DS 289: Numerical Solution of Differential Equations

Assignment 3

Instructor: Konduri Aditya — Due date: 30 April 2024
Total points: 100

Please follow the below instructions in preparing the solutions:

1. Provide solutions in the same order as questions.
2. The report should be in a PDF format with the necessary steps, plots, explanations, and discussions.
3. Compile all the solutions, including graphs, into a single PDF file.
4. Create a separate folder for each question that involves a code. Provide the code, makefile, input, and output files, which are used to obtain the solution, in the folder.
5. For submission, create a folder named *DS289_A2_firstname_lastname* (your first and last names) that includes the code folders and the report. Compress the folder in a ZIP format with the same name, and upload it into the MS Teams assignment portal.
6. Code submissions will be checked for plagiarism. Copying from external sources, including the Internet, is prohibited.
7. You may use LLMs and AI tools to generate code with proper acknowledgment, but using them to write the report is not allowed.

Questions

1. Consider the viscous Burgers' equation $u_t + uu_x = \alpha u_{xx}$, where $u(x,t)$ is the velocity component along the $x-$direction, and α is the kinematic viscosity. Solve this 1D equation in a periodic domain of size 1.0 for the following cases. Use $\Delta t = 0.0004$, $t_{end} = 0.075$, and $u(x,0) = sin(4\pi x) + sin(6\pi x) + sin(10\pi x)$.

   (a) Use Euler and first order upwind schemes to solve the equation with $\alpha = 0$ for a grid resolution of 64 and 1024. Compare the two results by plotting the solution for a few timesteps, and comment on the errors.

   (b) Use Euler and second order central difference schemes to solve the equation with $\alpha = 0.001$ for a grid resolution of 1024. Compare with part (a) solution and comment on the nature of solution.

2. Consider the linear wave equation $u_t + cu_x = 0$. Using leap frog method for time derivative and second order central difference for space derivative, obtain the stability condition. Derive the modified equation and comment of the dispersive and dissipative errors.

3. Construct the weak forms for the following equations.

   (a) Beam on elastic foundation:

$$\frac{d^2}{dx^2}\left(b\frac{d^2w}{dx^2}\right) + kw = f \quad \text{for} \quad 0 < x < L$$

$$w = b\frac{d^2w}{dx^2} = 0 \quad \text{at} \quad x = 0, L$$

   where $b = EI$ and f are functions of x, and k is a constant.

   (b) A nonlinear equation:

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \quad \text{for} \quad 0 < x < 1$$

$$\left.\frac{du}{dx}\right|_{x=0} = 0, \quad u(1) = \sqrt{2}$$

4. A version of the Poisson equation that occurs in mechanics is the following model for the vertical deflection of a bar with a distributed load $P(x)$:

$$A_c E \frac{d^2u}{dx^2} = P(x)$$

where A_c = cross-sectional area, E = Young's modulus, u = deflection, and x = distance measured along the bar's length. If the bar is rigidly fixed ($u = 0$) at both ends, use the finite-element method to model its deflections for $A_c = 0.1m^2$, $E = 200 \times 10^9 N/m^2$, $L = 10m$, and $P(x) = 100N/m$. Employ a value of $\Delta x = 0.5m$.

Note: You can use Python programming for question 4. Do not hesitate of contact me if you any questions or doubts.