STA 511 Homework #3

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October 22 2015

1. The density on [0,infty) is given by

$$f(x) = \frac{2}{\pi(1+x)\sqrt{x^2+2x}}$$

   (a) To prove $0 \le x \le \theta$

$$\frac{f(x)}{c_\theta g_\theta} = \frac{\frac{2}{\pi(1+x)\sqrt{x^2+2x}}}{\frac{2}{\pi\sqrt{2x}}}$$

Since the numerators are same , they can be ignored and the remaining equation is now considered,which leaves us with;

$$\frac{f(x)}{c_\theta g_\theta} = \frac{\sqrt{2x}}{(1+x)\sqrt{x^2+2x}}$$

For every value of $x > 0$ in $\frac{f(x)}{c_\theta g_\theta}$; the denominator is greater than the numerator,

So $\frac{f(x)}{c_\theta g_\theta} \le 1$

$\rightarrow f(x) \le c_\theta g_\theta$

Now to prove $x > \theta$

$$\frac{f(x)}{c_\theta g_\theta} = \frac{\frac{2}{\pi(1+x)\sqrt{x^2+2x}}}{\frac{2}{\pi x^2}}$$

Since the numerators are same , they can be ignored and the remaining equation is now considered,which leaves us with;

$$\frac{f(x)}{c_\theta g_\theta} = \frac{x}{(1+x)\sqrt{x^2+2x}}$$

For every value of $x > 0$ in $\frac{f(x)}{c_\theta g_\theta}$; the denominator is greater than the numerator,

So $\frac{f(x)}{c_\theta g_\theta} \leq 1$

$\rightarrow f(x) \leq c_\theta g_\theta$

Hence the $f(x) \leq c_\theta g_\theta$ stands true for both conditions

(b)

(c)

2. The Laplace distribution has pdf f(x) given by

$$f(x) = \frac{\theta}{2} e^{-\theta|x|} for \theta > 0 and for -\infty < x < \infty$$

(a) Here we assume
$\theta = 1$
$\mu = 3$

$$g(x) = \frac{\mu}{\pi(\mu^2 + x^2)}$$

To find the optimal rejection constant c we use the formula

$$c = sup \frac{f(x)}{g(x)}$$

$$c = \frac{\frac{\theta}{2} e^{-\theta|x|}}{\frac{\mu}{\pi(\mu^2 + x^2)}}$$

$$c = \frac{\frac{1}{2}e^{-}|x|}{\frac{3}{\pi}(9 + x^2)}$$

$$c = \frac{\pi}{6}e^{-|x|}(9 + x^2)$$

(b)

(c)

3. The beta distribution wih parameters $\alpha > 0$ and $\beta > 0$ has a continuous density given by

$$f(x) = \frac{1}{B(\alpha, \beta)}x^{\alpha-1}(1 - x)^{\beta-1} for 0 < x < 1$$

(a)