Embedded Control Laboratory

Ball on inclined plane

MECHATRONICS MASTERS

Report of lab exercise 1

WS 2019/20

Group 08

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09.11.2019

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1. Introduction

The main aim of the experiment is to balance the ball on the inclined plane by the Servo motor which acts as a actuator to change the angle of inclined plane with respect to the horizontal axis. The servo motor which controls the angle of the inclind plane is controlled by the Programmable Micro-controller.

2. Objective

- To find the relation between inclined plane angle and Servo motor angle such that the ball is balanced in the centre position of inclined plane.
- By Understanding the mathematical and geometrical relationship develop the mathematical equation for $\alpha = f(\beta)$.
- Develop the Scilab script for the obtained mathematical equation and plot the graph $\alpha = f(\beta)$ with α vs β.
- By the above obtained plot approximate linearization of the function $\alpha = f(\beta)$

3. Graphical Representation of the BOIP System

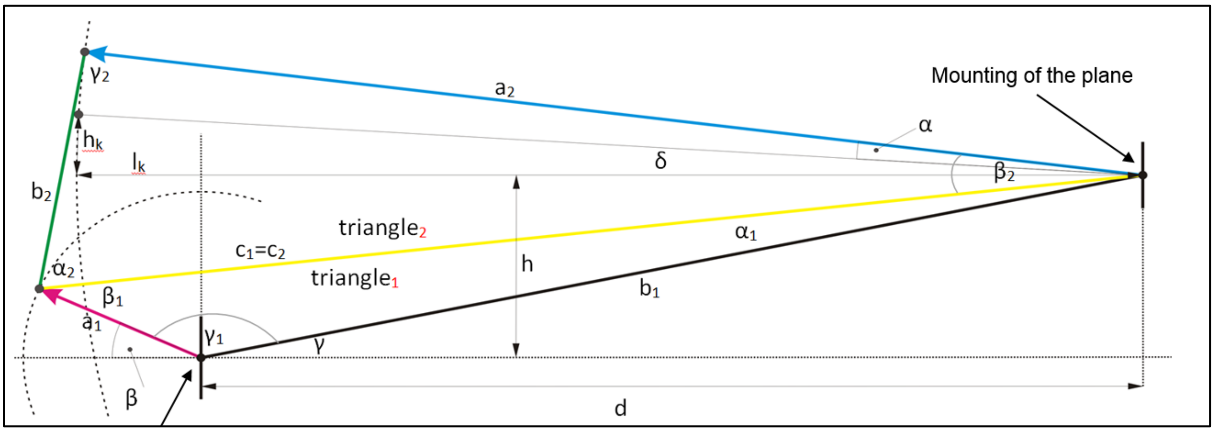


Figure 3.1 : Model of the Inclined plane

The graphical representation of ball placed on a inclined plane is described by the figure 3.1 . The ball is placed on the inclined plane which consits of two metal rods on its ends. The inclined plane is connected to the servo motor arm that controls the angle of the inclined plane. The tilting of the inclined plane results in the change of the ball position. So the ball placed on inclined plane starts moving from one end to another end. The ball is subjected to rotational, horizontal and gravitational forces. The angles and the different forces of the ball are considered in the mathematical calculation.

From the figure,
β = servo motor arm angle
α = inclined plane angle with respect to horizontal axis.
h_k = distance between hinge point and rolling surface
lk = length of rolling surface
a_1= length of the servo motor arm
a_2 = Half a plane
b_1 = imaginary line between the axis of the servo motor and the pivot point of the plane
b_2 = an imaginary connection between plane edge and one end of the motor arm

4. Mathematical Calculations

The following table shows characteristic values of the physical model.

Name	Value	SCILAB value
d	105mm	105 e-3
h	19mm	19e-3
l=a2	118.72mm	118.72e-3
r=a1	18mm	18e-3
c=b2	36.22mm	36.22e-3
hk	17mm	17e-3
lk	117.5mm	117.5e-3

Table 4.1: Characteristic values of the physical model

The following equations are helpful for building the SCILAB script for the model

• Constants:

$$b_1 = \sqrt{d^2 + h^2} = \sqrt[2]{(105^2 + 19^2)} = 106.70 \text{ mm} \text{ - - - - - - - - } (1)$$

$$\gamma = tan^{-1}\left(\frac{h}{d}\right) = \text{tan-1 } (17/117.5) = 8.23° \qquad \text{- - - - - - - - } (2)$$

$$\delta = \tan^{-1}\left(\frac{h_k}{l_k}\right) - - - - - - - - -(3)$$

where tan−1 denotes the inverse function of tangent, and often also named as arctan (scilab: atan())

- Sum of angles at the pivot point of the plane:

$$\alpha + \delta + \gamma = \alpha_1 + \beta_2 \quad \text{--------------- (4)}$$

- Law of cosines for two triangles:

$$c_1^2 = a_1^2 + b_1^2 - 2a_1 b_1 . \cos \gamma_1 \quad \text{--------------- (5)}$$

$$a_1^2 = c_1^2 + b_1^2 - 2c_1 b_1 \cdot \cos \alpha_1 \quad \text{--------------- (6)}$$

$$b_2^2 = a_2^2 + c_1^2 - 2a_2 c_1 . \cos \beta_2 \quad \text{--------------- (7)}$$

- Sum of angles at the servo motor arm:

$$\beta + \gamma_1 + \gamma = \pi \quad \text{--------------------- (8)}$$

$$\gamma_1 = \pi - (\beta + \gamma) \quad \text{--------------------- (9)}$$

- Solving above equations

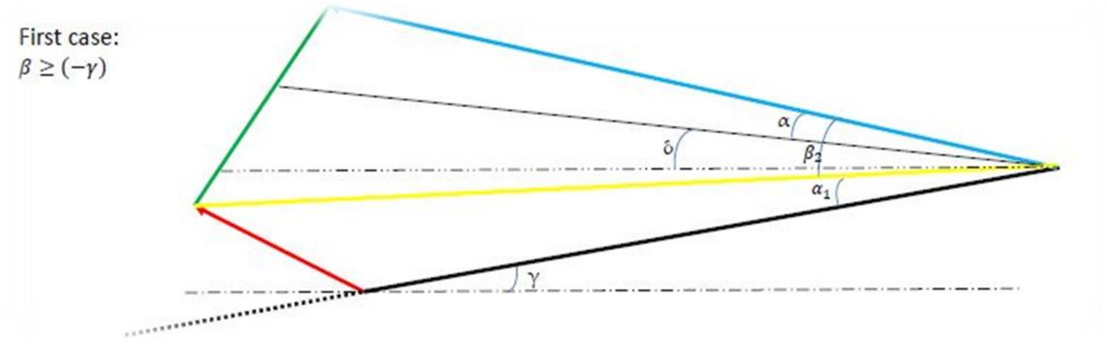
$$c_1^2 = a_1^2 + b_1^2 - 2a_1 b_1 \cos\big(\pi - (\beta + \gamma)\big)$$
$$c_1^2 = a_1^2 + b_1^2 + 2a_1 b_1 \cos(\beta + \gamma) \quad \text{---------------- (10)}$$

$$\cos \alpha_1 = \frac{c_1^2 + b_1^2 - a_1^2}{2c_1 b_1}$$

$$\alpha_1 = \cos^{-1}\left(\frac{c_1^2+b_1^2-a_1^2}{2c_1 b_1}\right) \quad \text{-------------------- (11)}$$

$$\cos\beta_2 = \frac{a_2^2 + c_1^2 - b_2^2}{2a_2 c_1}$$

$$\beta_2 = \cos^{-1}\left(\frac{a_2^2+c_1^2-b_2^2}{2a_2 c_1}\right) \quad \text{--------------------- (12)}$$



Case I: $\beta \geq (-\gamma)$

$$\alpha + \delta + \gamma = \alpha_1 + \beta_2 \quad \text{- -} \; (13)$$

Substituting the equations (2),(3),(11),(12) in Equation (13) we obtain "α" :

$$\alpha = \alpha_1 + \beta_2 - \delta - \gamma$$

$$\alpha = cos^{-1}\left(\frac{c_1^2+b_1^2-a_1^2}{2c_1b_1}\right) + \cos^{-1}\left(\frac{a_2^2+c_1^2-b_2^2}{2a_2c_1}\right) - tan^{-1}\left(\frac{h}{d}\right) - tan^{-1}\left({}^{h_k}/_{lk}\right) \quad \text{- -} \; (14)$$

The final equation for case I is obtained for "$\beta \geq (-\gamma)$"

Case II: $\boldsymbol{\beta} \leq (-\boldsymbol{\gamma})$

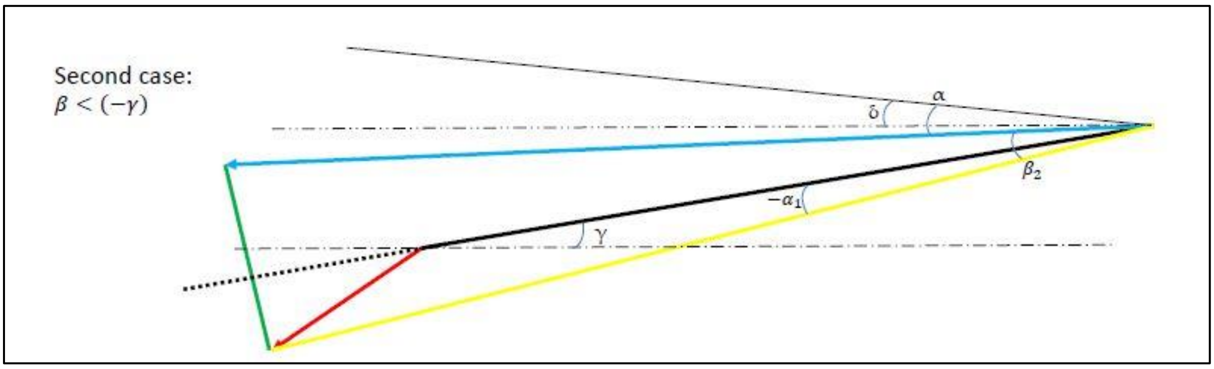


Figure 4.1: Variation of "alpha vs beta" along with "normal linearization".

From the above figure,

The servo motor arm is $\beta \leq -\gamma$ then the angle α_1 will be negative (-).

$$\alpha = -\alpha_1 + \beta_2 - \delta - \gamma \quad \text{- -} \; (15)$$

Substituting the equations (2),(3),(11),(12) in Equation (13) we obtain "α" :

$$\alpha = -\left(cos^{-1}\left(\frac{c_1^2+b_1^2-a_1^2}{2c_1b_1}\right)\right) + \cos^{-1}\left(\frac{a_2^2+c_1^2-b_2^2}{2a_2c_1}\right) - tan^{-1}\left(\frac{h}{d}\right) - tan^{-1}\left({}^{h_k}/_{lk}\right)$$

-------------------- (16)

The above two equations (14) and (16) determine α is function of β

$$\alpha = f(\beta).$$

The plot is obtained for the mathematical derived equation and the linearization

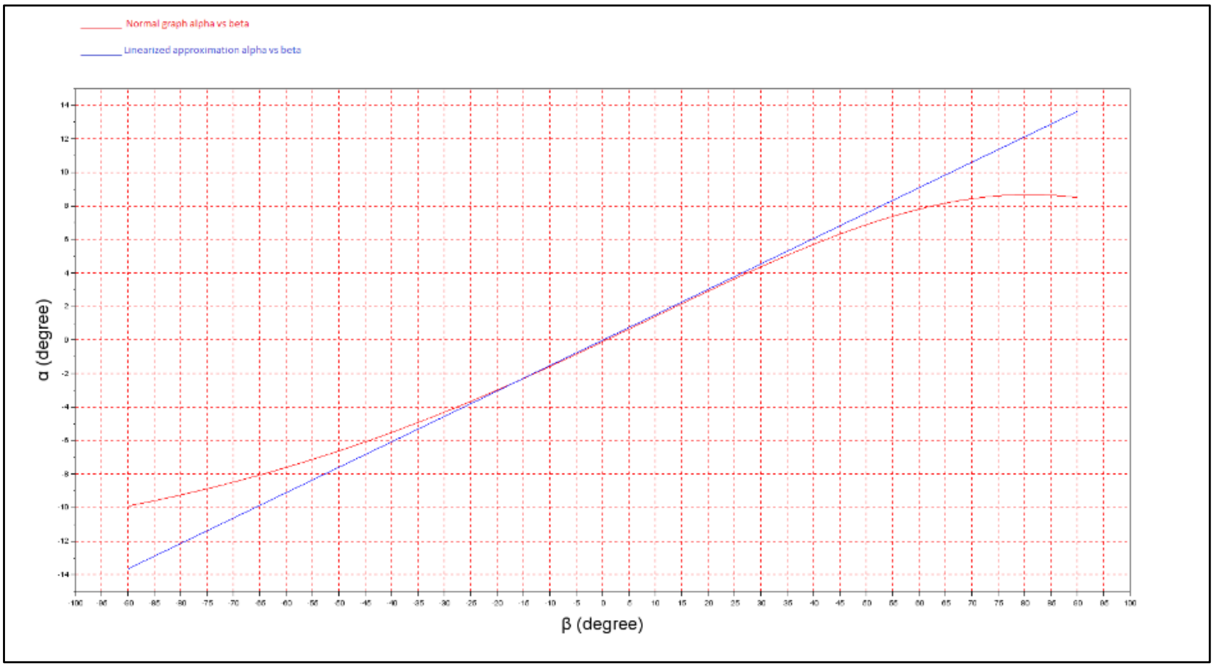


Figure 4.2: Variation of “alpha vs beta” along with “normal linearization”.

Determining the limits for angle β from the plot when the angle α is limited to ±5 degrees:
From the plot we can note that;
When α = +5° => β = 35.1°
When α = -5° => β = -35.3°

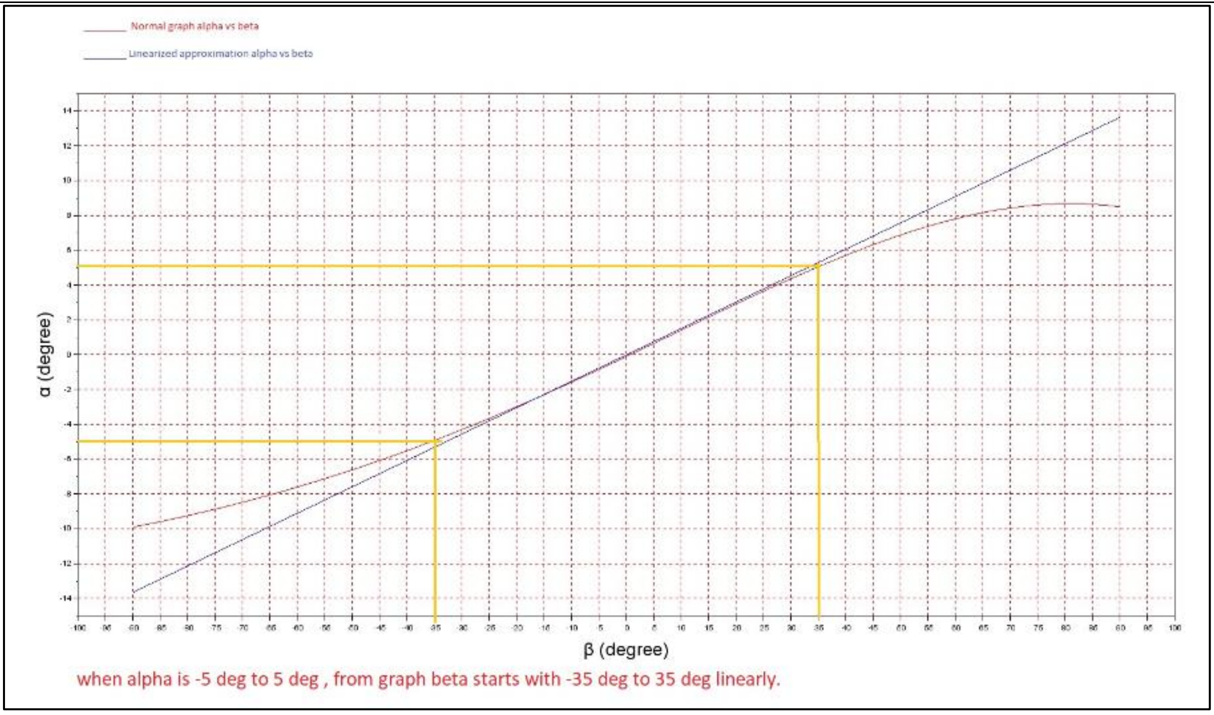


Figure 4.3: The limits of β when α is limited to +5° to -5°

Scilab Code

To calculate and plot exact curve and approximate curve for α for β from -90° to 90°:

```
// constants
d=0.105;
h=0.019;
a2=0.11872;
a1=0.018;
b2=0.03622;
hk=0.017;
Ik=0.1175;

// given equations
b1=sqrt(d^2+h^2);
gamma1=atand(h/d);
delta=atand(hk/Ik);

// Case1: beta<=(-gamma)
beta1= linspace (-(gamma1),-90,90);
c=sqrt(a1^2+b1^2+2*a1*b1*cosd(beta1+gamma1));
for n=1:90
alpha1(n) = acosd((c(n)^2+b1^2-a1^2)/(2*c(n)*b1));
//disp (alpha1)
beta2(n)= acosd((c(n)^2+a2^2-b2^2)/(2*a2*c(n)));
```

```
end
alpha = -alpha1 + beta2 - gamma1 - delta;
//disp (alpha)

//disp (beta1)
clf;
plot (beta1,alpha,'r')
xlabel("beta1","fontsize",5)
ylabel("alpha","fontsize",5)

alpha = (a1/a2)*beta1;
plot (beta1,alpha,'b')
xlabel("beta1","fontsize",5)
ylabel("alpha","fontsize",5)

// Case1: beta>=(-gamma)
beta1= linspace (+90,-(gamma1),90);
c=sqrt(a1^2+b1^2+2*a1*b1*cosd(beta1+gamma1));
//disp (c)
for n=1:90
alpha1(n) = acosd((c(n)^2+b1^2-a1^2)/(2*c(n)*b1));
//disp (alpha1)
beta2(n)= acosd((c(n)^2+a2^2-b2^2)/(2*a2*c(n)));
end
alpha = alpha1 + beta2 - gamma1 - delta;
disp (alpha)

//disp (beta1)
plot (beta1,alpha,'r')
xlabel("β (degree)","fontsize",5)
xgrid (5)
ylabel("α (degree)","fontsize",5)

beta1= linspace (-90,90,180);
alpha = (a1/a2)*beta1
plot (beta1,alpha,'b')
xlabel("β (degree)","fontsize",5)
xgrid (5)
ylabel("α (degree)","fontsize",5)
```

5. Explanation for the Given Questions:

5.1 Explain with reasons why the approximation is not so good if angle β greater than 40° or smaller than -40°.

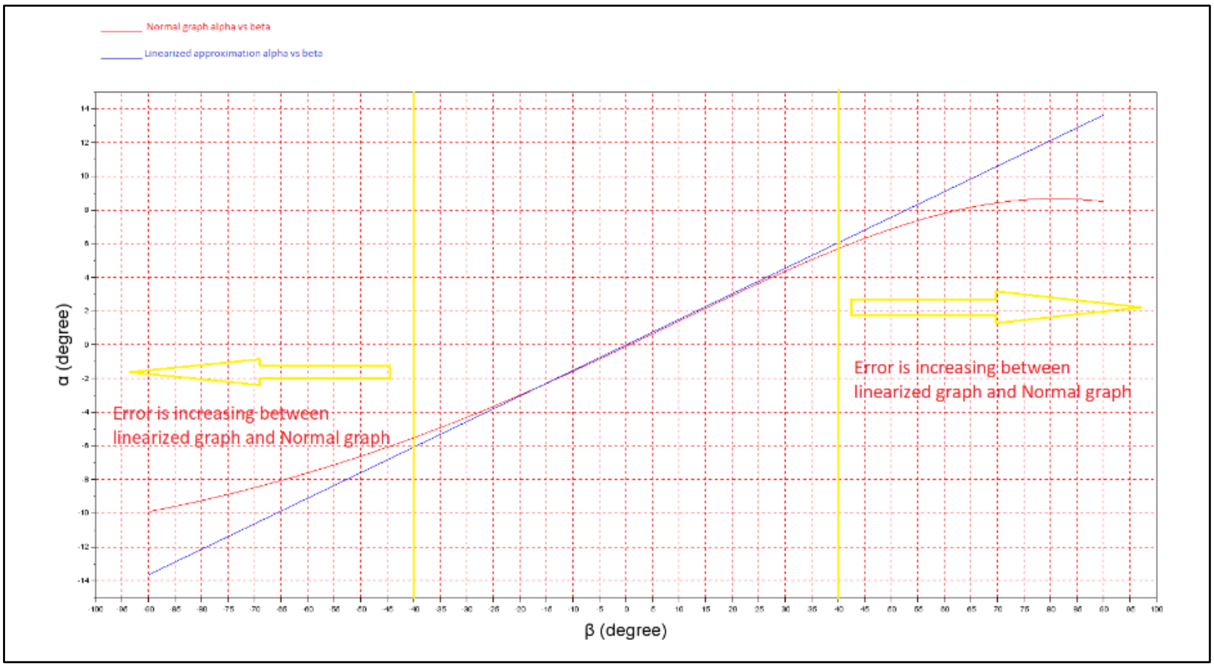


Figure 5.1: Angle β greater than 40° or smaller than -40°.

From the above graph (Figure 5.1), for the values of β beyond ± 40° the graph Starts deviating away from the linear approximation curve and the relationship between and becomes non-linear. But for the value of β between +40° and -40°, both linear approximation curve and the actual curve follow the same path and the relationship between β and becomes linear. So, the approximation is feasible only when β is between ±40°.

5.2 In your SCILAB simulation, maximize the range of the angle β to ±300 degrees, plot, and explain your output graph.

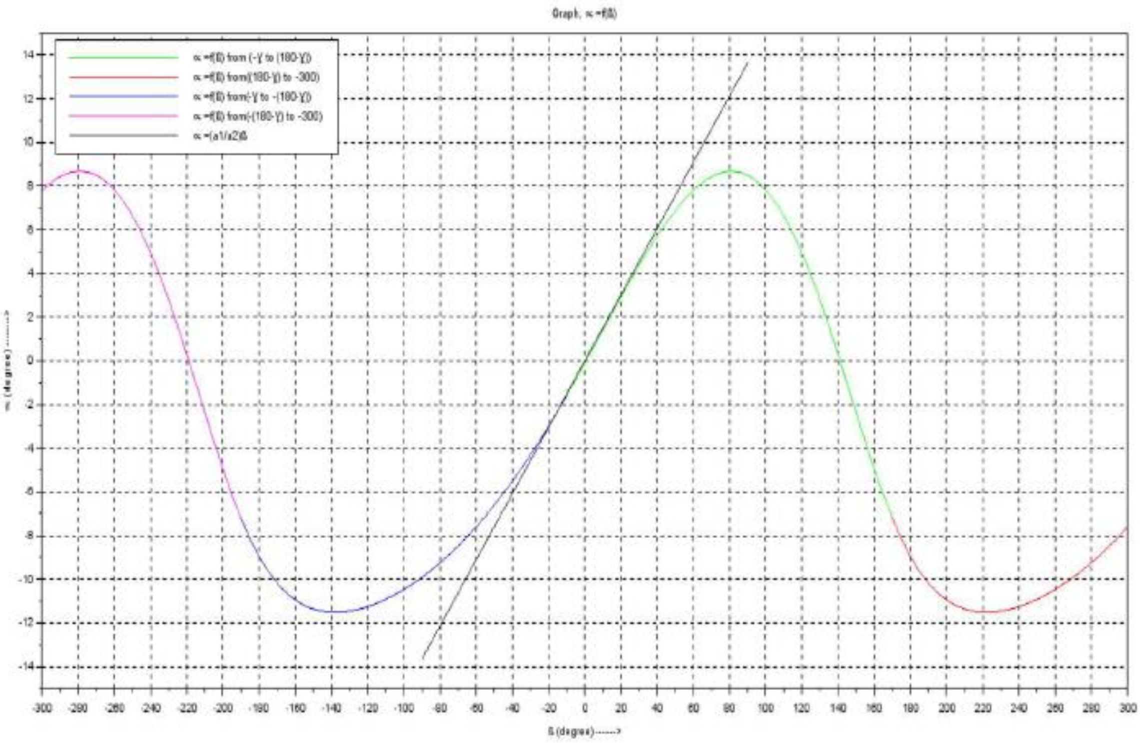


Figure 5.2: Variation of α Vs β in the range of -300° to +300°

From the above graph in figure 7, we can conclude that: For the values of β ranging between ±300, the angle α varies periodically and it changes from positive to negative when β varies between the intervals - γ to 180 - γ. When the servo arm crosses these angles, it is in line with the imaginary line joining the pivot point I and servo motor centre L. Balancing the ball becomes tough as the relationship between α and β is highly non-linear for larger and smaller values of β.

5.3 What are the mechanical problems when the black rod is too long or too short? (with plots and arguments)

Considering the arm which changes the angle of the inclined plane, when the arm length changes the positioning of inclined plane with respect to the arm also changes. Now considering two arm lengths short and long with respect to our normal arm length.

a) The black rod becomes too short. i.e., b_2 is small.
When the arm length is too small that is the rotation of the inclined plane remains to be short as the length of the arm is short compared to the inclined plane and the angle of rotation.

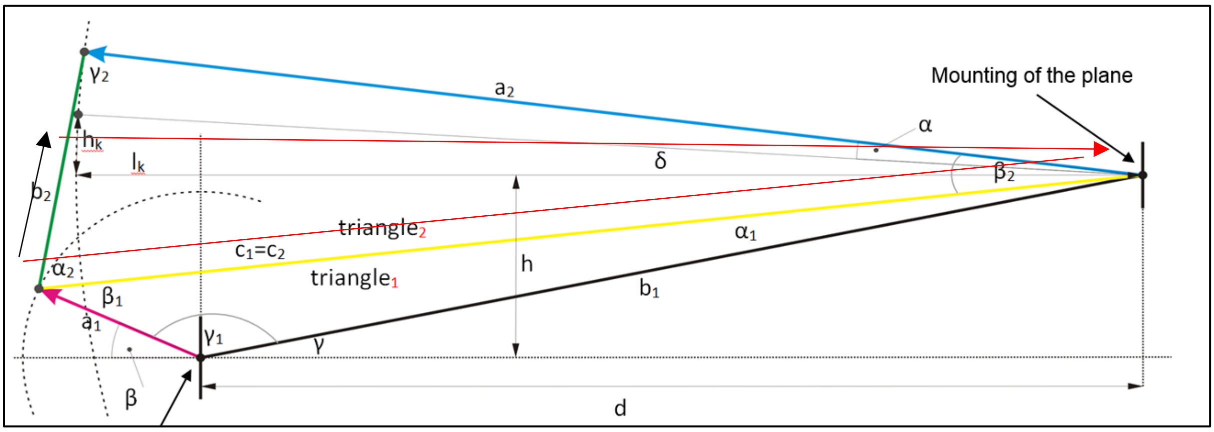


Figure 5.3: Reducing the arm length b2

From the above figure it is seen that as the rod length reduces the other two lengths of the plane that is the incline plane and the horizontal axis are too long, so the rotation of the whole system lays in the negative axis.
Considering length of the b2 as 0.01622.
b2 = 16.22mm

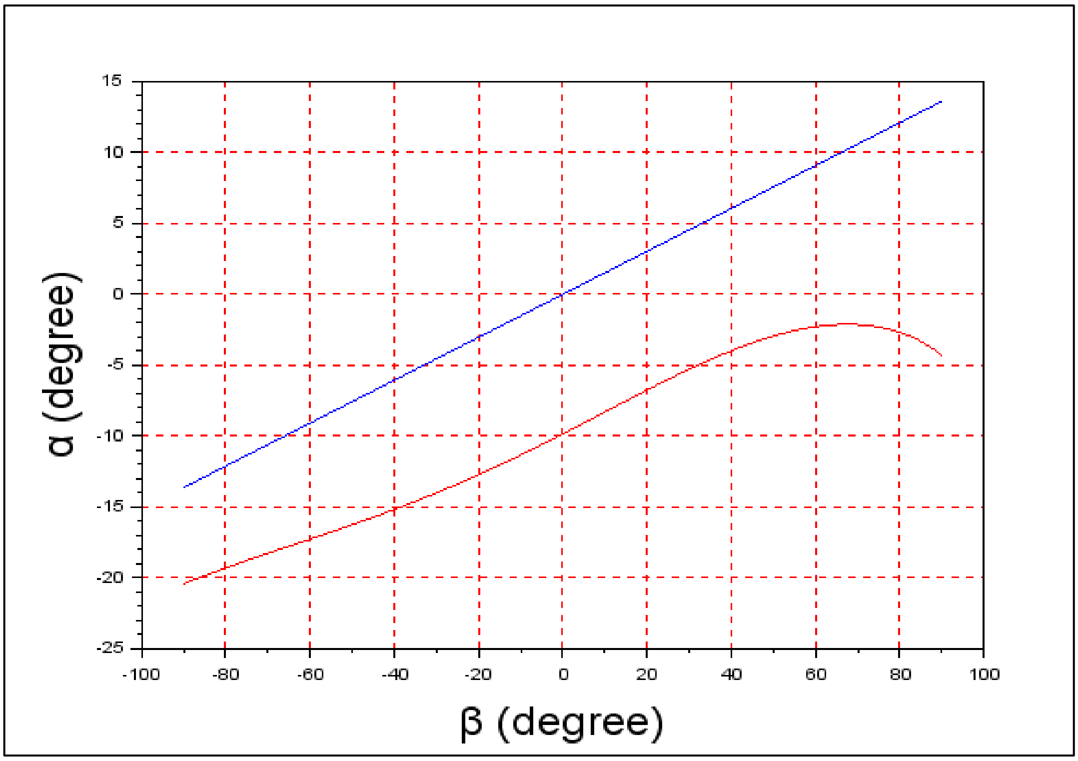


Figure 5.4: Arm length is too small vs linear polarization

b) The black rod becomes too long. i.e., b_2 is long.

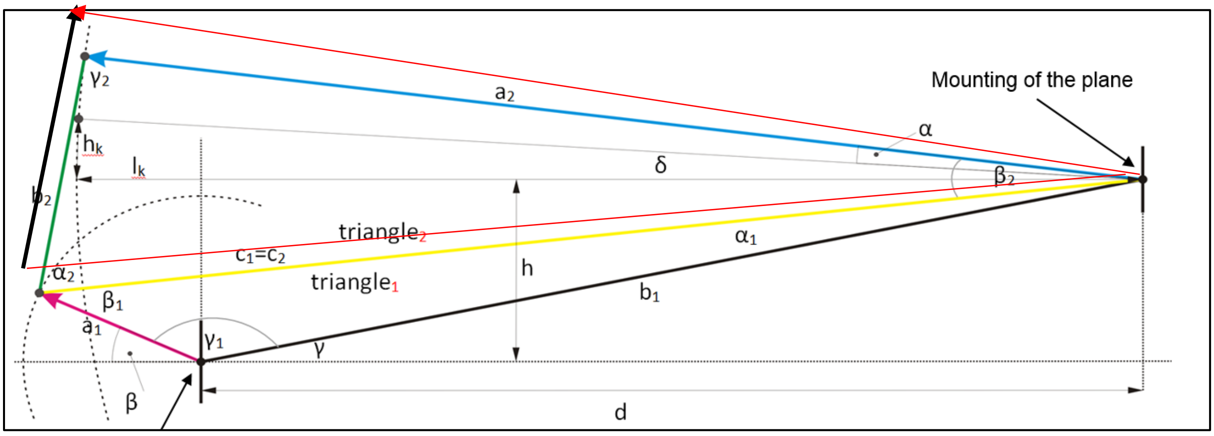


Figure 5.5: Increasing the arm length b2

From the above figure it is seen that as the rod length increase the other two lengths of the plane that is the incline plane and the horizontal axis are along with the rod, so the rotation of the whole system lays in the positive axis. Considering length of the b2 as 0.13622.
b2 = 136.22mm

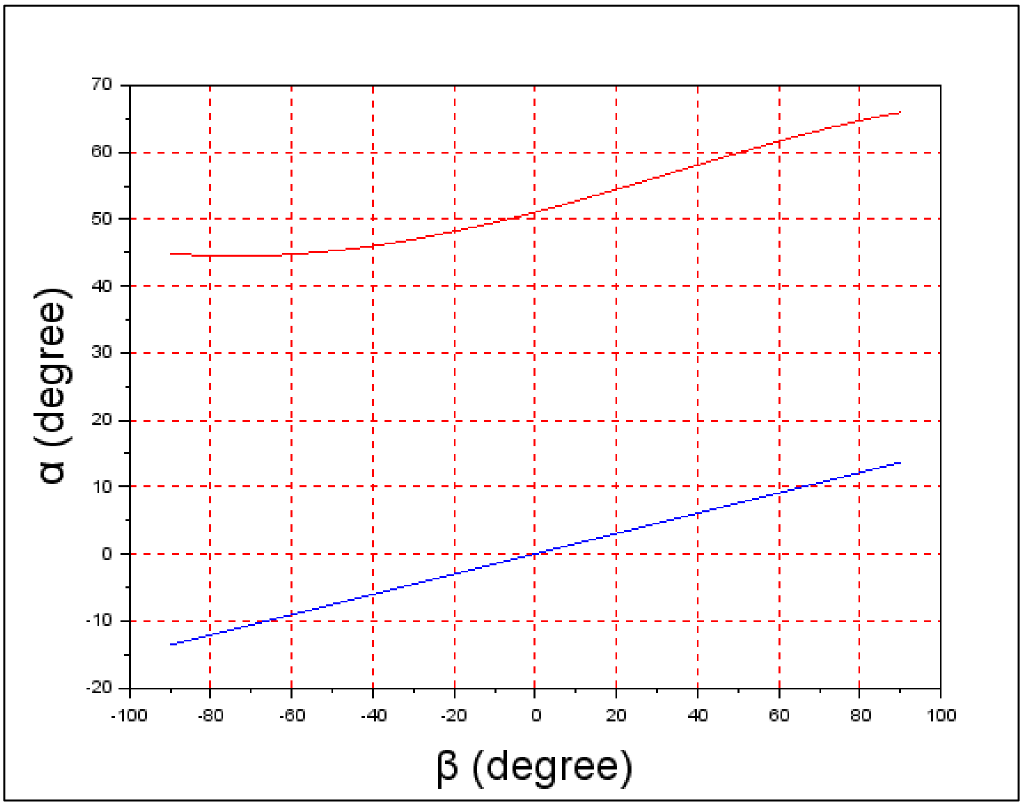


Figure 5.6: Arm length is long vs linear polarization.