Eqq. Maxwell (unità SI)

$$\bar\nabla\cdot\bar E = \frac{\rho}{\varepsilon_0} \qquad \bar\nabla\cdot\bar B = 0$$

$$\bar\nabla\wedge\bar E = -\frac{\partial\bar B}{\partial t} \qquad \bar\nabla\wedge\bar B = \mu_0\left(\bar J + \varepsilon_0\frac{\partial\bar E}{\partial t}\right)$$

ε_0 e μ_0 sono costanti del vuoto

Scegliamo unità di misura per $\{m, \ell, t, q\}$

corrente $I = \frac{dq}{dt}$, $\bar\nabla\cdot\bar J + \frac{\partial\rho}{\partial t} = 0$

① elettrostatica $F = k_1\frac{q_1 q_2}{r^2} \Rightarrow [k_1 q_1 q_2] = [F r^2] = N\ell^2 = m\ell t^{-2}\ell^2 = m\ell^3 t^{-2}$

$\Rightarrow [k_1] = m\ell^3 t^{-2} q^{-2}$

② campi $\mathcal{E} = \frac{F}{q_1} = k_1\frac{q_2}{r^2}$

forza tra due fili in cui passa corrente

③ $\frac{dF}{d\ell} = 2k_2\frac{I_1 I_2}{d} \Rightarrow [k_2 I_1 I_2] = m\ell t^{-2}$ $\qquad [I] = \left[\frac{dq}{dt}\right] = q t^{-1}$

$[k_2] = m\ell t^{-2} q^{-2} t^2$

Dunque $\left[\frac{k_1}{k_2}\right] = \frac{\cancel{m}\,\ell^{\cancel{3}\,2}\,\cancel{t^{-2}}\,\cancel{q^{-2}}}{\cancel{m}\,\cancel{\ell}\,\cancel{t^{-2}}\,\cancel{q^{-2}}\,t^2} = \ell^2 t^{-2} = [v^2]$

$B = 2k_2\alpha\frac{I}{d} \Rightarrow \left[\frac{E}{B}\right] = \frac{\ell}{t}\frac{1}{\alpha}$

$\bar\nabla\wedge\bar E + k_3\frac{\partial\bar B}{\partial t} = \bar 0 \Rightarrow [k_3] = \frac{1}{\alpha}$

Eqq Maxwell (unità generiche)

$$\bar\nabla\cdot\bar E = 4\pi k_1\rho \qquad \bar\nabla\cdot\bar B = 0$$

$$\bar\nabla\wedge\bar E + k_3\frac{\partial\bar B}{\partial t} = \bar 0 \qquad \bar\nabla\wedge\bar B = 4\pi k_2\alpha\bar J + \frac{k_2}{k_1}\alpha\frac{\partial\bar E}{\partial t}$$

senza sorgenti: $\bar J = \bar 0 \wedge \rho = 0$

$$\begin{cases} \bar{\nabla} \wedge \bar{E} + k_3 \dfrac{\partial \bar{B}}{\partial t} = \bar{0} \\ \bar{\nabla} \wedge \bar{B} = \dfrac{k_2}{k_1} \alpha \dfrac{\partial \bar{E}}{\partial t} \end{cases}$$

$$\bar{\nabla} \wedge (\bar{\nabla} \wedge \bar{E}) = \bar{\nabla}(\overbrace{\bar{\nabla} \cdot \bar{E}}^{=0}) - \nabla^2 \bar{E} = -\nabla^2 \bar{E}$$

$$k_3\, \bar{\nabla} \wedge \left(\frac{\partial \bar{B}}{\partial t} \right) = k_3 \frac{\partial}{\partial t} (\bar{\nabla} \wedge \bar{B}) = k_3 \frac{k_2}{k_1} \alpha \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\bar{\nabla} \wedge \left(\bar{\nabla} \wedge \bar{E} + k_3 \frac{\partial \bar{B}}{\partial t} \right) = \bar{\nabla} \wedge \bar{0} = \bar{0} \quad \Rightarrow \quad \nabla^2 \bar{E} - \frac{k_2 k_3}{k_1} \alpha \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

ma sappiamo che $\nabla^2 \bar{E} - \dfrac{1}{c^2} \dfrac{\partial^2 \bar{E}}{\partial t^2} = \bar{0} \Rightarrow \dfrac{k_2 k_3}{k_1} \alpha = \dfrac{1}{c^2}$

siccome $\dfrac{k_1}{k_2} = c^2 \Rightarrow \dfrac{1}{k_3 \alpha} = 1 \Rightarrow k_3 = \dfrac{1}{\alpha}$

SI:	$k_1 = \dfrac{1}{4\pi\varepsilon_0}$	$k_2 = \dfrac{1}{4\pi}$	$k_3 = 1$	$\alpha = 1$
Gauss:	$k_1 = 1$	$k_2 = \dfrac{1}{c^2}$	$k_3 = \dfrac{1}{c}$	$\alpha = c$

Trasf. galileiane sulle eqq. di Maxwell

$$\begin{cases} \bar{x}' = \bar{x} - \bar{u} t \\ t' = t \end{cases}$$ trasf. galileiane

$\bar{v}' = \bar{v} - \bar{u}$ velocità relativa tra SR

$\bar{a}' = \bar{a}$

$$\frac{\partial}{\partial t} \left((x')^i + u^i t \right) = \frac{\partial (x')^i}{\partial t} + u^i = (v')^i + u^i = v^i \quad ???$$

$$f(\bar{x}', t') = f(\bar{x}'(\bar{x}, t), t'(\bar{x}, t))$$

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'} \frac{\partial}{\partial t} + \frac{\partial x^i}{\partial t'} \frac{\partial}{\partial x^i} = \overbrace{\frac{\partial t}{\partial t'}}^{1} \frac{\partial}{\partial t} + \overbrace{\frac{\partial t}{\partial t'}}^{1} \underbrace{\frac{\partial x^i}{\partial t}}_{u^i} \frac{\partial}{\partial x^i} = \frac{\partial}{\partial t} + u^i \frac{\partial}{\partial x^i} \Rightarrow \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \bar{u} \cdot \bar{\nabla}$$

$$\frac{\partial}{\partial (x')^i} = \underbrace{\frac{\partial x^i}{\partial (x')^i}}_{1} \frac{\partial}{\partial x^i} + \cancelto{0}{\frac{\partial t}{\partial (x')^i}} \frac{\partial}{\partial t} = \frac{\partial}{\partial x^i} \Rightarrow \bar{\nabla}' = \bar{\nabla}$$

volume $\begin{cases} V' = V \\ q' = q \end{cases} \Rightarrow \rho' = \rho$ principio sperimentale $\dfrac{\partial \rho}{\partial t} = 0$

$$\bar{J} = \rho \bar{v} \Rightarrow \bar{J}' = \rho \bar{v}' = \rho (\bar{v} - \bar{u}) = \bar{J} - \rho \bar{u}$$

$$\bar{a}' = \bar{a} \Rightarrow F' = F \quad \because \quad m' = m$$

$\bar F = q(\bar E + \bar v \wedge \bar B)$ forza di Lorentz

$$\begin{cases} \bar F' = \bar F \\ q' = q \end{cases} \Longrightarrow \bar E + \bar v \wedge \bar B = \bar E' + \bar v' \wedge \bar B' = \bar E' + (\bar v - \bar u) \wedge \bar B'$$

vorremmo che la trasf. dipendesse solo da $\bar u$ quindi

$$\bar E + \bar v \wedge \bar B = \bar E' + (\bar v - \bar u) \wedge \bar B'$$

$$E + \bar v \wedge (\bar B - \bar B') = \bar E' - \bar u \wedge \bar B' \Longrightarrow \begin{cases} \bar B' = \bar B \\ \bar E' = \bar E + \bar u \wedge \bar B' \end{cases}$$

questo non e' detto che succeda

$$\begin{cases} \bar\nabla' \cdot \bar B' = 0 \\ \bar\nabla' = \bar\nabla \\ \bar B' = \bar B \end{cases} \Longrightarrow \bar\nabla \cdot \bar B = 0 \quad \checkmark$$

$$\bar\nabla' \wedge \bar E' + \frac{\partial \bar B'}{\partial t'} = \bar\nabla \wedge (\bar E + \bar u \wedge \bar B) + \left(\frac{\partial}{\partial t} + \bar u \cdot \bar\nabla\right) \bar B = \overbrace{\bar\nabla \wedge \bar E' + \frac{\partial \bar B}{\partial t}}^{=0} + \bar\nabla \wedge (\bar u \wedge \bar B) + (\bar u \cdot \bar\nabla) \bar B$$

$$= \bar u (\overbrace{\bar\nabla \cdot \bar B}^{=0}) - B(\cancel{\bar\nabla \cdot \bar u}) + \underbrace{(\bar B \cdot \bar\nabla)\bar u}_{=0} - \cancel{(\bar u \cdot \bar\nabla)\bar B} + \cancel{(\bar u \cdot \bar\nabla)\bar B} = 0 \quad \checkmark$$

$\bar u$ cost.

$$\bar\nabla' \cdot \bar E' - \frac{\rho'}{\varepsilon_0} = \bar\nabla \cdot (\bar E + \bar u \wedge \bar B) - \frac{\rho}{\varepsilon_0} = \overbrace{\bar\nabla \cdot \bar E - \frac{\rho}{\varepsilon_0}}^{=0} + \bar\nabla \cdot (\bar u \wedge \bar B) =$$

$$= \bar B \cdot (\underbrace{\bar\nabla \wedge \bar u}_{=0}) - \bar u \cdot (\bar\nabla \wedge \bar B) = \boxed{-\bar u \cdot (\bar\nabla \wedge \bar B)} \neq 0$$

in generale questi non valgono

$c^{-2} \doteq \mu_0 \varepsilon_0 = \text{cost.}$

$$\bar\nabla' \wedge \bar B' - \mu_0 \bar J' - \frac{1}{c^2} \frac{\partial \bar E'}{\partial t'} = \bar\nabla \wedge \bar B - \mu_0 (\bar J - \rho \bar u) - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + \bar u \cdot \bar\nabla\right)(\bar E + \bar u \wedge \bar B)$$

$$= \underbrace{\bar\nabla \wedge \bar B - \mu_0 \bar J - \frac{1}{c^2}\frac{\partial \bar E}{\partial t}}_{=0} + \boxed{\mu_0 \rho \bar u - \frac{1}{c^2}\left(\bar u \wedge \frac{\partial \bar B}{\partial t} + (\bar u \cdot \bar\nabla)(\bar u \wedge \bar B)\right)} \neq 0$$

$$\bar\nabla' \cdot \bar J' + \frac{\partial \rho'}{\partial t'} = \text{DIMOSTRA}$$