$$\Delta t\,\Delta\nu \sim 1$$

$$\bar{S} = \frac{c}{4\pi} E^2 \hat{n} \qquad \frac{dP}{d\Omega} = (\bar{S}\cdot\hat{n})\, R^2 \Big|_{\text{ritardato}} \sim (1-\bar{\beta}\cdot\hat{n})^{-5}$$

Definiamo $\displaystyle \frac{dP}{d\Omega} \doteq |\bar{a}(t)|^2 \implies \bar{a} \doteq \sqrt{\frac{c}{4\pi}}\, R\, \bar{E}\Big|_{\text{ritardato}}$

$$\bar{a}(t) = \sqrt{\frac{e^2}{4\pi c}}\ \frac{\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}}{(1-\hat{n}\cdot\bar{\beta})^3}\Bigg|_{rit}$$

$$\bar{a}(t) = \frac{1}{\sqrt{2\pi}}\int d\omega\, e^{-i\omega t}\,\bar{a}(\omega)\ , \qquad \bar{a}(\omega) = \frac{1}{\sqrt{2\pi}}\int dt\, e^{i\omega t}\,\bar{a}(t)$$

trasf. Fourier

$$\frac{dP}{d\Omega} = \frac{1}{2\pi}\iint d\omega\, d\omega'\, \bar{a}(\omega)\, e^{-i\omega t}\, \bar{a}^*(\omega')\, e^{i\omega' t}$$

energia per unità di angolo solido

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} \frac{dP}{d\Omega}\, dt$$

$$= \iint d\omega\, d\omega'\, \bar{a}(\omega)\, \bar{a}^*(\omega') \overbrace{\int_{-\infty}^{\infty} dt\, e^{i(\omega'-\omega)t}}^{\delta(\omega'-\omega)}$$

$$= \iint d\omega\, d\omega'\, \bar{a}(\omega)\, \bar{a}^*(\omega')\, \delta(\omega'-\omega)$$

$$= \int_{-\infty}^{\infty} d\omega\, |\bar{a}(\omega)|^2$$

$$= 2\int_0^{\infty} d\omega\, |\bar{a}(\omega)|^2$$

$$= \int_0^{\infty} d\omega\, \frac{dI(\omega)}{d\Omega}$$

$$\frac{dI}{d\Omega} \doteq 2|\bar{a}(\omega)|^2$$

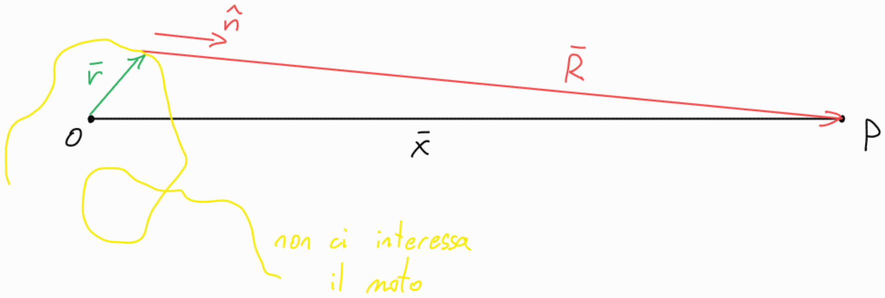
$$\frac{dI(\omega)}{d\Omega} = 2\,|\bar{a}(\omega)|^2$$

$$= 2\left|\frac{1}{\sqrt{2\pi}}\int dt\, e^{i\omega t}\,\bar{a}(t)\right|^2$$

$$= \left|\sqrt{\frac{e^2}{8\pi^2 c}}\int dt\, e^{i\omega t}\, \frac{\hat{n}\wedge(\hat{n}-\bar{\beta})\wedge\dot{\bar{\beta}}}{(1-\hat{n}\cdot\bar{\beta})^3}\Bigg|_{rit}\right|^2$$

$$= \left| \sqrt{\frac{e^2}{8\pi^2 c}} \int dt'\, e^{i\omega\left(t'+\frac{R}{c}\right)} \frac{\hat n \wedge (\hat n - \bar\beta)\wedge \dot{\bar\beta}}{(1-\hat n\cdot\bar\beta)^{\cancel{3}\,2}} \Bigg|_{rit} \cancel{(1-\hat n\cdot\bar\beta)} \right|$$

$$|\bar R| = |\bar x - \bar r| = |\bar x - \overbrace{\hat n \cdot \bar r\, \hat n}^{\text{cambia solo questo}}| \quad , \quad \bar R = R\,\hat n$$



$$\bar a(\omega) = \sqrt{\frac{e^2}{8\pi^2 c}} \int dt\, e^{i\omega\left(t - \frac{\hat n\cdot\bar r}{c}\right)} \frac{\hat n \wedge (\hat n - \bar\beta)\wedge \dot{\bar\beta}}{(1-\hat n\cdot\bar\beta)^2} \Bigg|_{rit} \qquad k \doteq 1-\hat n\cdot\bar\beta$$

$$\frac{d}{dt}\left(\frac{\hat n\wedge(\hat n\wedge\bar\beta)}{k}\right) = -\frac{\hat n\wedge(\hat n\wedge\bar\beta)}{k^2}\left(-\hat n\cdot\dot{\bar\beta}\right) + \frac{\hat n\wedge(\hat n\wedge\dot{\bar\beta})}{k^2}\left(1-\hat n\cdot\bar\beta\right)$$

$\frac{d\hat n}{dt}$ sono derivate del secondo ordine rispetto a $\dot{\bar\beta}$ (l'osservatore e' molto lontano)

$$= \frac{\hat n\wedge(\hat n\wedge\dot{\bar\beta})}{k^2} + \frac{\hat n\wedge\left(\hat n\wedge\overbrace{\left((\hat n\cdot\bar\beta)\dot{\bar\beta} - (\hat n\cdot\dot{\bar\beta})\bar\beta\right)}^{\hat n\wedge\bar\beta\wedge\dot{\bar\beta}}\right)}{k^2}$$

$$= \frac{\hat n\wedge(\hat n\wedge\dot{\bar\beta})}{k^2} + \frac{\hat n\wedge\left(\hat n\wedge\left(\hat n\wedge(\bar\beta\wedge\dot{\bar\beta})\right)\right)}{k^2} \qquad \hat n\wedge(\hat n\wedge\bar v) = -\bar v$$

$$= \frac{\hat n\wedge(\hat n\wedge\dot{\bar\beta})}{k^2} - \frac{\hat n\wedge\bar\beta\wedge\dot{\bar\beta}}{k^2}$$

$$= \frac{\hat n\wedge(\hat n-\bar\beta)\wedge\dot{\bar\beta}}{k^2}$$

$$\bar a(\omega) = \sqrt{\frac{e^2}{8\pi^2 c}} \int dt\, e^{i\omega\left(t - \frac{\hat n\cdot\bar r}{c}\right)} \frac{d}{dt}\left(\frac{\hat n\wedge(\hat n\wedge\bar\beta)}{k}\right)\Bigg|_{rit}$$

per parti

$$\dot{\bar r} = \bar\beta$$

$$= \sqrt{\frac{e^2}{8\pi^2 c}}\left(\cancel{\left[e^{i\omega\left(t-\frac{\hat n\cdot\bar r}{c}\right)}\left(\frac{\hat n\wedge(\hat n\wedge\bar\beta)}{k}\right)\right]_{-\infty}^{\infty}} - \int dt\, \frac{\hat n\wedge(\hat n\wedge\bar\beta)}{\cancel{k}}\, e^{i\omega\left(t-\frac{\hat n\cdot\bar r}{c}\right)}\, i\omega\cancel{(1-\hat n\cdot\bar\beta)}\right)$$

per $t=\pm\infty$ la particella e' fuori dal tempo in cui ha interagito

$$= \sqrt{\frac{e^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt\left(\hat n\wedge(\hat n\wedge\bar\beta)\right) i\omega\, e^{i\omega\left(t-\frac{\hat n\cdot\bar r}{c}\right)}$$

$$\frac{dI}{d\Omega} = 2\left|\bar{a}(\omega)\right|^2 = \frac{e^2\omega^2}{8\pi^2 c}\left|\int_{-\infty} dt \left(\hat{n}\wedge\left(\hat{n}\wedge\bar{\beta}\right)\right) e^{i\omega\left(t-\frac{n\cdot r}{c}\right)}\right|$$

raggio istantaneo di curvatura

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)\left(\frac{1}{\gamma^2}+\theta^2\right)\left(K_{2/3}(\xi)+\left(\frac{\theta^2}{\frac{1}{\gamma^2}+\theta^2}\right)K^2_{1/3}(\xi)\right), \qquad \xi \doteq \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}$$

funzioni di Bessel modificate

Per basse frequenze: $\frac{dI}{d\omega} \sim \frac{e^2}{c}\left(\frac{\omega\rho}{c}\right)^{1/3}, \qquad \omega \ll \omega_c \doteq \frac{3}{2}\gamma^3\frac{c}{\rho}$

Per alte frequenze: $\frac{dI}{d\omega} \sim \frac{e^2}{c}\gamma\sqrt{\frac{\omega}{\omega_c}}\, e^{-\frac{\omega}{\omega_c}}, \qquad \omega \gg \omega_c$

Dal punto di vista quantistico: $\frac{dI}{d\omega} = \sqrt{3}\,\frac{e^2}{c}\gamma\frac{\omega}{\omega_c}\int K_{5/3}(x)\,dx$ dunque

$$\frac{dN}{d\omega} = \frac{\sqrt{3}\,e^2\gamma}{\hbar c}\frac{\omega}{\omega_c}\int K_{5/3}(x)\,dx, \qquad N = \#\text{fotoni}$$