DS 284 - Numerical Linear Algebra
Assignment 1

N Surya (22938)

October 5, 2023

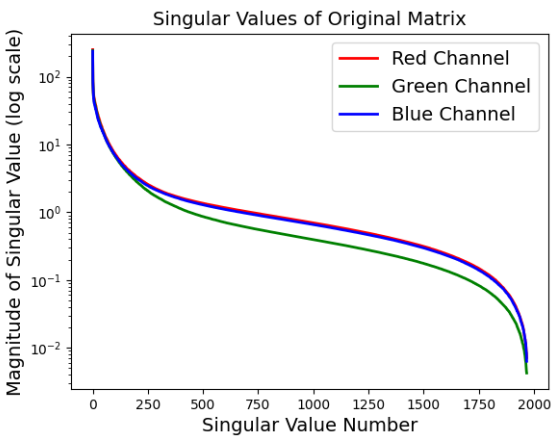
Problem 2

a) The Required Number of Singular Values:

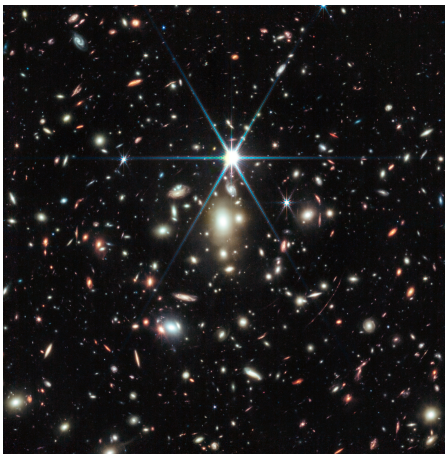
- Based on visual inspection and relative error (2-norm) calculations, the required number of singular values is chosen to be 190.
- After inspecting images for various numbers of singular values ranging from 10 to 1000 and tabulating the percentage relative errors in all the channels, this value was chosen as the relative error was less than 1.5 % for all the channels.
- Also from the plot, it can be clearly seen that beyond approximately 200 values, the magnitude of singular values drop significantly.

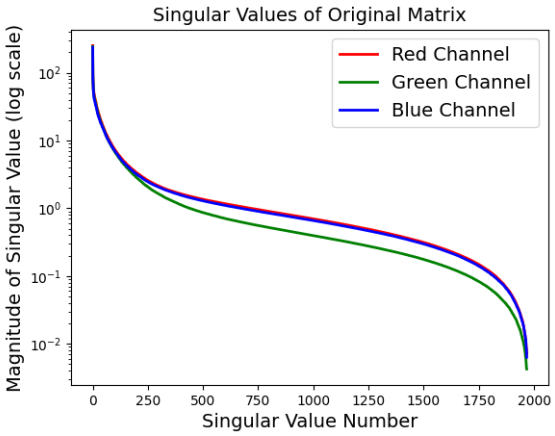
Percentage Relative Error vs Rank:

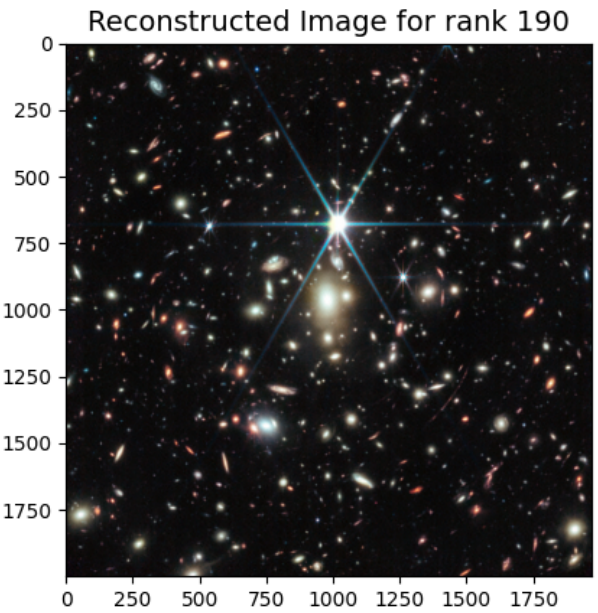
rank	Red Channel	Green Channel	Blue Channel
10	17.0595	16.0235	15.4503
20	12.0862	11.7280	11.3296
30	9.3395	8.9314	8.7796
40	7.4401	7.1143	7.2235
50	6.1244	6.0275	6.0074
60	5.0699	4.9641	5.0448
70	4.3271	4.2096	4.2926
80	3.7779	3.6244	3.7473
90	3.3186	3.2011	3.3205
100	2.9582	2.8027	2.9606
150	1.8559	1.7169	1.8513
160	1.6997	1.5615	1.7103
170	1.5977	1.4527	1.6091
180	1.4928	1.3354	1.5010
190	1.4117	1.2514	1.4104
200	1.3281	1.1564	1.3259
250	1.0263	0.8541	1.0361
300	0.8464	0.6654	0.8581
350	0.7301	0.5457	0.7425
400	0.6496	0.4611	0.6550
450	0.5887	0.3999	0.5914
500	0.5390	0.3561	0.5418
1000	0.2771	0.1627	0.2768

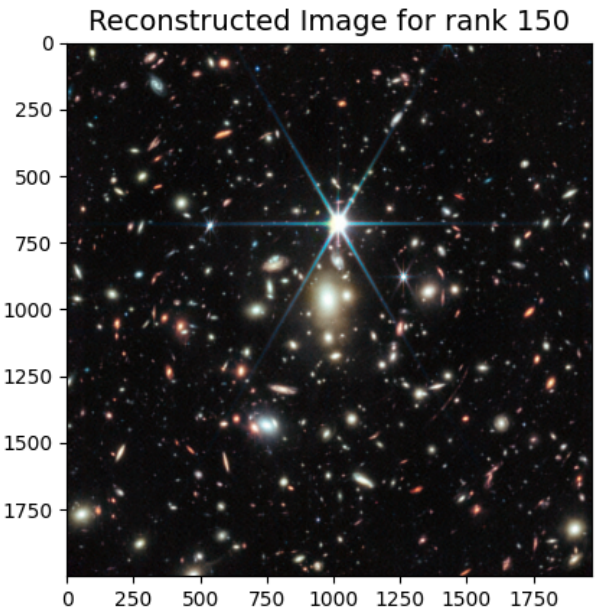


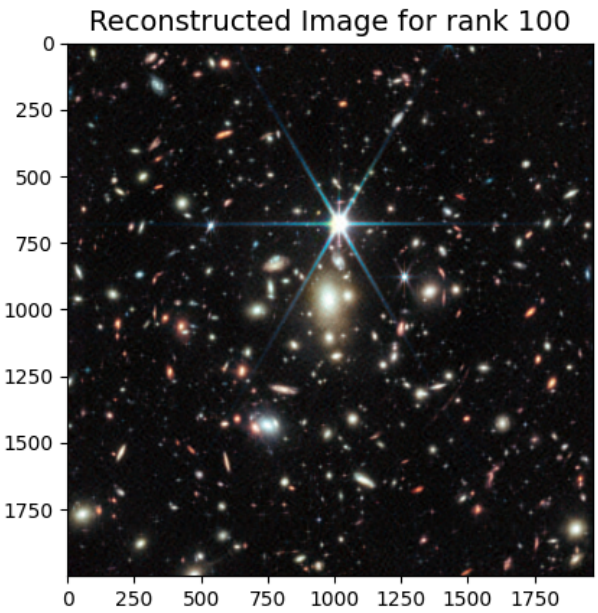
Plot and Images:

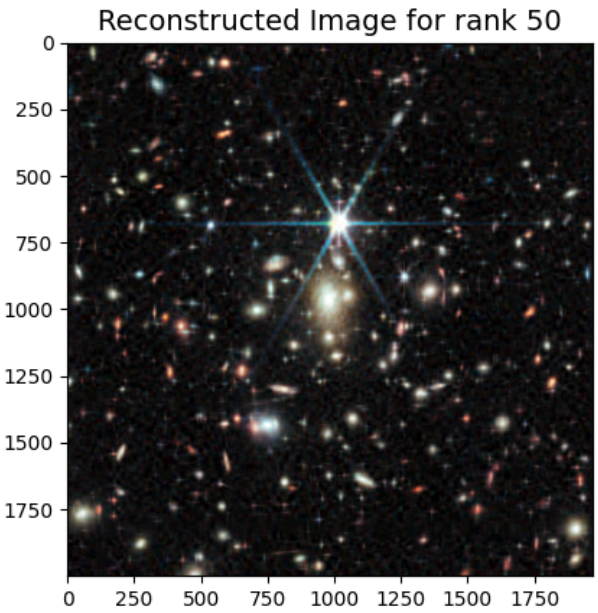












b) The Required Number of Entries:

- The required number of entries depend on the rank of the *reconstructed* image matrix and the dimensions of the given image matrix.
- The rank is same as the chosen number of singular values i. e. 190.
- The total number of entries is the count of all non-zero entries in the reduced SVD ($A = \hat{U}\ \Sigma\ \hat{V}^T$) form of the reconstructed image matrix.
- The $\hat{U}$ and $\hat{V}^T$ matrices with the selected number of singular values for all three channels give: $3 \times 190 \times (2000 + 1968 + 1)$.
- This corresponds to 2262330 non-zero entries to be transmitted.

c) Verification of Theorems:

- The required 2-norm and Frobenius errors are calculated and compared with the expected theoretical values.
- From the table below, it can be seen that the theorems hold true.

For Red:

r	$\|A - A_v\|_2$	σ_{v+1}	$\|A - A_v\|_F$	$\left(\sum_{i=v+1}^{r} {\sigma_i}^2\right)^{\frac{1}{2}}$
10	42.7928	42.7928	196.1066	196.1108
20	30.3176	30.3176	157.8531	157.8561
30	23.4275	23.4275	132.9128	132.9150
40	18.6631	18.6631	114.9895	114.9914
50	15.3626	15.3626	101.5154	101.5171
60	12.7175	12.7175	91.2128	91.2140
70	10.8543	10.8543	83.0011	83.0022
80	9.4767	9.4767	76.4366	76.4378
90	8.3244	8.3244	71.0136	71.0148
100	7.4204	7.4204	66.4862	66.4871
150	4.6554	4.6554	51.8052	51.8059
200	3.3314	3.3314	43.6887	43.6893
250	2.5745	2.5745	38.4209	38.4215
300	2.1231	2.1231	34.6668	34.6673
350	1.8313	1.8313	31.7499	31.7502
400	1.6294	1.6294	29.3024	29.3028
450	1.4768	1.4768	27.1737	27.1740
500	1.3519	1.3519	25.2670	25.2673
1000	0.6951	0.6951	12.0013	12.0014

For Green:

r	$\|A - A_v\|_2$	σ_{v+1}	$\|A - A_v\|_F$	$\left(\sum_{i=v+1}^{r} {\sigma_i}^2\right)^{\frac{1}{2}}$
10	38.8456	38.8456	180.3711	180.3746
20	28.4320	28.4320	144.2461	144.2493
30	21.6524	21.6524	120.6269	120.6291
40	17.2472	17.2472	103.4226	103.4246
50	14.6124	14.6124	90.2088	90.2101
60	12.0343	12.0343	79.7239	79.7252
70	10.2052	10.2052	71.4958	71.4970
80	8.7865	8.7865	64.8743	64.8753
90	7.7603	7.7603	59.3269	59.3278
100	6.7945	6.7945	54.6604	54.6612
150	4.1622	4.1622	39.3700	39.3706
200	2.8034	2.8034	30.9795	30.9799
250	2.0706	2.0706	25.8241	25.8244
300	1.6130	1.6130	22.3583	22.3586
350	1.3230	1.3230	19.8029	19.8032
400	1.1179	1.1179	17.8147	17.8149
450	0.9694	0.9694	16.2196	16.2197
500	0.8632	0.8632	14.8734	14.8736
1000	0.3943	0.3943	6.7611	6.7612

For Blue:

r	$\|A - A_v\|_2$	σ_{v+1}	$\|A - A_v\|_F$	$\left(\sum_{i=v+1}^{r} {\sigma_i}^2\right)^{\frac{1}{2}}$
10	36.6428	36.6428	176.7835	176.7866
20	26.8700	26.8700	144.2146	144.2175
30	20.8220	20.8220	123.1174	123.1195
40	17.1317	17.1317	107.6448	107.6466
50	14.2475	14.2475	95.4848	95.4863
60	11.9645	11.9645	85.9440	85.9452
70	10.1806	10.1806	78.5186	78.5197
80	8.8874	8.8874	72.4380	72.4391
90	7.8752	7.8752	67.3548	67.3557
100	7.0215	7.0215	63.1131	63.1139
150	4.3907	4.3907	49.2260	49.2266
200	3.1445	3.1445	41.5172	41.5177
250	2.4573	2.4573	36.5312	36.5316
300	2.0351	2.0351	32.9360	32.9364
350	1.7610	1.7610	30.0907	30.0911
400	1.5535	1.5535	27.7170	27.7173
450	1.4025	1.4025	25.6707	25.6710
500	1.2850	1.2850	23.8491	23.8493
1000	0.6566	0.6566	11.3515	11.3516