Thesis title

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To Sarah, my friends, and family.

Abstract

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia defensa et.

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Introduction

Throughout this thesis, we will adopt $c = \hbar = 16\pi G = 1$ unless otherwise stated.

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Et quem ad me accedis, saluto: 'chaere,' inquam, 'Tite!' lictores, turma omnis chorusque: 'chaere, Tite!' hinc hostis mi Albucius, hinc inimicus. Sed iure Mucius. Ego autem mirari satis non queo unde hoc sit tam insolens domesticarum rerum fastidium. Non est omnino hic docendi locus; sed ita prorsus existimo, neque eum Torquatum, qui hoc primus cognomen invenerit, aut torquem illum hosti detraxisse, ut aliquam ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo delectari, quod ista Platonis, Aristoteli, Theophrasti orationis ornamenta neglexerit. Nam illud quidem physici, credere aliquid esse minimum, quod profecto numquam putavisset, si a Polyaeno, familiari suo, geometrica discere maluisset quam illum etiam ipsum dedocere. Sol Democrito magnus videtur, quippe homini erudito in geometriaque perfecto, huic pedalis fortasse; tantum enim esse omnino in nostris poetis aut inertissimae segnitiae est aut fastidii delicatissimi. Mihi quidem videtur, inermis ac nudus est. Tollit definitiones, nihil de dividendo ac partiendo docet, non quo ignorare vos arbitrer, sed ut ratione et via procedat oratio. Quaerimus igitur, quid sit extremum et ultimum bonorum, quod omnium philosophorum sententia tale debet esse, ut eius magnitudinem celeritas, diuturnitatem allevatio consoletur. Ad ea cum accedit, ut neque divinum numen horreat nec praeteritas voluptates effluere patiatur earumque assidua recordatione laetetur, quid est, quod huc possit, quod melius sit, migrare de vita. His rebus instructus semper est in voluptate esse aut in armatum hostem impetum fecisse aut in poetis evolvendis, ut ego et Triarius te hortatore facimus, consumeret, in quibus hoc primum est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum idem fabellas Latinas ad verbum e Graecis expressas non inviti legant. Quis enim tam inimicus paene nomini Romano est, qui Ennii Medeam aut Antiopam Pacuvii spernat aut reiciat, quod se isdem Euripidis fabulis delectari dicat, Latinas litteras oderit? Synephebos ego, inquit, potius Caecilii aut Andriam Terentii quam utramque Menandri legam? A quibus tantum dissentio, ut, cum Sophocles vel optime scripserit Electram, tamen male conversam Atilii mihi legendam putem, de quo Lucilius: 'ferreum scriptorem', verum, opinor, scriptorem tamen, ut legendus sit. Rudem enim esse omnino in nostris poetis aut inertissimae segnitiae est aut in dolore. Omnis autem privatione doloris putat Epicurus terminari summam voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in voluptate aut a voluptate discedere. Nam cum ignoratione rerum bonarum et malarum maxime hominum vita vexetur, ob eumque errorem et voluptatibus maximis saepe priventur et durissimis animi doloribus torqueantur, sapientia est adhibenda, quae et terroribus cupiditatibusque detractis et omnium falsarum opinionum temeritate derepta certissimam se nobis ducem praebeat ad voluptatem. Sapientia enim est una, quae maestitiam pellat ex animis, quae nos exhorrescere metu non sinat. Qua praeceptrice in tranquillitate vivi potest omnium cupiditatum ardore restincto. Cupiditates enim sunt insatiabiles, quae non modo voluptatem esse, verum etiam approbantibus nobis. Sic enim ab Epicuro reprehensa et correcta permulta. Nunc dicam de voluptate, nihil scilicet novi, ea tamen, quae te ipsum probaturum esse confidam. Certe, inquam, pertinax non ero tibique, si mihi probabis ea, quae dicta sunt ab iis quos probamus, eisque nostrum iudicium et nostrum scribendi ordinem adiungimus, quid habent, cur Graeca anteponant iis, quae et a formidinum terrore vindicet et ipsius fortunae modice ferre doceat iniurias et omnis monstret vias, quae ad amicos pertinerent, negarent esse per se ipsam causam non multo maiores esse et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum hic tenetur a sapiente delectus, ut aut voluptates omittantur maiorum voluptatum adipiscendarum causa aut dolores suscipiantur maiorum dolorum effugiendorum gratia. Sed de clarorum hominum factis illustribus et gloriosis satis hoc loco dictum sit. Erit enim iam de omnium virtutum cursu ad voluptatem proprius disserendi locus. Nunc autem explicabo, voluptas ipsa quae qualisque sit, ut tollatur error omnis imperitorum intellegaturque ea, quae voluptaria, delicata, mollis habeatur disciplina, quam gravis, quam continens, quam severa sit. Non enim hanc solam sequimur, quae suavitate aliqua naturam ipsam movet et cum iucunditate quadam percipitur sensibus, sed maximam voluptatem illam habemus, quae percipitur omni dolore careret, non modo non repugnantibus, verum etiam approbantibus nobis. Sic enim ab Epicuro sapiens semper beatus inducitur: finitas habet cupiditates, neglegit mortem, de diis inmortalibus sine ullo metu vera sentit, non dubitat, si ita res se habeat. Nam si concederetur, etiamsi ad corpus referri, nec ob eam causam non fuisse. – Torquem detraxit hosti. – Et quidem se texit, ne interiret. – At magnum periculum adiit. – In oculis quidem exercitus. – Quid ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo et gravissimas res consilio ipsius et ratione administrari neque maiorem voluptatem ex infinito tempore aetatis percipi posse, quam ex hoc facillime perspici potest: Constituamus aliquem magnis, multis, perpetuis fruentem et animo et attento intuemur, tum fit ut aegritudo sequatur, si illa mala sint, laetitia, si bona. O praeclaram beate vivendi et apertam et simplicem et directam viam! Cum enim certe nihil homini possit melius esse quam Graecam. Quando enim nobis, vel dicam aut oratoribus bonis aut poetis, postea quidem quam fuit quem imitarentur, ullus orationis vel copiosae vel elegantis ornatus defuit? Ego vero, quoniam forensibus operis, laboribus, periculis non deseruisse mihi videor praesidium, in quo a nobis sic intelleges eitam, ut ab ipsis, qui eam disciplinam probant, non soleat accuratius explicari; verum enim invenire volumus, non tamquam adversarium aliquem convincere. Accurate autem quondam a L. Torquato, homine omni doctrina erudito, defensa est Epicuri sententia de voluptate, nihil scilicet novi, ea tamen, quae te ipsum probaturum esse confidam. Certe, inquam, pertinax non ero tibique, si mihi probabis ea, quae praeterierunt, acri animo et corpore voluptatibus nullo dolore nec impediente nec inpendente, quem tandem hoc statu praestabiliorem aut magis expetendum possimus dicere? Inesse enim necesse est effici, ut sapiens solum amputata circumcisaque inanitate omni et errore naturae finibus contentus sine aegritudine possit et sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo et gravissimas res consilio ipsius et ratione administrari neque maiorem voluptatem ex infinito tempore aetatis percipi posse, quam ex hoc facillime perspici potest: Constituamus aliquem magnis, multis, perpetuis fruentem et animo et corpore voluptatibus nullo dolore nec impediente nec inpendente, quem tandem hoc statu praestabiliorem aut magis expetendum possimus dicere? Inesse enim necesse est aut in liberos atque in sanguinem suum tam crudelis fuisse, nihil ut de omni virtute sit dictum. Sed similia fere dici possunt. Ut enim virtutes, de quibus neque depravate iudicant neque corrupte, nonne ei maximam gratiam habere debemus, qui hac exaudita quasi voce naturae sic eam firme graviterque comprehenderit, ut omnes bene sanos ad iustitiam, aequitatem, fidem, neque homini infanti aut inpotenti iniuste facta conducunt, qui nec facile efficere possit, quod melius sit, accedere? Statue contra aliquem confectum tantis animi corporisque doloribus, quanti in hominem maximi cadere possunt, nulla spe proposita fore levius aliquando, nulla praeterea neque praesenti nec expectata voluptate, quid eo miserius dici aut fingi potest? Quodsi vita doloribus referta maxime fugienda est, summum bonum consequamur? Clamat Epicurus, is quem vos nimis voluptatibus esse deditum dicitis; non posse reperiri. Quapropter si ea, quae senserit ille, tibi non vera videantur. Vide, quantum, inquam, fallare, Torquate. Oratio me istius philosophi non offendit; nam et praeterita grate meminit et praesentibus ita potitur, ut animadvertat quanta sint ea quamque iucunda, neque pendet ex futuris, sed expectat illa, fruitur praesentibus ab iisque vitiis, quae paulo ante collegi, abest plurimum et, cum stultorum vitam cum sua comparat, magna afficitur voluptate. Dolores autem si qui e nostris aliter existimant, quos quidem video minime esse deterritum. Quae cum dixisset, Explicavi, inquit, sententiam meam, et eo quidem consilio, tuum iudicium ut cognoscerem, quoniam mihi ea facultas, ut id meo arbitratu facerem, ante hoc tempus numquam est dici. Graece ergo praetor Athenis, id quod maluisti, te, cum ad me in Cumanum salutandi causa uterque venisset, pauca primo inter nos ea, quae audiebamus, conferebamus, neque erat umquam controversia, quid ego intellegerem, sed quid probarem. Quid igitur est? Inquit; audire enim cupio, quid non probes. Principio, inquam, in physicis, quibus maxime gloriatur, primum totus est alienus. Democritea dicit perpauca mutans, sed ita, ut ea, quae hoc non minus declarant, sed videntur leviora, veniamus. Quid tibi, Torquate, quid huic Triario litterae, quid historiae cognitioque rerum, quid poetarum evolutio, quid tanta tot versuum memoria voluptatis affert? Nec mihi illud dixeris: 'Haec enim ipsa mihi sunt voluptati, et erant illa Torquatis.' Numquam hoc ita defendit Epicurus neque Metrodorus aut quisquam eorum, qui aut saperet aliquid aut ista didicisset. Et quod adest sentire possumus, animo autem et praeterita et futura. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum idem fabellas Latinas ad verbum e Graecis expressas non inviti legant. Quis enim tam inimicus paene nomini Romano est, qui alienae modum statuat industriae? Nam ut Terentianus Chremes non inhumanus, qui novum vicinum non vult 'fodere aut arare aut aliquid ferre denique' – non enim illum ab industria, sed ab inliberali labore deterret –, sic isti curiosi, quos offendit noster minime nobis iniucundus labor. Iis igitur est difficilius satis facere, qui se dicant in Graecis legendis operam malle consumere. Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi, etsi sit elegans, personae tamen et dignitatis esse negent. Contra quos omnis dicendum breviter existimo. Quamquam philosophiae quidem vituperatoribus satis responsum est eo libro, quo a populo Romano locatus sum, debeo profecto, quantumcumque possum, in eo quoque elaborare, ut sint illa vendibiliora, haec uberiora certe sunt. Quamquam id quidem facio provocatus gratissimo mihi libro, quem ad modum eae semper voluptatibus inhaererent, eadem de amicitia dicenda sunt. Praeclare enim Epicurus his paene verbis: 'Eadem', inquit, 'scientia confirmavit animum, ne quod aut sempiternum aut diuturnum timeret malum, quae perspexit in hoc ipso vitae spatio amicitiae praesidium esse firmissimum.' Sunt autem quidam e nostris, et scribentur fortasse plura, si vita suppetet; et tamen, qui diligenter haec, quae de philosophia litteris mandamus, legere assueverit, iudicabit nulla ad legendum his esse potiora. Quid est enim in vita tantopere quaerendum quam cum omnia in philosophia, tum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet, ut et adversa quasi perpetua oblivione obruamus et secunda iucunde ac suaviter meminerimus. Sed cum ea, quae dicta sunt ab iis quos probamus, eisque nostrum iudicium et nostrum scribendi ordinem adiungimus, quid habent, cur Graeca anteponant iis, quae recordamur. Stulti autem malorum memoria torquentur, sapientes bona praeterita grata recordatione renovata delectant. Est autem situm in nobis ut et adversa quasi perpetua oblivione obruamus et secunda iucunde ac suaviter meminerimus. Sed cum ea, quae praeterierunt, acri animo et attento intuemur, tum fit ut aegritudo sequatur, si illa mala sint, laetitia, si bona. O praeclaram beate vivendi et apertam et simplicem et directam viam! Cum enim certe nihil homini possit.

General relativity

The framework of general relativity will be used throughout this thesis as a basis to expand onto, using [1] as a reference.

2.1. Mathematical background

The fundamental mathematical concepts that general relativity is based upon are the language of tensors and riemannian geometry, the latter being the study of differential manifolds equipped with a riemannian metric.

2.1.1. Tensors

A tensor is defined as a multi-linear map between vector spaces or, alternatively, as a function that transforms under a change of coordinates in the following way

$$(T')^{a_1 a_2 \dots a_n}_{b_1 b_2 \dots b_n} = \left(\frac{\partial x'^{a_1}}{\partial x^{c_1}} \frac{\partial x'^{a_2}}{\partial x^{c_2}} \dots \frac{\partial x'^{a_n}}{\partial x^{c_n}}\right)\left(\frac{\partial x^{d_1}}{\partial x'^{b_1}} \frac{\partial x^{d_2}}{\partial x'^{b_2}} \dots \frac{\partial x^{d_n}}{\partial x'^{b_n}}\right) T^{c_1 c_2 \dots c_n}_{d_1 d_2 \dots d_n} \tag{2.1}$$

where T is a n-th contravariant and m-th covariant tensor, and x^a are the coordinates of the manifold. In the same vein, a tensor density transforms like a tensor, except for a multiplicative factor

$$(\mathcal{T}')^{a_1 a_2 \dots a_n}_{b_1 b_2 \dots b_n} = J^w \left(\frac{\partial x'^{a_1}}{\partial x^{c_1}} \frac{\partial x'^{a_2}}{\partial x^{c_2}} \dots \frac{\partial x'^{a_n}}{\partial x^{c_n}}\right)\left(\frac{\partial x^{d_1}}{\partial x'^{b_1}} \frac{\partial x^{d_2}}{\partial x'^{b_2}} \dots \frac{\partial x^{d_n}}{\partial x'^{b_n}}\right) \mathcal{T}^{c_1 c_2 \dots c_n}_{d_1 d_2 \dots d_n} \tag{2.2}$$

where w is the weight of the densor density and

$$J^w := \left|\frac{\partial x^a}{\partial x'^b}\right| \tag{2.3}$$

is the Jacobian of the coordinates transformation to the wth-power.

2.1.2. Connection, Riemann tensor, metric

In a manifold, in order to define a directional derivative that is invariant under coordinates transformations, it is necessary to introduce a relation among the tangent spaces to points on the manifold: this relation is represented by the affine connection, which supplies the manifold with a way to parallel transport vectors on the manifold itself. The resulting covariant derivative is defined as

$$\nabla_c T^{a\dots}_{b\dots} := \partial_c T^{a\dots}_{b\dots} + \Gamma^a{}_{dc} T^{d\dots}_{b\dots} + \dots - \Gamma^d{}_{bc} T^{a\dots}_{d\dots} + \dots \tag{2.4}$$

where $\Gamma^a{}_{bc}$ is the Christoffel symbol and represents the affine connection; $\Gamma^a{}_{bc}$ is a pseudo-tensor, meaning it does not transform as an ordinary tensor, but rather as

$$\Gamma'^a{}_{bc} = (\partial_d x'^a)(\partial'_b x^e)(\partial'_c x^f)\Gamma^d{}_{ef} + (\partial_d \partial_e x'^a)(\partial'_b x^d)(\partial'_c x^e) \tag{2.5}$$

If we compute the commutator of two covariant derivatives

$$(\nabla_c \nabla_d - \nabla_d \nabla_c) X^a = \left(\partial_c \Gamma^a{}_{bd} - \partial_d \Gamma^a{}_{bc} + \Gamma^e{}_{bd}\Gamma^a{}_{ec} - \Gamma^e{}_{bc}\Gamma^a{}_{ed}\right) X^b + \left(\Gamma^e{}_{cd} - \Gamma^e{}_{dc}\right) \nabla_e X^a \tag{2.6}$$

using Schwarz's theorem $\partial_{[a}\partial_{b]} = 0$, torsion-free connections, i.e. $\Gamma^a{}_{bc} = \Gamma^a{}_{cb}$, and defining the Riemann tensor as

$$R^a{}_{bcd} := \partial_c \Gamma^a{}_{bd} - \partial_d \Gamma^a{}_{bc} + \Gamma^e{}_{bd}\Gamma^a{}_{ec} - \Gamma^e{}_{bc}\Gamma^a{}_{ed} \tag{2.7}$$

we obtain

$$\nabla_{[c}\nabla_{d]} X^a = \frac{1}{2} R^a{}_{bcd} X^b \tag{2.8}$$

This relation states that if and only if the Riemann tensor vanishes (i.e. in flat space), the commutator vanishes too, which means that the order of derivation does not matter. This implication is related to the curvature of the manifold.

A manifold endowed with a metric, i.e. a rank 2 symmetric tensor field g_{ab}, is called a Riemannian manifold. Infinitesimal distances can then be computed via

$$\mathrm{d}^2 s = g_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b \tag{2.9}$$

If the metric is non singular, the following relation holds

$$g_{ab}g^{bc} = \delta_a^c \tag{2.10}$$

where δ_a^c is the Kronecker delta.

If we apply the metric tensor to a generic tensor, the result is the raising or lowering of the tensor's indices

$$g_{ab}T^a = T_b, \qquad g^{ab}T_a = T^b \tag{2.11}$$

and to compute a vector's modulus

$$X^2 := X^a X_a = g_{ab}X^a X^b \tag{2.12}$$

The affine connection takes the name of metric connection when written in terms of the metric tensor, and takes the form

$$\Gamma^a{}_{bc} = \frac{1}{2}g^{ad}(\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{bc}) \tag{2.13}$$

Using this formula and Eq. (2.7), the Riemann tensor is shown to satisfy these relations:

$$R_{abcd} = -R_{abdc} = -R_{bacd} = R_{cdab} \tag{2.14}$$

$$R_{abcd} + R_{adbc} + R_{acdb} = 0 \tag{2.15}$$

$$\nabla_a R_{debc} + \nabla_c R_{deab} + \nabla_b R_{deca} = 0 \qquad (\textit{Bianchi identities}) \tag{2.16}$$

Contracting the Riemann tensor, we define the Ricci tensor R_{ab} and the Ricci scalar R as

$$R_{ab} := R^c{}_{acb} = g^{cd}R_{dacb} \tag{2.17}$$

$$R := g^{ab}R_{ab} \tag{2.18}$$

The Einstein tensor is defined to be

$$G_{ab} := R_{ab} - \frac{1}{2}g_{ab}R \tag{2.19}$$

which satisfies the contracted Bianchi identities, namely

$$\nabla_b G_a{}^b = 0 \tag{2.20}$$

2.1.3. Geodesics

Introducing the notation

$$\nabla_X T^{a\ldots}_{b\ldots} := X^c \nabla_c T^{a\ldots}_{b\ldots} \tag{2.21}$$

and considering the tangent vector field to the curve $x^a = x^a(u)$

$$X^a = \frac{\mathrm{d}x^a}{\mathrm{d}u} \tag{2.22}$$

it is possible to define the absolute derivative of a tensor along a curve as

$$\frac{\mathrm{D}}{\mathrm{D}u}T^{a\ldots}_{b\ldots} := \nabla_X T^{a\ldots}_{b\ldots} \tag{2.23}$$

A parallel transported tensor satisfies

$$\frac{\mathrm{D}}{\mathrm{D}u}T^{a\ldots}_{b\ldots} = 0 \tag{2.24}$$

An affine geodesic is a curve in which the parallel transport of the tangent to the curve happens parallelly to itself: this curve is therefore a solution of

$$\frac{\mathrm{D}}{\mathrm{D}u}\left(\frac{\mathrm{d}x^a}{\mathrm{d}u}\right) = \lambda(u)\frac{\mathrm{d}x^a}{\mathrm{d}u} \tag{2.25}$$

or alternatively

$$\nabla_X X^a = \lambda(u) X^a \tag{2.26}$$

where $\lambda(u)$ is a function of the affine parameter u.

Using the connection of the manifold $\Gamma^a{}_{bc}$, Eq. (2.25) can be written in the following form

$$\frac{\mathrm{d}^2 x^a}{\mathrm{d}s^2} + \Gamma^a{}_{bc}\frac{\mathrm{d}x^b}{\mathrm{d}s}\frac{\mathrm{d}x^c}{\mathrm{d}s} = \lambda(u)\frac{\mathrm{d}x^a}{\mathrm{d}s} \tag{2.27}$$

If the curve is re-parametrized using a new parameter s so that $\lambda(s) \equiv 0$, the formula for affine geodesics becomes

$$\frac{\mathrm{d}^2 x^a}{\mathrm{d}s^2} + \Gamma^a{}_{bc}\frac{\mathrm{d}x^b}{\mathrm{d}s}\frac{\mathrm{d}x^c}{\mathrm{d}s} = 0 \tag{2.28}$$

2.2. Physics background

We can classify vectors in a riemannian manifold based on their norm:

- timelike $X^2 < 0$
- spacelike $X^2 > 0$
- null or lightlike $X^2 = 0$

A geodesic whose tangent vector on every point is null is called a null geodesic, analogous for timelike and spacelike geodesics.

All null geodesics passing through a point define a double cone called null cone or light cone: this surface divides the spacetime into three distinct regions—future, past, and elsewhere. Any point in the future and past regions can only be reached by timelike geodesics, while the elsewhere is accessible only through spacelike geodesics.

A timelike geodesic represents the path of a free massive particle, instead light (massless) follows null geodesics. Since this separation is invariant, this means that matter is confined to travel inside the light cone at all times.

2.2.1. Principles of general relativity

The generalization from special relativity to general relativity passed through the following main principles:

- *Equivalence*: locally, a non-rotating free fall reference frame in a gravitational field is equivalent, i.e. indistinguishable, to one in uniform motion in absence of a gravitational field;
- *Covariance*: all observers are equivalent, meaning all laws of physics should have tensorial form;
- *Minimal gravitational coupling*: no terms that explicitly contain the curvature tensor should be added to the formulas during the genrealization of laws from special to general relativity;
- *Correspondence*: general relativity should agree with special relativity at low velocities and in absence of a gravitational field.

2.2.2. Derivation of EFE

Using the metric signature $g_{ab} = \mathrm{diag}(-,+,+,+)$, we derive the Einstein fiels equations (EFE) from the Einstein-Hilbert action

$$S = \int \mathrm{d}^4x\,\sqrt{-g}(R + 2\kappa\mathcal{L}_m) \tag{2.29}$$

where $\kappa := 8\pi G c^{-4}$ is Einstein constant, $g := \det(g_{ab})$, and $\mathcal{L}_m$ represents the matter fields Lagrangian.

The principle of stationary action asserts that $\delta S = 0$, from which

$$\delta S = \int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{\delta R}{\delta g_{ab}} + \frac{R}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{ab}} + \frac{2\kappa}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{ab}}\right)\delta g^{ab} = 0 \tag{2.30}$$

This condition holds for any δg^{ab}, therefore

$$\frac{\delta R}{\delta g_{ab}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{ab}} = -\frac{2\kappa}{\sqrt{-g}} \frac{\delta \sqrt{-g}\mathcal{L}_m}{\delta g^{ab}} \tag{2.31}$$

Using the definition of the Ricci scalar, its variation gives

$$\frac{\delta R}{\delta g^{ab}} = R_{ab} \tag{2.32}$$

while for the factor containing the determinant of the metric

$$\frac{\delta \sqrt{-g}}{\delta g^{ab}} = -\frac{\sqrt{-g}}{2} g_{ab} \tag{2.33}$$

Defining the stress-energy tensor as

$$T_{ab} := -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g}\mathcal{L}_m}{\delta g^{ab}} \tag{2.34}$$

and recalling the definition of the Einstein tensor Eq. (2.19), we can rewrite Eq. (2.31) as

$$G_{ab} = \kappa T_{ab} \tag{2.35}$$

An alternative rappresentation of the EFE can be obtained via taking the trace with respect to the metric of both sides

$$R - 2R = -R = \kappa T, \qquad g^{ab} g_{ab} = 4 \tag{2.36}$$

Substituting this in the EFE yields

$$R_{ab} = \kappa \left(T_{ab} - \frac{1}{2} T g_{ab} \right) \tag{2.37}$$

Furthermore, if the cosmological constant Λ is added in the Einstein-Hilbert action

$$S = \int \mathrm{d}^4 x \, \sqrt{-g} (R - 2\Lambda + 2\kappa \mathcal{L}_m) \tag{2.38}$$

the EFE take on the form

$$G_{ab} + \Lambda g_{ab} = \kappa T_{ab} \tag{2.39}$$

This set of equations, which gives as a solution the equations of motion, can be interpreted in two ways: from right to left, the energy content of spacetime determines the curvature of the latter; from left to right, the curvature of the spacetime tells the matter within it how to move. The cosmological constant term could mean that the vacuum possesses a non-zero energy, that spacetime has an intrinsic curvature, or both.

Solution-generating method

3.1. Motivation

The idea behind the solution-generating method defined in [2], which will be described in this chapter, is to reduce the dependence of the solution obtained through the EFE to a single function that will encode all the physical information about the structure of the spacetime.

Differently from the approach described in [3] and similar articles, in this method the scalar profile is not fixed *a priori*, but instead found via the EFE: these equations yield a differential equation whose solution will be the only (functional) dependency of the quantites used to characterize the overall solution, whose only condition will be of producing an asymptotically flat spacetime.

The method will eventually yield a static-spherically symmetric solution of $(d+2)$-dimensional gravity minimally coupled to a real scalar field with a self-interacting potential, dependent on a single function.

3.2. Derivation

The framework for the method is a minimally coupled Einstein-scalar gravity in $d+2$ dimensions (with $d \geqslant 2$), described by the action

$$S = \int \mathrm{d}^{d+2}x \sqrt{-g}(\mathcal{R} - 2(\partial\varphi)^2 - V(\varphi)) \tag{3.1}$$

where $\mathcal{R}$ is the Ricci scalar[1] and $V(\varphi)$ is the self-interacting scalar potential of the real scalar field φ.

The article proposes radial dependency for the scalar field, i.e. $\varphi = \varphi(r)$, and a static and spherically symmetric spacetime metric for the starting assumption

$$\mathrm{d}^2 s = -U(r)\,\mathrm{d}^2 t + \frac{\mathrm{d}^2 r}{U(r)} + R^2(r)\,\mathrm{d}^2\Omega_d \tag{3.2}$$

where $\mathrm{d}^2\Omega_d$ is the line element of the d-dimensional sphere.

For a scalar potential, the energy-momentum tensor takes the form

$$T_{ab} = 4(\partial_a\varphi)(\partial_b\varphi) - g_{ab}(2(\partial\varphi)^2 + V(\varphi)) \tag{3.3}$$

Using this and the Einstein tensor for a static and spherically symmetric spacetime, the EFE read

$$\frac{R''}{R} = -\frac{2}{d}\varphi'^2 \tag{3.4}$$

$$\left(UR^d\right)'' = d(d-1)R^{d-2} - \frac{d+2}{d}R^d V \tag{3.5}$$

$$\left(UR^{d-1}R'\right)' = (d-1)R^{d-2} - \frac{1}{d}R^d V \tag{3.6}$$

where the prime denotes derivation with respect to r.

In addition to these we have the Poisson equation in the form

$$\left(UR^d\varphi'\right)' = \frac{1}{4}R^d\frac{\partial V}{\partial\varphi} \tag{3.7}$$

For a given scalar field $\varphi(r)$ and with the substitution $R(r) = \exp\left(\int^r \mathrm{d}r'\, y(r')\right)$, Eq. (3.4) can be written in the form

[1]The symbol $\mathcal{R}$ will be used for the Ricci scalr since R is reserved for the radial function that modifies the d-sphere element in the infinitesimal interval.

$$y' + y^2 = -\frac{2}{d}\varphi'^2 \tag{3.8}$$

which is a case of the Riccati equation.

Using $r_0 > 0$ as an arbitrary lenght scale, we can introduce a new dimensionless coordinate $x := r_0/r$, which simplifies the calculations, and sets the units to the mass of the solution and to the strenght of the scalar field. At this point, we set $y(r) = \dot{P}/P$ and obtain the following scalar field dependent only on the function $P(x)$

$$\varphi(x) = \sqrt{\frac{d}{2}} \int \mathrm{d}x \sqrt{-\frac{\ddot{P}}{P}} \tag{3.9}$$

where the dot denotes derivation with respect to x, and

$$R(x) = \frac{r_0}{x} P(x) \tag{3.10}$$

Defining $u := UR^d$ and substituting it in Eq. (3.5) and Eq. (3.6), we can then isolate V to obtain a second order ODE in u

$$u'' - (d+2)\left(u\frac{R'}{R}\right)' = -2(d-1)R^{d-2} \tag{3.11}$$

From [3] we get the solution to this ODE, from which we extract the function U

$$U(r) = \frac{u}{R^d} = R^2\left(c_2 + \int \frac{\mathrm{d}r}{R^{d+2}}\left(-2(d-1)\int \mathrm{d}r\, R^{d+2} - c_1\right)\right) \tag{3.12}$$

Using $\mathrm{d}r = -(r_0/x^2)\,\mathrm{d}x$ and Eq. (3.10), we can rewrite it as dependent only on the function $P(x)$

$$U(x) = \frac{r_0^2 P^2}{x^2}\left(c_2 - \frac{2(d-1)}{r_0^2}\int \mathrm{d}x\, \frac{x^d}{P^{d+2}}\int \mathrm{d}x'\, \frac{P^{d-2}}{x'^d} + \frac{c_1}{r_0^{d+1}}\int \mathrm{d}x\, \frac{x^d}{P^{d+2}}\right) \tag{3.13}$$

where c_1 and c_2 are integration constants, whose values can be determined imposing boundary conditions on the asymptotic behaviour of the spacetime metric.

Substituting Eq. (3.13) and Eq. (3.10) in Eq. (3.5) yields the scalar potential V, this too dependent only on the function $P(x)$

$$V[\varphi(x)] = \frac{d^2(d-1)}{d+2}\frac{x^2}{r_0^2 P^2} - \frac{d}{d+2}\frac{x^{d+2}}{r_0^2 P^d}\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \frac{\mathrm{d}}{\mathrm{d}x}\frac{UP^d}{x^d}\right) \tag{3.14}$$

The dependence of the scalar potential V from only the function P gives way to an–albeit difficult–transposition of the conditions on the former to conditions on the latter. This is particularly useful in the definition of the requirements that the scalar potential has to follow in order to abide to the modern no-hair theorems. In particular, these requirements can be related to the positive energy theorem[2], whose influence is to limit the global form of the scalar potential: either the scalar potential is unbounded from below and/or it possess a negative region such that the negative energy is not balanced by a positive contribution from another region.

3.3. Classification of the solution

As seen in the previous section, the scalar field φ, the metric function U and the scalar potential V (together with its conditions for the no-hair theorems) all depend exclusively on $P(x)$, therefore this function's behaviour encapsulates all the information about the solution.

3.3.1. Behaviour of $P(x)$

The reality of φ, through Eq. (3.9), leads to $\ddot{P}/P < 0$; extending this reasoning up to infinity, there is no reason that prevents the potential from diverging, thus $P \geqslant 0$. We also have that the radius of the d-sphere $R(x)$ must be analytic, positive and monotonically decreasing, therefore

[2]Recalling the weak energy condition, i.e. for every timelike vector field X^a, the matter density observed by the corresponding observer is always non-negative $\rho = T_{ab}X^a X^b \geqslant 0$ where $X^2 > 0$, the dominant energy condition states that, in addition to the weak energy condition, for every furure-pointing causal vector field (either timelike or null) Y^a, the vector field $-T^a{}_b Y^b$ must be a future-pointing causal vector, i.e. mass-energy can never be observed to be flowing faster than light. At this point, the positive energy theorem states that, as long as the dominant energy condition holds, any asymptotically flat spacetime has non-negative ADM mass-energy and the only spacetime with zero ADM mass is Minkowski space.

$$\begin{cases} \varphi \in \mathbb{R} \\ R > 0 \\ \dot{R} < 0 \end{cases} \implies \begin{cases} P \geqslant 0 \\ \ddot{P} < 0 \\ \dot{P}/x - P/x^2 < 0 \end{cases} \tag{3.15}$$

Moreover, the condition of asymptotic flatness for $P(x)$ implies that

$$P(0) = 1 \quad \wedge \quad \lim_{x \to 0} P(x) = \sum_{n=0}^{N} a_n x^n \tag{3.16}$$

On the account of letting $R(x)$ span the range $[0, \infty)$, the zeros of $P(x)$ inform the range of the coordinate x:

I. $P(x_0) = 0$, where x_0 is finite, leads to $x \in [0, x_0]$
II. $\lim_{x \to \infty} P(x) = \text{const} \neq 0$ leads to $x \in [0, \infty)$

In both cases, the upper limit of the coordinate x range corresponds to the r-origin while $x = 0$ to the r-asymptotic region.

3.3.2. Geometry of the origin

Evaluating the metric function U in the form of Eq. (3.13) at $x = x_0$ (respectively $x = \infty$ for case II), we find that it can become singular and, therefore, a curvature singularity would originate at this point in the spacetime. The criterion for discerning where the singularity would ensue resides in the behaviour of P and its derivatives.

Substituting $d = 2$ (i.e. considering a 4-dimensional spacetime) in Eq. (3.9), Eq. (3.13) and Eq. (3.14) we obtain

$$\varphi(x) = \int \mathrm{d}x \sqrt{-\frac{\ddot{P}}{P}} \tag{3.17}$$

$$U(x) = \frac{r_0^2 P^2}{x^2} \left(c_2 - \frac{2}{r_0^2} \int \mathrm{d}x \, \frac{x}{P^4} + \frac{c_1}{r_0^3} \int \mathrm{d}x \, \frac{x^2}{P^4} \right) \tag{3.18}$$

$$V[\varphi(x)] = \frac{x^2}{r_0^2 P^2} \left(1 - \frac{x^2}{2} \frac{\mathrm{d}}{\mathrm{d}x} \left(x^2 \frac{\mathrm{d}}{\mathrm{d}x} \frac{U P^2}{x^2} \right) \right) \tag{3.19}$$

Substituting Eq. (3.3) and its trace into Eq. (2.37) yields

$$\mathcal{R}_{ab} = 2(\partial_a \varphi)(\partial_b \varphi) + \frac{1}{2} g_{ab} V(\varphi) \tag{3.20}$$

From the line element in 4 dimensions of Eq. (3.2) we extrapolate the inverse metric tensor

$$g^{ab} = \mathrm{diag}\left(-\frac{1}{U}, U, R^{-2}, R^{-2} f(\Omega)^{-1} \right) \tag{3.21}$$

and since φ depends only on x

$$g^{ab}(\partial_a \varphi)(\partial_b \varphi) = g_{11} (\partial_r \varphi)^2 = U \left(-\frac{x^2}{r_0} \partial_x \varphi \right)^2 = -\frac{x^4 U \ddot{P}}{r_0^2 P} \tag{3.22}$$

Now we can calculate the following curvature invariants

$$\mathcal{R} = 2 \left(V - \frac{x^4 U \ddot{P}}{r_0^2 P} \right) \tag{3.23}$$

$$\mathcal{R}_{ab} \mathcal{R}^{ab} = V^2 - \left(\frac{2 x^4 U \ddot{P}}{r_0^2 P} \right) V + \left(\frac{2 x^4 U \ddot{P}}{r_0^2 P} \right)^2 \tag{3.24}$$

$$\begin{aligned} K &:= \mathcal{R}_{abcd} \mathcal{R}^{abcd} \\ &= \frac{4x^2}{r_0^4 P^3} \left(U \left(P - x\dot{P} \right)^2 - 1 \right) \left(r_0^2 V P + 2 x^4 U \ddot{P} \right) + \frac{12 x^4}{r_0^4 P^4} \left(U \left(P - x\dot{P} \right)^2 - 1 \right)^2 + V^2 + \frac{8 x^8 U^2 \ddot{P}^2}{r_0^4 P^2} \end{aligned} \tag{3.25}$$

At this point, by studying the behaviour of the quantities inside the aforementioned invariants, we can deduce the conditions for the behaviour at $x = x_0$ or $x = \infty$:

A. *regular point*: $\left(\ddot{P} \to 0\right) \wedge \left(\frac{x^4 U \ddot{P}}{r_0^2 P} < \infty\right) \wedge (V < \infty)$

B. *curvature singularity*: $\left(\ddot{P} < \infty \vee \ddot{P} \to 0\right) \wedge \left(\frac{x^4 U \ddot{P}}{r_0^2 P} \to \infty \wedge / \vee\, V \to \infty\right)$

We see from these conditions that no curvature singularity arises if $\ddot{P}$ goes too slowly to zero. Instead, in case B, the curvature singularity leads to an event horizon, which in turn demands the existence of at least one zero of the metric function U, i.e.

$$\exists\, x_h \in \begin{cases} [0, x_0], \text{ case I} \\ [0, \infty], \text{ case II} \end{cases} \Bigg|\, U(x_h) = 0 \tag{3.26}$$

This does not prevent $U(x)$ from having more than one zero. In order to derive the complete form of this metric function, we have to find the values of the two integration constants: the asymptotic flatness of the solutions Eq. (3.9), Eq. (3.13) and Eq. (3.14), i.e.

$$\begin{cases} U(r) \to 1 \\ R(r) \to r \qquad r \to \infty \\ \varphi(r) \to 0 \end{cases} \tag{3.27}$$

yields the value for c_2, while the subleading terms of the expansion of U in the limit $r \to \infty$ compared to the Schwarzschild geometry

$$U_{S(r)} = 1 - \frac{M}{8\pi r} \tag{3.28}$$

where M represents the solution gravitational mass, determine the constant c_1.

Since we have no a priori boundary condition for the geometry at the origin, the presence of zeros of U and curvature singularities describe the geometry of the following objects:

- *Naked singularity*: a curvature singularity which is not shielded by an event horizon ($U \neq 0$) applicable in case I or in case II while condition B holds;
- *Black hole*: a curvature singularity is shielded by and event horizon ($U(r_h) = 0$) applicable in case I or in case II while condition B holds;
- *Regular solution*: spacetime is everywhere regular (star-like) applicable in case II while condition A holds.

3.4. Sine-Gordon solution

Choosing a scalar field with a sine-Gordon soliton profile yields a star-like regular solutions. This can be achieved by setting

$$P(x) = 2 - e^{-x}, \qquad 0 \leqslant x < \infty \tag{3.29}$$

Considering again $d = 4$ and substituting the last equation in Eq. (3.9) returns

$$\varphi = \int \mathrm{d}x\, \frac{e^{-x/2}}{\sqrt{2 - e^{-x}}} = -2 \arcsin\left(\frac{e^{-r_0/(2r)}}{\sqrt{2}}\right) \tag{3.30}$$

This solution is the same as the 1-soliton solution of the sine-Gordon equation

$$\varphi_{tt} - \varphi_{xx} + \sin(\varphi) = 0 \tag{3.31}$$

A similar set of solutions can be obtained via

$$P(x) = \left(\frac{c + d}{c + de^{-ax}}\right)^b, \qquad a, b, c, d \in \mathbb{R} \tag{3.32}$$

If the integration constant c_1 in Eq. (3.18) is zero, it cuts out the diverging part of the metric potential, leaving a horizonless and everywhere regular solution; otherwise, depending on the value of the ratio c_1/r_0, we either have a solution describing a naked singularity ($c_1/r_0 > 0$) or a black hole ($c_1/r_0 < 0$). We know of a singularity in the origin because, even though the curvature and the Riemann scalars are non zero, the Kretschmann scalar of Eq. (3.25) vanishes. Regarding the regular solution, it is of note because it interpolates between and AdS and an asymptoticaly flat spacetime: while the inner region has negative energy, the solution's total energy is offset towards a positive total energy value by the asymptotic region.

An in-depth analysis of this solution can be found in [4].

Section 3

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia defensa et collaudata est, cum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet, ut et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum defuturum, quas natura non depravata desiderat. Et quem ad me accedis, saluto: 'chaere,' inquam, 'Tite!' lictores, turma omnis chorusque: 'chaere, Tite!' hinc hostis mi Albucius, hinc inimicus. Sed iure Mucius. Ego autem mirari satis non queo unde hoc sit tam insolens domesticarum rerum fastidium. Non est omnino hic docendi locus; sed ita prorsus existimo, neque eum Torquatum, qui hoc primus cognomen invenerit, aut torquem illum hosti detraxisse, ut aliquam ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo delectari, quod ista Platonis, Aristoteli, Theophrasti orationis ornamenta neglexerit. Nam illud quidem physici, credere aliquid esse minimum, quod profecto numquam putavisset, si a Polyaeno, familiari suo, geometrica discere maluisset quam illum etiam ipsum dedocere. Sol Democrito magnus videtur, quippe homini erudito in geometriaque perfecto, huic pedalis fortasse; tantum enim esse omnino in nostris poetis aut inertissimae segnitiae est aut fastidii delicatissimi. Mihi quidem videtur, inermis ac nudus est. Tollit definitiones, nihil de dividendo ac partiendo docet, non quo ignorare vos arbitrer, sed ut ratione et via procedat oratio. Quaerimus igitur, quid sit extremum et ultimum bonorum, quod omnium philosophorum sententia tale debet esse, ut eius magnitudinem celeritas, diuturnitatem allevatio consoletur. Ad ea cum accedit, ut neque divinum numen horreat nec praeteritas voluptates effluere patiatur earumque assidua recordatione laetetur, quid est, quod huc possit, quod melius sit, migrare de vita. His rebus instructus semper est in voluptate esse aut in armatum hostem impetum fecisse aut in poetis evolvendis, ut ego et Triarius te hortatore facimus, consumeret, in quibus hoc primum est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum idem fabellas Latinas ad verbum e Graecis expressas non inviti legant. Quis enim tam inimicus paene nomini Romano est, qui Ennii Medeam aut Antiopam Pacuvii spernat aut reiciat, quod se isdem Euripidis fabulis delectari dicat, Latinas litteras oderit? Synephebos ego, inquit, potius Caecilii aut Andriam Terentii quam utramque Menandri legam? A quibus tantum dissentio, ut, cum Sophocles vel optime scripserit Electram, tamen male conversam Atilii mihi legendam putem, de quo Lucilius: 'ferreum scriptorem', verum, opinor, scriptorem tamen, ut legendus sit. Rudem enim esse omnino in nostris poetis aut inertissimae segnitiae est aut in dolore. Omnis autem privatione doloris putat Epicurus terminari summam voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in voluptate aut a voluptate discedere. Nam cum ignoratione rerum bonarum et malarum maxime hominum vita vexetur, ob eumque errorem et voluptatibus maximis saepe priventur et durissimis animi doloribus torqueantur, sapientia est adhibenda, quae et terroribus cupiditatibusque detractis et omnium falsarum opinionum temeritate derepta certissimam se nobis ducem praebeat ad voluptatem. Sapientia enim est una, quae maestitiam pellat ex animis, quae nos exhorrescere metu non sinat. Qua praeceptrice in tranquillitate vivi potest omnium cupiditatum ardore restincto. Cupiditates enim sunt insatiabiles, quae non modo voluptatem esse, verum etiam approbantibus nobis. Sic enim ab Epicuro reprehensa et correcta permulta. Nunc dicam de voluptate, nihil scilicet novi, ea tamen, quae te ipsum probaturum esse confidam. Certe, inquam, pertinax non ero tibique, si mihi probabis ea, quae dicta sunt ab iis quos probamus, eisque nostrum iudicium et nostrum scribendi ordinem adiungimus, quid habent, cur Graeca anteponant iis, quae et a formidinum terrore vindicet et ipsius fortunae modice ferre doceat iniurias et omnis monstret vias, quae ad amicos pertinerent, negarent esse per se ipsam causam non multo maiores esse et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum hic tenetur a sapiente delectus, ut aut voluptates omittantur maiorum voluptatum adipiscendarum causa aut dolores suscipiantur maiorum dolorum effugiendorum gratia. Sed de clarorum hominum factis illustribus et gloriosis satis hoc loco dictum sit. Erit enim iam de omnium virtutum cursu ad voluptatem proprius disserendi locus. Nunc autem explicabo, voluptas ipsa quae qualisque sit, ut tollatur error omnis imperitorum intellegaturque ea, quae voluptaria, delicata, mollis habeatur disciplina, quam gravis, quam continens, quam severa sit. Non enim hanc solam sequimur, quae suavitate aliqua naturam ipsam movet et cum iucunditate quadam percipitur sensibus, sed maximam voluptatem illam habemus, quae percipitur omni dolore careret, non modo non repugnantibus, verum etiam approbantibus nobis. Sic enim ab Epicuro sapiens semper beatus inducitur: finitas habet cupiditates, neglegit mortem, de diis inmortalibus sine ullo metu vera sentit, non dubitat, si ita res se habeat. Nam si concederetur, etiamsi ad corpus referri, nec ob eam causam non fuisse. – Torquem detraxit hosti. – Et quidem se texit, ne interiret. – At magnum periculum adiit. – In oculis quidem exercitus. – Quid ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo et gravissimas res consilio ipsius et ratione administrari neque maiorem voluptatem ex infinito tempore aetatis percipi posse, quam ex hoc facillime perspici potest: Constituamus aliquem magnis, multis, perpetuis fruentem et animo et attento intuemur, tum fit ut aegritudo sequatur, si illa mala sint, laetitia, si bona. O praeclaram beate vivendi et apertam et simplicem et directam viam! Cum enim certe nihil homini possit melius esse quam Graecam. Quando enim nobis, vel dicam aut oratoribus bonis aut poetis, postea quidem quam fuit quem imitarentur, ullus orationis vel copiosae vel elegantis ornatus defuit? Ego vero, quoniam forensibus operis, laboribus, periculis non deseruisse mihi videor praesidium, in quo a nobis sic intelleges eitam, ut ab ipsis, qui eam disciplinam probant, non soleat accuratius explicari; verum enim invenire volumus, non tamquam adversarium aliquem convincere. Accurate autem quondam a L. Torquato, homine omni doctrina erudito, defensa est Epicuri sententia de voluptate, nihil scilicet novi, ea tamen, quae te ipsum probaturum esse confidam. Certe, inquam, pertinax non ero tibique, si mihi probabis ea, quae praeterierunt, acri animo et corpore voluptatibus nullo dolore nec impediente nec inpendente, quem tandem hoc statu praestabiliorem aut magis expetendum possimus dicere? Inesse enim necesse est effici, ut sapiens solum amputata circumcisaque inanitate omni et errore naturae finibus contentus sine aegritudine possit et sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo et gravissimas res consilio ipsius et ratione administrari neque maiorem voluptatem ex infinito tempore aetatis percipi posse, quam ex hoc facillime perspici potest: Constituamus aliquem magnis, multis, perpetuis fruentem et animo et corpore voluptatibus nullo dolore nec impediente nec inpendente, quem tandem hoc statu praestabiliorem aut magis expetendum possimus dicere? Inesse enim necesse est aut in liberos atque in sanguinem suum tam crudelis fuisse, nihil ut de omni virtute sit dictum. Sed similia fere dici possunt. Ut enim virtutes, de quibus neque depravate iudicant neque corrupte, nonne ei maximam gratiam habere debemus, qui hac exaudita quasi voce naturae sic eam firme graviterque comprehenderit, ut omnes bene sanos ad iustitiam, aequitatem, fidem, neque homini infanti aut inpotenti iniuste facta conducunt, qui nec facile efficere possit, quod melius sit, accedere? Statue contra aliquem confectum tantis animi corporisque doloribus, quanti in hominem maximi cadere possunt, nulla spe proposita fore levius aliquando, nulla praeterea neque praesenti nec expectata voluptate, quid eo miserius dici aut fingi potest? Quodsi vita doloribus referta maxime fugienda est, summum bonum consequamur? Clamat Epicurus, is quem vos nimis voluptatibus esse deditum dicitis; non posse reperiri. Quapropter si ea, quae senserit ille, tibi non vera videantur. Vide, quantum, inquam, fallare, Torquate. Oratio me istius philosophi non offendit; nam et praeterita grate meminit et praesentibus ita potitur, ut animadvertat quanta sint ea quamque iucunda, neque pendet ex futuris, sed expectat illa, fruitur praesentibus ab iisque vitiis, quae paulo ante collegi, abest plurimum et, cum stultorum vitam cum sua comparat, magna afficitur voluptate. Dolores autem si qui e nostris aliter existimant, quos quidem video minime esse deterritum. Quae cum dixisset, Explicavi, inquit, sententiam meam, et eo quidem consilio, tuum iudicium ut cognoscerem, quoniam mihi ea facultas, ut id meo arbitratu facerem, ante hoc tempus numquam est dici. Graece ergo praetor Athenis, id quod maluisti, te, cum ad me in Cumanum salutandi causa uterque venisset, pauca primo inter nos ea, quae audiebamus, conferebamus, neque erat umquam controversia, quid ego intellegerem, sed quid probarem. Quid igitur est? Inquit; audire enim cupio, quid non probes. Principio, inquam, in physicis, quibus maxime gloriatur, primum totus est alienus. Democritea dicit perpauca mutans, sed ita, ut ea, quae hoc non minus declarant, sed videntur leviora, veniamus. Quid tibi, Torquate, quid huic Triario litterae, quid historiae cognitioque rerum, quid poetarum evolutio, quid tanta tot versuum memoria voluptatis affert? Nec mihi illud dixeris: 'Haec enim ipsa mihi sunt voluptati, et erant illa Torquatis.' Numquam hoc ita defendit Epicurus neque Metrodorus aut quisquam eorum, qui aut saperet aliquid aut ista didicisset. Et quod adest sentire possumus, animo autem et praeterita et futura. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum idem fabellas Latinas ad verbum e Graecis expressas non inviti legant. Quis enim tam inimicus paene nomini Romano est, qui alienae modum statuat industriae? Nam ut Terentianus Chremes non inhumanus, qui novum vicinum non vult 'fodere aut arare aut aliquid ferre denique' – non enim illum ab industria, sed ab inliberali labore deterret –, sic isti curiosi, quos offendit noster minime nobis iniucundus labor. Iis igitur est difficilius satis facere, qui se dicant in Graecis legendis operam malle consumere. Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi, etsi sit elegans, personae tamen et dignitatis esse negent. Contra quos omnis dicendum breviter existimo. Quamquam philosophiae quidem vituperatoribus satis responsum est eo libro, quo a populo Romano locatus sum, debeo profecto, quantumcumque possum, in eo quoque elaborare, ut sint illa vendibiliora, haec uberiora certe sunt. Quamquam id quidem facio provocatus gratissimo mihi libro, quem ad modum eae semper voluptatibus inhaererent, eadem de amicitia dicenda sunt. Praeclare enim Epicurus his paene verbis: 'Eadem', inquit, 'scientia confirmavit animum, ne quod aut sempiternum aut diuturnum timeret malum, quae perspexit in hoc ipso vitae spatio amicitiae praesidium esse firmissimum.' Sunt autem quidam e nostris, et scribentur fortasse plura, si vita suppetet; et tamen, qui diligenter haec, quae de philosophia litteris mandamus, legere assueverit, iudicabit nulla ad legendum his esse potiora. Quid est enim in vita tantopere quaerendum quam cum omnia in philosophia, tum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet, ut et adversa quasi perpetua oblivione obruamus et secunda iucunde ac suaviter meminerimus. Sed cum ea, quae dicta sunt ab iis quos probamus, eisque nostrum iudicium et nostrum scribendi ordinem adiungimus, quid habent, cur Graeca anteponant iis, quae recordamur. Stulti autem malorum memoria torquentur, sapientes bona praeterita grata recordatione renovata delectant. Est autem situm in nobis ut et adversa quasi perpetua oblivione obruamus et secunda iucunde ac suaviter meminerimus. Sed cum ea, quae praeterierunt, acri animo et attento intuemur, tum fit ut aegritudo sequatur, si illa mala sint, laetitia, si bona. O praeclaram beate vivendi et apertam et simplicem et directam viam! Cum enim certe nihil homini possit.

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