# Are Worked Examples an Effective Feedback Mechanism During Problem Solving?

### Prawal Shrestha (prawalshrestha@gmail.com)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

## Ashish Maharjan (maharjan.ashish@gmail.com)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

## Xing Wei (wx6872@gmail.com)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

### Leena Razzaq (leenar@cs.wpi.edu)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

#### Neil T. Heffernan (nth@wpi.edu)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

#### Cristina Heffernan (ch@wpi.edu)

Worcester Polytechnic Institute, Department of Computer Science 100 Institute Road, Worcester, MA 01609 USA

#### Abstract

Current research in learning technologies has found both interactive tutored problem solving and presenting worked examples to be effective in helping students learn math. However, which information presentation method is more effective is still being debated among the cognitive science and intelligent tutoring societies and there is no widely accepted answer. This study compares the relative effectiveness between these two strategies when they are used as a feedback mechanism. Controlling for the number of problems, we presented both strategies to groups of students in local middle schools and the results showed significant learning in both conditions. In addition, our results are more in favor of the tutored problem solving condition as it showed significantly higher learning. We propose that the level of interactivity plays a role in which strategy is more effective.

**Keywords:** Tutored problem solving; Worked examples; Intelligent tutoring systems; Cognitive load.

#### Introduction

Students are often taught new material in mathematics by first being introduced to the principles needed to understand the new material, then worked examples that show how to use the principles to solve related problems and finally, practice problems for the students to work on. Traditionally, teachers often present only a few examples and assign a large number of practice problems. Likewise, learning technologies for mathematics often focus heavily on tutoring step-by-step problem solving with positive learning results (i.e. Cognitive Tutors (Anderson, Corbett, Koedinger, & Pelletier, 1995), Andes (VanLehn, Graesser,

Jackson, Jordan, Olney, & Rose, 2005) and the ASSISTment System (Razzaq, et al., 2007)) rather than presenting information about principles or presenting many worked examples.

Cognitive scientists have been interested in the role of worked examples in reducing cognitive load and helping students to learn, and there have been numerous studies on the effectiveness of worked examples (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001; Graesser, Moreno, Marineau, Adcock, Olney, & Person, 2003). Sweller and Cooper (1985) presented evidence that supported their hypothesis that worked examples helped novices to acquire "schemas" which they defined as "mental constructs that allow patterns or configurations to be recognized as belonging to a previously learned category and which specify what moves are appropriate for that category." It appears that novices who have not learned the required schemas have to depend on superficial search strategies in solving problems (Larkin, McDermott, Simon, & Simon, 1980) while experts can choose the next appropriate step based on their ability to correctly categorize the problem.

Sweller and Cooper's work suggested that problem-solving practice did not help students to acquire schemas as efficiently as the use of worked examples perhaps because of the change of focus from "goal-directed problem-solving" to "problem-state configurations." Kalyuga, Ayres, Chandler & Sweller (2001) presented results that point to a benefit of using worked examples with novice students and then using problem-solving practice later as students show more understanding.

Tutored problem solving helps students solve a problem by providing feedback and help on each step of a problem and is more interactive than reading worked examples. Several studies in the literature have found evidence of the benefit of greater interaction. Comparing Socratic and didactic tutoring strategies, Core, Moore and Zinn (2003) found that the more interactive (based on words produced by students) Socratic tutorial dialogs correlated more with learning. Chi, Siler, Jeong, Yamauchi and Hausmann (2001) found that students who engaged in a more interactive style of human tutoring were "able to transfer their knowledge better than the students in the didactic style of tutoring." Evens and Michaels (2006) compared expert human tutoring to reading a text book with the same material and found that the tutored students got significantly higher scores on a post-test. Results that support greater interaction have also been found in studies of intelligent tutoring systems (Graesser, Moreno, Marineau, Adcock, Olney, & Person, 2003; VanLehn, Graesser, Jackson, Jordan, Olney, & Rose, 2005). Additionally, there is evidence that tutored problem solving is more effective for less proficient students than less interactive methods of tutoring (Razzaq, Heffernan, & Lindeman, 2007).

Other researchers have been interested in comparing tutored problem solving to worked examples. Schwonke, Wittwer, Aleven, Salden, Krieg & Renkel (2007) found that students learned more from gradually fading worked examples to tutored problem solving than from tutored problem solving alone. In Schwonke et al.'s work, the fading of worked examples was the same for all students and did not depend on their demonstration of understanding.

Salden, Aleven, Renkl and Schwonke (2008) experimented with an adaptive fading scheme where worked examples were gradually faded when students showed understanding based on their self-explanations. Salden et al. found evidence that adaptively fading worked examples was more effective than fixed fading.

This study investigates whether students in a classroom setting will benefit more from interactive tutored problem solving than from worked examples given as a feedback mechanism. We also attempt to determine whether students will differ by their math proficiency. We expect that less proficient students will benefit more from tutored problem solving than more proficient students. We used the ASSISTment System, described in the next section, to test our hypothesis.

## The ASSISTment System

The ASSISTment System (Razzaq, et al., 2007) offers instruction to students while providing a more detailed evaluation of their abilities to teachers. The ASSISTment System is able to identify the difficulties individual students are having as well as the class as a whole. Teachers are able to use this detailed data on their students to tailor their instruction to focus on the areas that students are struggling with as identified by the system. Unlike other assessment

systems, the ASSISTment system also provides students with tutoring assistance while the assessment information is being collected.

The ASSISTment System is primarily used by middleand high-school teachers throughout Massachusetts who are preparing students for the Massachusetts Comprehensive Assessment System (MCAS) tests. Currently, there are over 3,000 students and 50 teachers using the ASSISTment System as part of their regular math classes. We have had over 30 teachers use the system to create content.

# **Experiment**

# **Participants**

This experiment was conducted with 8<sup>th</sup> grade students in three local middle schools located in central Massachusetts. One of the schools was suburban, while the other two were urban. Over 80% of the students who participated were from a school which according to its state test scores is in the bottom 5% in the state and has been labeled by the *No Child Left Behind Act* as not making adequate yearly progress. The experiment took place in the months of April and May of 2008 at the computer labs of the respective schools. The students who participated in this experiment were exposed to both conditions: tutored problem solving, and worked examples. They were given problem sets to work on and their actions were logged which was later analyzed.

#### Material

For the experiment we created nine problem sets, each consisting of four to five ASSISTments. All of the main questions of the ASSISTments were taken from 6<sup>th</sup> Grade MCAS tests for Mathematics (2001 – 2007) focusing on the *Patterns, Relations and Algebra* section, which concentrates on different mathematics skills: populating a table from a relation, finding a missing value in a table, using fact families, determining equations for relations, substituting values into variables, interpreting relations from number patterns, and finding values from a graph.

## **Procedure**

Each problem set in this study was a collection of ASSISTments grouped into three sections: pre-test, experiment, and post-test. For the experiment, students were considered to have completed a problem set only if they finished every part of it. We used the gain score from pre-test to post-test to determine whether students had learned anything from the conditions.

When students start a problem set, they are first given a pre-test problem. The pre-test is an ASSISTment with a single question, and does not include any form of help or hint.

In order to make sure that the students understand what is happening, we inform them that the question was a pre-test and that they will not receive feedback on whether their answer is right or wrong. They are also informed that the question will be repeated at the end of the problem set.

In the pretest, students are allowed only one attempt to answer the question, so the first answer they provide is considered as the final answer for the pre-test and it cannot be changed. After the one question pre-test, students are presented with the first question from a randomly chosen condition. The computer randomly assigns either the tutored problem solving or the worked example condition to the students. This part consists of two or three ASSISTments all in the same condition. Within the two conditions, students do the same number of questions, so the content of the questions were held constant between the conditions. Finally, when the students finished all of the ASSISTments in the experiment section, they were given the post-test which is the pre-test question repeated. Also similar to the pre-test, the first response of the student is recorded and used for analysis. However, unlike the pre-test, we do inform the students regarding the correctness of their answer. Learning can be assessed by comparing the results of the pre-test and the post-test.

In the worked example condition, when a student gives an incorrect answer or presses on the "Break this problem into steps" button, a problem that is similar to the main question is shown solved step by step. As such, the students will have a pattern to follow in order to solve the problem. The worked example condition is shown in Figure 1. The student is asked to read through the worked example and choose "I have read the example and now I am ready to try again" when he/she is done. The student is then asked to do the original problem again.

In the tutored problem solving condition, students who get a problem wrong are asked to answer a set of questions that break down the main problem into steps, shown in Figure 2. If the student provides a wrong answer or presses on the "Break this problem into steps" button, he/she will be directed to the first scaffolding question, which helps the student to understand the first step to solve the original problem. Students can ask for hints on each step if they need more help. If the student presses on the "Show me a hint" button, hints will be shown one by one until the student reaches a "bottom-out hint" which is typically the answer to this scaffolding question. After this, the student is directed to the next scaffolding question. The number of scaffolding questions depends on the complexity of the original question. At the end, the student is expected to understand how to do the original problem step by step.

During the experiment, teachers introduced the problem sets as a regular assignment. As such, students were not aware of the randomized controlled experiment. They were neither briefed about the problem set structure nor the number of ASSISTments in a problem set. Thus, students might not have been aware that they were taking a pre-test until they submitted an answer, as we tell them that the question they answered was a pre-test only after answer submission.

We do not distinguish the experiment section from the post-test with any specific instruction or notice like we do in

the pre-test. The only way a student can know that they are in the post-test is if they realized that the pre-test question has been repeated. It should be noted that there is a possibility that some students were not exposed to either of the conditions since conditions are introduced only when a student makes a mistake in the first response. If students answered all of the ASSISTments in a problem set correctly in their first attempt then they would not have been exposed to any of the conditions and their performance on that problem set were not included in the study.

Karen purchased a new camera for \$60. She also purchased 5 rolls of film The total cost of the camera and the rolls of film was \$90. Karen's purchase is represented by the equation below.						
In the equation, $f$ stands for the cost of each roll of film. 5f + 60 = 90						
What was the cost of each roll of film that Karen purchased?						
Break this problem into steps Type your answer below (mathematical expression):						
Submit Answer						
Let's move on and figure out this problem.						
Let's look at the solution for a problem similar to the one above: Simond purchased a new play-station for \$100. He also purchased 4 game CDs.						
The total cost of the game and the CDs was \$200. Simond's purchase is represented by the equation below.						
In the equation, f stands for the cost of each CD.  4f + 100 = 200						
What was the cost of each CD that Simond purchased? Solution:						
Let us first try to find out the amount Simond spends to buy CDs. Simond spends \$100 out of \$200 on the play-station and the rest on CDs. So to find the amount he spends on CDs we need to subtract 100 from 200.						
200-100 = 100.						
So he spends \$100 on cds. Now, we know that Simond spends \$100 on CDs						
and that he buys a total of 4 CDs.						
Let us try to find the cost of each CD.						
Simond bought 4 CDs for \$100. So if f represents the cost of each CD, we add f 4 times to get \$100.						
f + f + f + f = 100 We need to find a number that when added 4 times to itself gives us 100.						
This number can be found by dividing 100 by 4.						
100 ÷ 4= 25.						
So, \$25 must be the cost of each CD.						
I have read the example and now I am ready to try again.						
Submit Answer						
✓ Correct!						
Karen purchased a new camera for \$60. She also purchased 5 rolls of filn The total cost of the camera and the rolls of film was \$90. Karen's purchase is represented by the equation below. In the equation, $f$ stands for the cost of each roll of film. $5f + 60 = 90$ What was the cost of each roll of film that Karen purchased?						
The cost of each roll of film is \$6.						
Type in 6.  Comment on this hint  Type your answer below (mathematical expression):						
6						
Submit Answer						
✓ Correct!						
You are done with this assignment!						

Figure 1: The worked example condition requires students to read the example and then try to answer the question again.

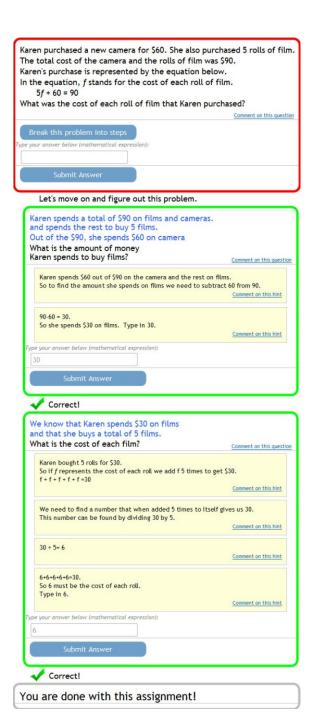


Figure 2: The tutored problem solving condition requires students to work through each step of the problem.

# Results

Our experiment used a repeated measures design where students participated in a different number of experiments, and each time the student started an experiment, he/she was randomly assigned to one of the two conditions. For the analysis, we only considered the students who had completed at least one problem set in both of the conditions and ignored all other students who were exposed to only one

condition. Problem sets that were not completed were ignored. In addition, we also ignored students who correctly answered both the pre-test and the post-test questions, as we assumed the student had mastered that material. Since repeated measure design suffers from ordering effects, we relied on the random assignment of conditions as a control for that effect.

Out of a total of 186 participants, 166 students completed at least one problem set and we had data from a total of 866 attempts at completing a problem set. We then ignored data where both pre-test and post-test answers were correct. We also ignored data from students who completed only one of the two conditions. We then had a total of 68 students who participated in both tutoring conditions. So this means each of the 68 students completed at least one problem set where they were given tutored problem solving and at least one problem set where they were given worked examples.

For each student, the average learning gain from tutored problem solving and the average learning gain from worked examples were calculated. Learning gain for a problem set was defined to be the post-test score minus the pre-test score. Average learning gain for the tutored problem solving condition was defined to be the average of the learning gains for the entire problem sets that the student did when they were assigned to the tutored problem solving condition. Similarly, the average learning gain for worked examples was the average of the gains for all of the problem sets that the students did when they were assigned to the worked examples condition. There was no need to check if both groups were balanced at pre-test since our experiment was a repeated measure design and each student participated in both conditions.

There was a significant effect for condition with tutored problem solving receiving higher gain scores than worked examples (35% average gain vs. 13% average gain), t(67) = 2.38, p = 0.02. These results are shown in Table 1.

Table 1: Mean gain scores for both conditions.

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair	Scaffold	.3498	68	.53799	.06524
1	Worked	.1306	68	.52168	.06326

To determine whether there was an aptitude-treatment interaction we calculated a student math proficiency score using an Item Response Theory (IRT) model which takes into account difficulty of ASSISTments and how students performed on ASSISTments throughout the school year.

We did not have IRT scores for five students, so this analysis was done on data from 63 students. We did a median split on the IRT scores to categorize students as either high or low proficiency. We did not find a significant difference based on math proficiency (F(1, 61) = .158, p = 0.69). Both high proficiency and low proficiency students learned more from the tutored problem solving condition

than from the worked example condition. High proficiency students had a mean gain of 47% with tutored problem solving and 22% with worked examples (t(29) = 1.599, p = 0.12). Low proficiency students had a mean gain of 20% with tutored problem solving and 3% with worked examples (t(32) = 1.404, p = 0.17).

Because the tutored problem solving is more interactive, it does consume more time. Tutored problem solving (M = 244 seconds) took significantly more time on average than worked examples (M = 166 seconds), (t(66) = 2.93, p = 0.002).

## **Discussion and Contributions**

Our study compared the effectiveness of tutored problem solving versus worked examples when used as feedback. Students participated in the study in a classroom environment and the problems were presented as classroom assignments. Our results indicate that tutored problem solving is significantly better than worked examples in terms of the average gain of students in each condition. Furthermore, we did not find an aptitude-treatment interaction.

Our study differed from previous studies in that we compared worked examples to tutored problem solving rather than untutored problem solving. We also differ in that we presented worked examples as feedback after students unsuccessfully attempted to solve a problem rather than presenting them before they attempted problem solving.

We speculate that many studies that have found positive results for worked examples were done in lab settings, where an adult lab attendant provided the extra focusing attention that a classroom environment does not provide. Perhaps in the classroom setting, the more interactive tutored problem solving condition was superior due to the fact that the higher interactivity level required from tutored problem solving better engages students' focus. This theory suggests that students with greater focus might yield results that would be more in line with the current literature.

Salden et al. (2008) thought of their results as an instance of the Assistance Dilemma coined by Koedinger and Aleven (2007) which studies the dilemma of when to give assistance to students versus when to withhold information in an attempt to get students to generate information on their own. The Assistance Dilemma would consider worked examples to be "high assistance" while tutored problemsolving to be "low assistance". However, this does not seem to consider that these may be seen differently by students depending on how well-focused they are. For instance, Chi, Bassok, Lewis, Reimann, & Glaser (1989) found a difference in the way that students used worked examples based on their proficiency in problem-solving: "... we find that the Good students use the examples in a very different way from the Poor students. In general, Good students, during problem solving, use the examples for a specific reference, whereas Poor students reread them as if to search for a solution." Recently we (Razzaq, Heffernan, &

Lindeman, 2007) found that students who received workedout *solutions* to problems rather than tutored problemsolving learned more only if they were above average students. Below average students did better with tutored problem-solving. (We believe that our use of *worked-out solutions* is similar to worked examples in that they do not withhold information.) This is important because it raises the question about whether worked examples are always a better thing to do before problem solving for all students.

We think our theory can explain the current results in this area. In particular, we speculate that the students in the recent Salden et al. study (2008) might have been just the right type of well focused students that could benefit from reading worked examples. However, if you want to help the less focused student then tutored problem solving is superior.

This conclusion is reasonable in a few respects. Firstly, these two conditions have different degrees of interactivity. In the worked examples condition, a student is shown a completely solved example problem which is similar to the main problem. The student is only one click away from answering the original problem again. In contrast, the tutored problem solving condition asks several subsequent questions pertaining to the main problem, all of which have to be completed before returning to the main problem again. For most students, it is reasonable to assume that answering questions frequently keeps them more focused than reading off of a screen.

It is possible that our results can be explained by cognitive load theory: perhaps the tutored problem solving reduces cognitive load even more than worked examples as students are walked through problems step by step and subgoals are set for them. There may be a tradeoff in that students may lose the big picture by working on pieces of a problem at a time and are not asked to induce principles, but sub-goal learning has been found to help guide problem solving by helping learners focus on the steps (Catrambone, 1998).

According to Sweller and Cooper (1985), "The use of worked example problems may redirect attention away from the problem goal and toward problem-state configurations and their associated moves." Perhaps using worked examples as feedback increased cognitive load as students tried to read the example and solve the problem at the same time. McLaren et al. (2008) found little difference in learning gains between tutored problem solving alone and tutored problem solving interleaved with worked examples so we believe this theory makes sense.

A logical follow up study would be if we controlled for the level of interactivity in the two conditions by asking students questions about the worked examples they read. Another logical follow up would be to control for time on task.

In conclusion, the results of our study show that worked examples as feedback are not more effective than tutored

problem solving. The key may be in how interactive the tutoring strategy is.

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