A^μ si trasforma come un vettore di Lorentz:

Boost x

$$\begin{cases} \varphi' = \gamma\left(\varphi - \frac{v}{c}A_x\right) \\ (A')_x = \gamma\left(A_x - \frac{v}{c}\varphi\right) \\ (A')_y = A_y \\ (A')_z = A_z \end{cases}$$

$$(x')^\mu = \Lambda^\mu{}_\nu x^\nu \implies (F')^{\mu\nu} = \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma F^{\rho\sigma}$$

Boost x: $\Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\qquad E^i = F^{i0}$

$$(F')^{i0} = \Lambda^i{}_\rho \Lambda^0{}_\sigma F^{\rho\sigma}$$

$$\begin{aligned}
(F')^{10} &= \Lambda^1{}_\rho \Lambda^0{}_\sigma F^{\rho\sigma} \\
&= \Lambda^1{}_0 \Lambda^0{}_\sigma F^{0\sigma} + \Lambda^1{}_1 \Lambda^0{}_\sigma F^{1\sigma} + \cancel{\overbrace{\Lambda^1{}_2}^{0} \Lambda^0{}_\sigma F^{2\sigma}} + \cancel{\overbrace{\Lambda^1{}_3}^{0} \Lambda^0{}_\sigma F^{3\sigma}} \\
&= \cancel{\Lambda^1{}_0 \Lambda^0{}_0 \underbrace{F^{00}}_{0}} + \Lambda^1{}_0 \Lambda^0{}_1 F^{01} + \cancel{\Lambda^1{}_0 \overbrace{\Lambda^0{}_2}^{0} F^{02}} + \cancel{\Lambda^1{}_0 \overbrace{\Lambda^0{}_3}^{0} F^{03}} \\
&\quad + \Lambda^1{}_1 \Lambda^0{}_0 F^{10} + \cancel{\Lambda^1{}_1 \Lambda^0{}_1 \underbrace{F^{11}}_{0}} + \cancel{\Lambda^1{}_1 \overbrace{\Lambda^0{}_2}^{0} F^{12}} + \cancel{\Lambda^1{}_1 \overbrace{\Lambda^0{}_3}^{0} F^{13}}
\end{aligned}$$

$F^{01} = -F^{10}$

$$\begin{aligned}
&= -\beta\gamma(-\beta\gamma)F^{01} + \gamma(\gamma)F^{10} \\
&= -\beta^2\gamma^2 F^{10} + \gamma^2 F^{10} \\
&= \cancel{\gamma^2}\cancel{(1-\beta^2)}F^{10} \qquad \gamma^2 = \frac{1}{1-\beta^2} \\
&= F^{10}
\end{aligned}$$

$$\begin{cases} E'_x = E_x \\ E'_y = \gamma(E_y - \beta B_z) \\ E'_z = \gamma(E_z + \beta B_y) \end{cases}$$

le componenti si mischiano (boost x)

$$\begin{cases} B'_x = B_x \\ B'_y = \gamma(B_y + \beta E_z) \\ B'_z = \gamma(B_z - \beta E_y) \end{cases}$$

Per una direzione generica: $\begin{cases} \bar{E}'_{\parallel} = \bar{E}_{\parallel} \\ \bar{E}'_{\perp} = \gamma(\bar{E}_{\perp} + \bar{\beta} \wedge \bar{B}) \end{cases}$ $\begin{cases} \bar{B}'_{\parallel} = \bar{B}_{\parallel} \\ \bar{B}'_{\perp} = \gamma(\bar{B}_{\perp} - \bar{\beta} \wedge \bar{E}) \end{cases}$

Se in un SR $\bar{E}_{\perp} = \bar{0}$, possiamo trovare un boost per cui $\bar{E}'_{\perp} = \bar{\beta} \wedge \bar{B} \neq \bar{0}$

Se $\bar{E} \cdot \bar{B} = 0 \longrightarrow \begin{cases} |\bar{E}| < |\bar{B}| \implies \bar{E}' = \bar{0} \\ |\bar{E}| > |\bar{B}| \implies \bar{B}' = \bar{0} \end{cases}$

Ancora più in generale $\begin{cases} \bar{E}' = \gamma(\bar{E} + \bar{\beta} \wedge \bar{B}) - \frac{\gamma - 1}{\beta^2}(\bar{E} \cdot \bar{\beta})\bar{\beta} \\ \bar{B}' = \gamma(\bar{B} - \bar{\beta} \wedge \bar{E}) - \frac{\gamma - 1}{\beta^2}(\bar{B} \cdot \bar{\beta})\bar{\beta} \end{cases}$

Per piccole velocità $(v \ll c)$ $\begin{cases} \bar{E}' = \bar{E} + \bar{\beta} \wedge \bar{B} \\ \bar{B}' = \bar{B} + \bar{\beta} \wedge \bar{E} \end{cases}$