POSITIVE ENERGY IN ANTI-DE SITTER BACKGROUNDS AND GAUGED EXTENDED SUPERGRAVITY ☆

Peter BREITENLOHNER [1] and Daniel Z. FREEDMAN
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received 26 May 1982

The energy functional of a field theory with anti-de Sitter background geometry can be positive for scalar potentials which are unbounded below and for vacua corresponding to critical points which are maxima or saddle points.

Anti-de Sitter space occurs as a natural background geometry in field theories of gravity coupled to lower spin fields whenever the scalar potential $V(\phi^i)$ has a critical point at ϕ_0^i with $V(\phi_0^i) < 0$. This situation can occur in both grand unified field theories and in supergravity. Two related problems of field theory in an AdS background are stability which has been recently analyzed [1] and the lack of a global Cauchy hypersurface which can be resolved with suitable boundary conditions [2].

We report here on research [‡1] which applies and extends the approaches of refs. [1] and [2]. The major new conclusion is that an AdS background is stable for fluctuations which vanish sufficiently fast at spatial infinity, not only when the critical point is a relative minimum, but also when it is a maximum or a saddle provided the eigenvalues of $(\partial/\partial\phi^i)\,\partial V(\phi)/\partial\phi^j$ are not too negative. Stability can occur even when $V(\phi)$ is unbounded below. All of this is surprising from a viewpoint based on flat-space intuition.

One important application is to gauged extended supergravity theories [4,5] for $N \geqslant 4$ where scalar potentials are unbounded below, which was heretofore considered a serious problem. Now, there appears to be a stable AdS "ground state" and a well-defined perturbative phase of these field theories. This vacuum with large cosmological term is still unphysical, but one can begin to study non-perturbative mechanisms such as formation of space–time foam [6] which could change the picture.

We assume that we have a theory of gravity coupled to lower spin fields and that fields of spin $\frac{1}{2}$, spin 1, and spin $\frac{3}{2}$ vanish in the background field configuration. Any other background will be less symmetric than the one obtained under these assumptions. Thus, the action restricted to the gravitational field $g_{\mu\nu}(x)$ and scalar fields $\phi^i(x)$ is relevant for the background. This action and its equations of motion are (with a convenient but unconventional normalization)

$$I = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,[-\tfrac{1}{2}R + g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i - 2V(\phi)]\ , \tag{1}$$

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = T_{\mu\nu} = 2\partial_\mu\phi^i\partial_\nu\phi^i - g_{\mu\nu}[\partial^\sigma\phi^i\partial_\sigma\phi^i - 2V(\phi)]\ , \tag{2}$$

$$(-g)^{-1/2}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi^i) + \partial V/\partial\phi^i = 0\ . \tag{3}$$

We look for a background solution with constant scalar fields, $\phi^i(x) = \phi_0^i$, so that (3) requires that we are at a critical point. The Einstein equation reduces to

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R - g_{\mu\nu}2V(\phi_0) = 0\ , \tag{4}$$

and we choose the maximally symmetric solution, viz. O(4, 1) de Sitter space if $V(\phi_0) > 0$, Minkowski space if $V(\phi_0) = 0$, and O(3, 2) anti-de Sitter space if $V(\phi_0) < 0$.

Actually, we take the covering space CAdS. This

☆ Supported in part by NSF Grant PHY8107394.

[1] Permanent address: Max-Planck-Institut für Physik, 8000 Munich 40, Federal Republic of Germany.

[‡1] A more detailed version will be published elsewhere [3].

0 031-9163/82/0000–0000/$02.75 © 1982 North-Holland

space has a global static metric in coordinates t, ρ, θ, ϕ given by the line element

$$\mathrm{d}s^2 = (a^2 \cos^2\rho)^{-1}$$
$$\times [\mathrm{d}t^2 - \mathrm{d}\rho^2 - \sin^2\rho(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2)] ,$$
$$-\infty < t < \infty , \quad 0 \leqslant \theta < \pi ,$$
$$0 \leqslant \rho < \pi/2 , \quad 0 \leqslant \phi < 2\pi ,$$
$$a^2 = -\tfrac{2}{3} V(\phi_0) . \tag{5}$$

The scalar curvature is $R = 12a^2$. The lack of a global Cauchy surface in this space–time [2,7] is closely related to the fact, evident from (5), that a radial light-signal from the origin, $\rho = 0$, propagates to spatial infinity, $\rho = \pi/2$, in finite time $t = \pi/2$.

A static field configuration is stable if the associated conserved energy functional is positive for a suitable function space of fluctuations, namely those for which the energy integral converges. When coupling to gravity is included, the energy functional depends on the background metric and can be defined most clearly if there is a global time-like Killing vector which generates the time translation isometry. The use of the Killing vector to define the energy functional is known for the asymptotically flat case and has recently been generalized to de Sitter and AdS backgrounds [1]. The metric and scalar fields are split into background plus deviation as $g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$ and $\phi^i(x) = \phi_0^i + \lambda^i(x)$. If $\xi^\mu(x)$ is the time-like Killing vector, then the energy functional, defined by expanding the Einstein field equation around the background, is given by an integral over a hypersurface at fixed time,

$$E = \int \mathrm{d}^3x \sqrt{-\bar{g}}\, [t^{0\nu} + T^{0\nu} - \bar{g}^{0\nu} 2V(\phi_0)]\, \xi_\nu . \tag{6}$$

Here, $T^{\mu\nu}$ is the stress tensor of (2) and $t^{\mu\nu}$ is the gravitational pseudotensor which contains terms of second and higher order in $h_{\mu\nu}$. The energy functional is formally conserved in time by construction and actually conserved if the deviations $h_{\mu\nu}(x)$ and $\lambda^i(x)$ vanish sufficiently fast at spatial infinity. In the Minkowski and AdS cases, the time translation Killing vector is globally time-like [as is clear for AdS, since $\xi^\nu = (1, 0, 0, 0)$ in the coordinates of (5)], and there is no difficulty in applying (6). In de Sitter space, the Killing vector is not globally time-like. Although one can investigate stability [1] for fluctuations localized within the horizon of any observer, we do not discuss this case further [‡2].

[‡2] For a global treatment of stability in de Sitter space, see ref. [8].

In interpreting (6) one should note that there are constraint equations which relate $h_{\mu\nu}(x)$ and $\lambda^i(x)$ on the hypersurface of integration. Partly because of this, it is difficult to prove positivity of E to all orders in the fluctuations, although one may hope that the techniques [9] applied to the asymptotically flat case can be generalized to AdS. Further, there is a happy feature of supergravity, discussed below, which gives an independent stability argument. For the moment, we establish the stability of the AdS background for small fluctuations, i.e., we work to quadratic order in $h_{\mu\nu}$ and λ^i. The energy functional then splits into two independent parts:

$$E = E(t_{\mu\nu}) + E(T_{\mu\nu}) , \tag{7}$$
$$E(t_{\mu\nu}) = \int \mathrm{d}^3x \sqrt{-\bar{g}}\, t^{0\nu} \xi_\nu , \tag{8}$$
$$E(T_{\mu\nu}) = \int \mathrm{d}^3x \sqrt{-\bar{g}}\, [T^{0\nu} - \bar{g}^{0\nu} 2V(\phi_0)]\, \xi_\nu$$
$$= \frac{1}{a^2} \int \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}\rho\, \tan^2\rho\, \{(\partial_t \lambda^i)^2$$
$$+ (1/\sin^2\rho)[(\partial_\theta \lambda^i)^2 + (1/\sin^2\theta)(\partial_\phi \lambda^i)^2]$$
$$+ (\partial_\rho \lambda^i)^2 + (a^2/\cos^2\rho)\, V_{ij} \lambda^i \lambda^j\} , \tag{9}$$
$$V_{ij} = (\partial/\partial\phi^i)\, \partial V(\phi)/\partial\phi^j |_{\phi=\phi_0} .$$

It is convenient to diagonalize the "mass matrix" V_{ij} and to denote its eigenvalues by $-a^{-2}\alpha^i$ and eigenvectors by $h^i(x)$. Small fluctuation positivity for $E(t_{\mu\nu})$ and for $E(T_{\mu\nu})$ with positive eigenvalues ($\alpha^i \leqslant 0$) is known [1], and it is sufficient for us to consider a scalar fluctuation $h(x)$ of negative eigenvalue ($\alpha > 0$). The energy functional then reduces to

$$E(T_{\mu\nu}) = \frac{1}{a^2} \int \mathrm{d}\Omega\, \mathrm{d}\rho\, \tan^2\rho\, \{(\partial_t h)^2$$
$$+ (1/\sin^2\rho)[(\partial_\theta h)^2 + (1/\sin^2\theta)(\partial_\phi h)^2]$$
$$+ (\partial_\rho h)^2 - (\alpha/\cos^2\rho)\, h^2\} . \tag{10}$$

To establish positivity, we let $h(x) = (\cos\rho)^\mu h'(x)$ where μ is a real parameter to be determined. After some manipulation, the radial derivative and potential terms in (10) can be rewritten as

$$\frac{1}{a^2} \int \mathrm{d}\Omega\, \mathrm{d}\rho\, \tan^2\rho\, (\cos\rho)^{2\mu} \{(\partial_\rho h')^2$$
$$- [(\alpha - 3\mu + \mu^2 \sin^2\rho)/\cos^2\rho]\, (h')^2\}$$
$$- \frac{\mu}{a^2} \int \mathrm{d}\Omega\, \tan^3\rho\, (\cos\rho)^{2\mu} (h')^2 |_{\rho=0}^{\rho=\pi/2} . \tag{11}$$

We now choose μ to satisfy $\mu^2 - 3\mu + \alpha = 0$, i.e., $\mu_\pm = \frac{3}{2} \pm \frac{1}{2}(9 - 4\alpha)^{1/2}$, so that the volume term in (11) becomes

$$\frac{1}{a^2} \int d\Omega \, d\rho \, \tan^2\rho (\cos\rho)^{2\mu} [(\partial_\rho h')^2 + \mu^2 (h')^2] \,, \quad (12)$$

which is positive. We now observe that the integrals converge and the surface term vanishes for functions $h(x)$ which vanish faster than $(\cos\rho)^{3/2}$ at spatial infinity and the energy is positive for such fluctuations. The condition $\alpha \leqslant \frac{9}{4}$ is required for reality. The qualitative reason that positivity holds in spite of the negative unbounded potential is that the volume factor and metric of AdS appear in (10), so that fluctuations are required to vanish fast enough at spatial infinity that the positive kinetic terms in (10) dominate the negative potential.

The characterization of the allowed fluctuations is not quite complete because we have not taken into account an ambiguity in the definition of the stress tensor $T_{\mu\nu}$ of the scalar perturbations. The improved stress tensor

$$\hat{T}_{\mu\nu} = T_{\mu\nu} - \bar{g}_{\mu\nu} 2V(\phi_0) + \beta(\bar{g}_{\mu\nu}\Box - D_\mu \partial_\nu + \bar{R}_{\mu\nu}) h^2 \quad (13)$$

is also conserved for arbitrary β where $\Box$ is the covariant wave operator and $\bar{R}_{\mu\nu}$ is the Ricci tensor of AdS. The improved energy functional

$$E(\hat{T}_{\mu\nu}) = \int d^3x \sqrt{-\bar{g}} \, \hat{T}^{0\nu} \quad (14)$$

differs from $E(T_{\mu\nu})$ by the surface term

$$E(\hat{T}_{\mu\nu}) - E(T_{\mu\nu})$$
$$= \beta \int d^3x \, \partial_i [\sqrt{-\bar{g}} \, \xi^0 D^i(h^2) - (D^i \xi^0) h^2]$$
$$= -\frac{\beta}{a^2} \int d\Omega \tan^2\rho (\partial_\rho - \tan\rho) h^2 |_0^{\pi/2} \,. \quad (15)$$

From (10) and (12) with $\mu = \mu_-$, one finds that a fluctuation $h(x)$ with asymptotic falloff $h(x) \sim (\cos\rho)^{\mu_-}$ has positive convergent energy if $\frac{1}{2} < \mu_- < \frac{3}{2}$. The combined surface terms in (11) and (15) diverge for general β, but the surface terms vanish for the special value $\beta = \mu_-/(2\mu_- + 1)$ which we choose. For $\alpha = 2$ and $\mu_- = 1$, $\hat{T}_{\mu\nu}$ is the conformal stress tensor.

The reason one can improve the definition of the energy of linearized scalar fluctuations is that it is inherently ambiguous. If one starts from the original action (1) [which we now restrict for purposes of illustration to a single scalar field $\phi(x)$] and makes the Weyl transformation $g_{\mu\nu} = \Lambda^{-2} g'_{\mu\nu}$ of the metric with $\Lambda = 1 + \frac{1}{2}\beta(\phi - \phi_0)^2$, then the theory described in terms of $g'_{\mu\nu}(x)$ and $\phi(x)$ has the same critical point $V(\phi_0)$ and the same background AdS metric as before. To linearized order, the only effect of the Weyl transformation is that the improved $\hat{T}_{\mu\nu}$ appears in the matter energy functional (9).

The wave equation for small scalar fluctuations in a CAdS background is

$$\Box h - a^2 \alpha h = 0 \quad (16)$$

and the energy criterion for stability is closely related to the question of a unique solution of the Cauchy problem for this equation. The scalar product

$$(h_1, h_2) = \mathrm{i} \int d^3x \sqrt{-\bar{g}} \, \bar{g}^{\mu\nu} (\bar{h}_1 \overleftrightarrow{\partial}_\nu h_2) \quad (17)$$

and the energy functionals $E(T_{\mu\nu})$ and $E(\hat{T}_{\mu\nu})$ are formally conserved for solutions of this equation, and we define the appropriate Hilbert space of solutions by imposing boundary conditions so that these formally conserved quantities are actually conserved, i.e., no flux crosses the boundary at spatial infinity. This procedure [3] is a modification of that of ref. [2] and leads to a complete set of positive frequency modes

$$\phi_{\omega lm}(x) = e^{-\mathrm{i}\omega t} Y_l^m(\theta, \phi) R_{\omega l}(\rho) \,, \quad (18)$$

in terms of which the general solution can be expanded as

$$h(x) = \sum_{\omega lm} [a_{\omega lm} \phi_{\omega lm}(x) + a^*_{\omega lm} \bar{\phi}_{\omega lm}(x)] \quad (19)$$

with expansion coefficients determined from initial data as expected, i.e.,

$$a_{\omega lm} = (\phi_{\omega lm}, h) \,. \quad (20)$$

By standard boundary value analysis of the radial wave equation from (16) one finds that, for $\alpha \leqslant \frac{9}{4}$, there are two possible boundary conditions, denoted here by $\pm$, each of which leads to a complete set of modes:

$$R^\pm_{\omega l}(\rho) \sim (\sin\rho)^l (\cos\rho)^{\mu_\pm} P_n^{[\pm(9-4\alpha)^{1/2}/2, \, l+\frac{1}{2}]}(\cos 2\rho)$$
$$\omega = \mu_\pm + l + 2n \,, \quad (21)$$

so that frequencies are quantized. It is easy to see that the scalar product (17) leads to the standard orthogonality integral for the Jacobi polynomials in (21). The modes $R^{+}_{\omega l}$ give conservation of both energy functionals, while the $R^{-}_{\omega l}$ require the improved energy $E(\hat{T}_{\mu\nu})$. The two allowed boundary conditions correspond to Dirichlet and Neumann boundary conditions in boundary value problems of mathematical physics. Each leads to a unique solution of the wave equation and results are compatible with the previous analysis of positivity of energy. For $\alpha > \frac{9}{4}$, there does not seem to be any way to achieve positive conserved energy.

Since the isometry group of CAdS is the group SO(3, 2), one expects that the perturbative dynamics in the background is SO(3, 2) invariant. Indeed, see ref. [10], the two sets of modes $\phi^{\pm}_{\omega lm}(x)$ carry the unitary irreducible representations $D(\mu^{\pm}, 0)$ of SO(3, 2).

To see how the present analysis applies to gauged extended supergravity, let us consider the gauged $N = 4$ theory, which describes the vierbein $V_{a\mu}(x)$, 4 spin $\frac{3}{2}$ fields $\psi^{i}_{\mu}(x)$, 6 SO(4) Yang–Mills potentials $A^{ij}_{\mu}(x)$, 4 spin $\frac{1}{2}$ fields $\chi^{i}(x)$, and a real scalar $A(x)$ and pseudoscalar $B(x)$. In terms of a complex field $z(x) = \kappa\,[A(x) + \mathrm{i}B(x)]$, the scalar and gravity part of the action is

$$I = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\,\sqrt{-g}$$
$$\times\left[-\tfrac{1}{2}R + \frac{g^{\mu\nu}\partial_\mu\bar{z}\,\partial_\nu z}{(1-\bar{z}z)^2} + \frac{e^2}{\kappa^2}\left(3 + \frac{2\bar{z}z}{1-\bar{z}z}\right)\right]. \qquad (22)$$

The scalar kinetic term illustrates a well-known feature [11] of extended supergravity, namely the scalar dynamics is that of a nonlinear σ-model on the homogeneous space SU(1, 1)/U(1). The scalar potential is proportional to the Yang–Mills coupling e^2. It is unbounded below, and it has a smaller internal symmetry, namely U(1) instead of SU(1, 1), than the kinetic term. Because z is a coordinate of SU(1, 1)/U(1), its range is $0 \leqslant |z| < 1$.

The only critical point of the scalar potential in (22) is the global maximum at $z = 0$, and this leads to a CAdS background (5) with de Sitter constant $a^2 = e^2/\kappa^2$. One sees that the linearized scalar fluctuations in the background have mass matrix eigenvalue parameter $\alpha = 2$ which corresponds to conformally coupled scalar fields. Conformal coupling implies that the Green's function of the scalar excitations has support only on local light cones of CAdS. In this sense, they are massless as would be expected in supergravity [12]. The previous analysis of the energy functional shows that the background is stable against small fluctuations.

However, much more is true because the full background configuration (including the zero background values for χ^{i}, A^{ij}_{μ}, and ψ^{i}_{μ}) is invariant under the global supersymmetry of the supergroup OSp(4, 4). The transformations of the group are obtained from the local supersymmetry variations of the full theory by restricting spinor parameters $\epsilon(x)$ to satisfy

$$(\partial_\mu + \tfrac{1}{2}\,\omega_{\mu ab}\,\sigma^{ab} + \tfrac{1}{2}\mathrm{i}a\,V_{a\mu}\gamma^a)\,\epsilon(x) = 0\,, \qquad (23)$$

where $\omega_{\mu ab}$ and $V_{a\mu}$ are the spin connection and vierbein of the AdS background. Such spinors $\epsilon(x)$ are called Killing spinors [1] because they determine generalized isometries of the background and are, in fact, closely related to Killing vectors. It is known that (23) is integrable [13] and that there are 4 linearly independent Killing spinors which may be expressed as

$$\epsilon_\alpha(x) = S(x)_{\alpha\beta}\,\theta_\beta\,, \qquad (24)$$

where θ_β is a constant spinor and $S(x)$ is known [14, 3]. A background configuration in supergravity which is invariant under Killing spinor transformations can be called a supersymmetric background.

By a procedure [1] analogous to the definition of the energy functional one can define conserved quantities corresponding to the SO(3, 2) generators L_{AB} (in a five-dimensional notation) of which the Killing energy previously discussed is $E = L_{04}$. Further, one can define conserved supersymmetry charges Q^{i}_{α}, and these satisfy the expected OSp anti-commutation relations

$$\{Q^{i}_{\alpha}, \bar{Q}^{i}_{\beta}\} = \gamma^{a}_{\alpha\beta}L_{a4} + \mathrm{i}\sigma^{ab}_{\alpha\beta}L_{ab}\,, \qquad (25)$$

for any fixed i. {We simplify to the OSp(1, 4) algebra here, but the OSp(N, 4) case is easily treated, ref. [3].} By tracing with $\gamma^{0}_{\alpha\beta}$, one finds the standard positivity statement

$$L_{04} = E = \frac{1}{4}\sum_{\alpha}\,(Q^{i}_{\alpha}Q^{i+}_{\alpha} + Q^{i+}_{\alpha}Q^{i}_{\alpha})\,. \qquad (26)$$

Thus, energy would be expected to be nonnegative to all orders for fluctuations which vanish sufficiently fast at spatial infinity. Further, the case of neutral stability can be ruled out, since any zero energy configuration would be annihilated by Q^i_α and therefore by L_{AB}. Since the SO(3, 2) group acts transitively on CAdS, any invariant scalar field configuration is constant. It is either the original vacuum configuration or not within the allowed space of fluctuations.

Since the vacuum is OSp(N, 4) invariant, one expects that the perturbative dynamics is supersymmetric, and it is. The action of the massive supermultiplet in a CAdS background with de Sitter constant a^2 is

$$I = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-\bar{g}}\,[\bar{g}^{\mu\nu}\partial_\mu A\partial_\nu A + \bar{g}^{\mu\nu}\partial_\mu B\partial_\nu B + \mathrm{i}\bar{\chi}\gamma^\mu D_\mu\chi + (2a^2 + am - m^2)A^2 + (2a^2 - am - m^2)B^2 - m\bar{\chi}\chi]\,. \tag{27}$$

It is invariant under transformation rules with Killing spinor parameters, namely

$$\delta A = \bar{\epsilon}(x)\,\chi\,, \quad \delta B = \bar{\epsilon}(x)\,\mathrm{i}\gamma_5\chi\,,$$
$$\delta\chi = -[\mathrm{i}\not{\partial}(A + \mathrm{i}\gamma_5 B) + a(A - \mathrm{i}\gamma_5 B) + m(A + \mathrm{i}\gamma_5 B)]\,\epsilon(x)\,. \tag{28}$$

Note that the mass parameters are different for all 3 fields. The value $m = 0$ gives the small fluctuation action in gauged extended supergravity. Further, for all m one has α_A and $\alpha_B \leqslant \frac{9}{4}$ so that the stability condition is satisfied.

Supersymmetry implies that scalar modes transform into spinor modes and back into scalar modes and this process must "close". If one starts with a complete set of modes for $A(x)$ and a complete set for $B(x)$, then one must come back to the same modes after two transformations. It is fairly easy to see [3] that not all choices of boundary conditions are compatible with supersymmetry. For $|\mu| > \frac{1}{2}$, one must choose $R^+_{\omega l}(\rho)$ wave functions for both $A(x)$ and $B(x)$, while for $|\mu| < \frac{1}{2}$, one must have $R^+_{\omega l}(\rho)$ for $A(x)$ and $R^-_{\omega l}(\rho)$ for $B(x)$ or vice versa. Only then are energy eigenvalues spaced by $\frac{1}{2}$ as required by the OSp(1, 4) algebra. For $|\mu| < \frac{1}{2}$, supersymmetry forces us to use the improved stress term $\hat{T}_{\mu\nu}$ discussed earlier.

The scalar potentials [5] of gauged extended supergravity for $N \geqslant 5$ are more complicated than for $N = 4$ but still unbounded below. There is always an SO(N) symmetric local maximum which gives a supersymmetric AdS background to which the stability analysis above applies. However, there are other critical points, for example, a saddle point in the $N = 5$ theory, at which SO(5) is spontaneously broken to SO(3). The mass eigenvalue parameters for the 10 real scalars are $\alpha = -\frac{20}{7}, 0, 0, 0, 0, 0, 0, 0, \frac{12}{7}, \frac{12}{7}$. The 7 zeroes are Goldstone modes for the broken generators of SO(5). The critical point is not supersymmetric since these eigenvalues cannot be paired as required by (27). Further, there are no Killing spinors. However, the associated AdS background, with $a^2 = 7e^2/\kappa^2$, is stable against small fluctuations since $\alpha \leqslant \frac{9}{4}$ for all eigenvalues.

In a field theory where the scalar potential has several critical points, there is an interesting consequence of the fact that the energy functional depends on the background metric and converges only for asymptotically vanishing fluctuations. According to present ideas, there is no way to compare the "energies" of different vacua. Indeed, several different vacua may be stable according to the stability criterion associated with the proper energy functional. This phenomenon has been exemplified recently [15,16] in $N = 1$ supersymmetric grand unified theories where decay of a Minkowski vacuum by bubble formation [17] is not allowed even when there is a lower minimum of the scalar potential.

[1] L. Abbott and S. Deser, Nucl. Phys. B195 (1982) 76.
[2] S.J. Avis, C.J. Isham and D. Storey, Phys. Rev. D18 (1978) 3565.
[3] P. Breitenlohner and D.Z. Freedman, to be published.
[4] E. Cremmer and J. Scherk, Nucl. Phys. B127 (1977) 259; A. Das, M. Fischler and M. Roček, Phys. Rev. D16 (1977) 3427;
E. Cremmer, J. Scherk and S. Ferrara, unpublished.
[5] B. de Wit and H. Nicolai, Nucl. Phys. B188 (1981) 98; Phys. Lett. 108B (1982) 285.
[6] S.W. Hawking, Nucl. Phys. B144 (1978) 349.
[7] S.W. Hawking and G.F.R. Ellis, The large scale structure of space–time (Cambridge, 1973).
[8] P. Ginsparg and M. Perry, to be published.
[9] P. Schoen and S.-T. Yao, Commun. Math. Phys. 65 (1979) 45;
E. Witten, Commun. Math. Phys. 80 (1981) 381.
[10] C. Fronsdal, Phys. Rev. D12 (1975) 3819.
[11] E. Cremmer and B. Julia, Nucl. Phys. B159 (1979) 141.
[12] S.M. Christensen, M.J. Duff, G.W. Gibbons and M. Roček, Phys. Rev. Lett. 45 (1980) 161.
[13] P.K. Townsend, Phys. Rev. D15 (1977) 2802.
[14] F. Gursey and T.D. Lee, Proc. Nat. Acad. Sci. 49 (1963) 179.
[15] S. Weinberg, University of Texas preprint (1982).
[16] A. Guth, private communication.
[17] S. Coleman and F. DeLuccia, Phys. Rev. D21 (1980) 3305.