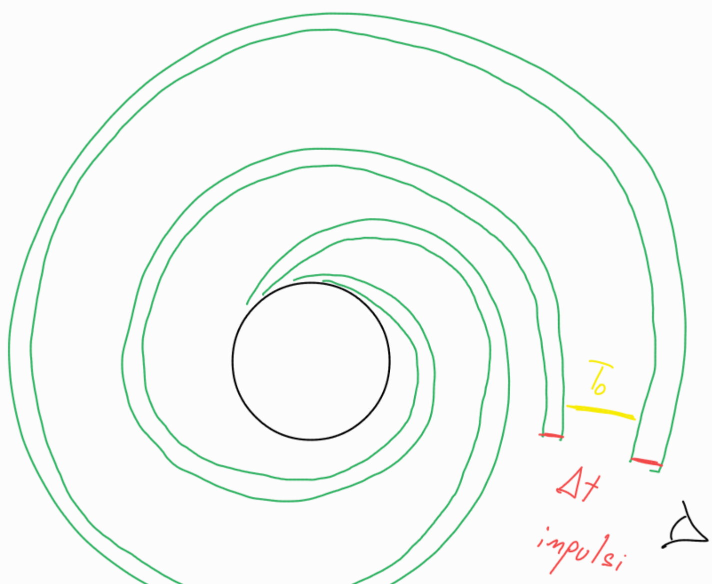
Siccome $\Delta t \Delta\nu \sim 1 \implies \Delta\nu \sim \frac{c}{\rho}\gamma^3$

In un sincrotrone:



periodo $T_0 = \frac{\rho}{v}$

$$\nu \sim \frac{c}{\rho}\gamma^3 = \nu_0\gamma^3$$



Esempio: $\mathcal{E}_{sinc} = 200\,MeV$ elettrone

$$\gamma = \frac{\mathcal{E}}{mc^2} = \frac{200\,MeV}{0.5\,MeV} = 400$$

$$\nu_0 = 3\cdot 10^8\,Hz$$ micro onde

$$\implies \nu = \gamma^3\nu_0 \simeq 2\cdot 10^{16}\,Hz$$ raggi X