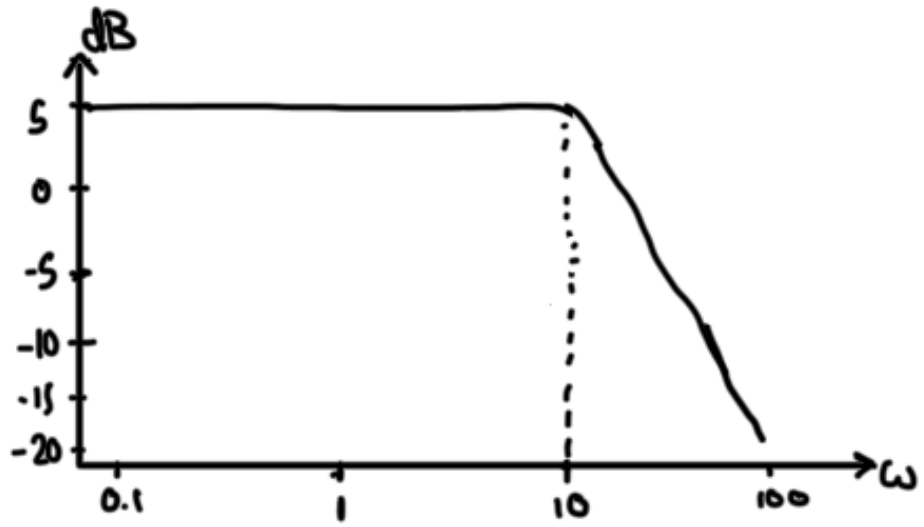
Assignment 1

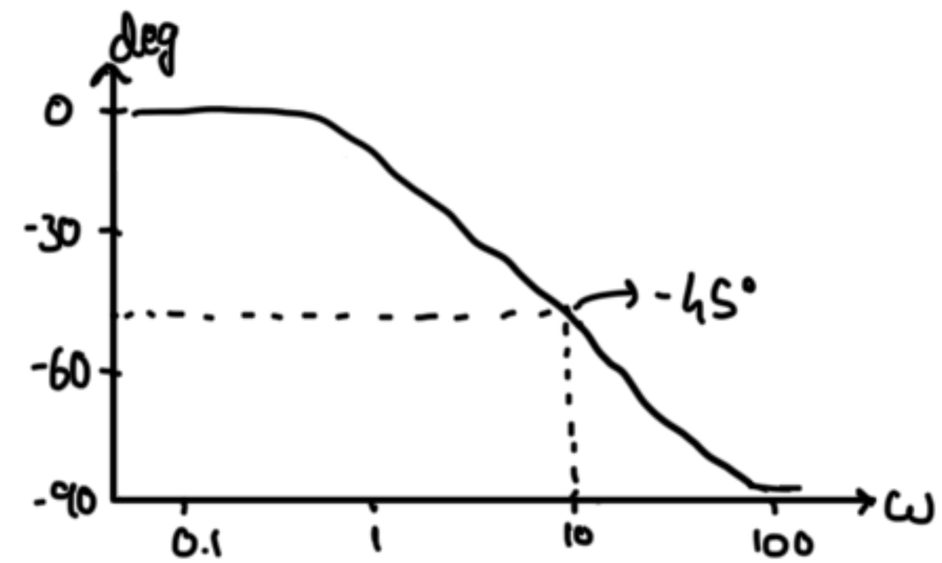
1.1 $G_1(s) = \dfrac{10}{s+10}$

1. pole $\Rightarrow |G_1(s)| \longrightarrow \infty \Longrightarrow s = -10$ is the only pole

DC gain $= \lim\limits_{\omega \to 0} G_1(j\omega) = G_1(0) = 1$

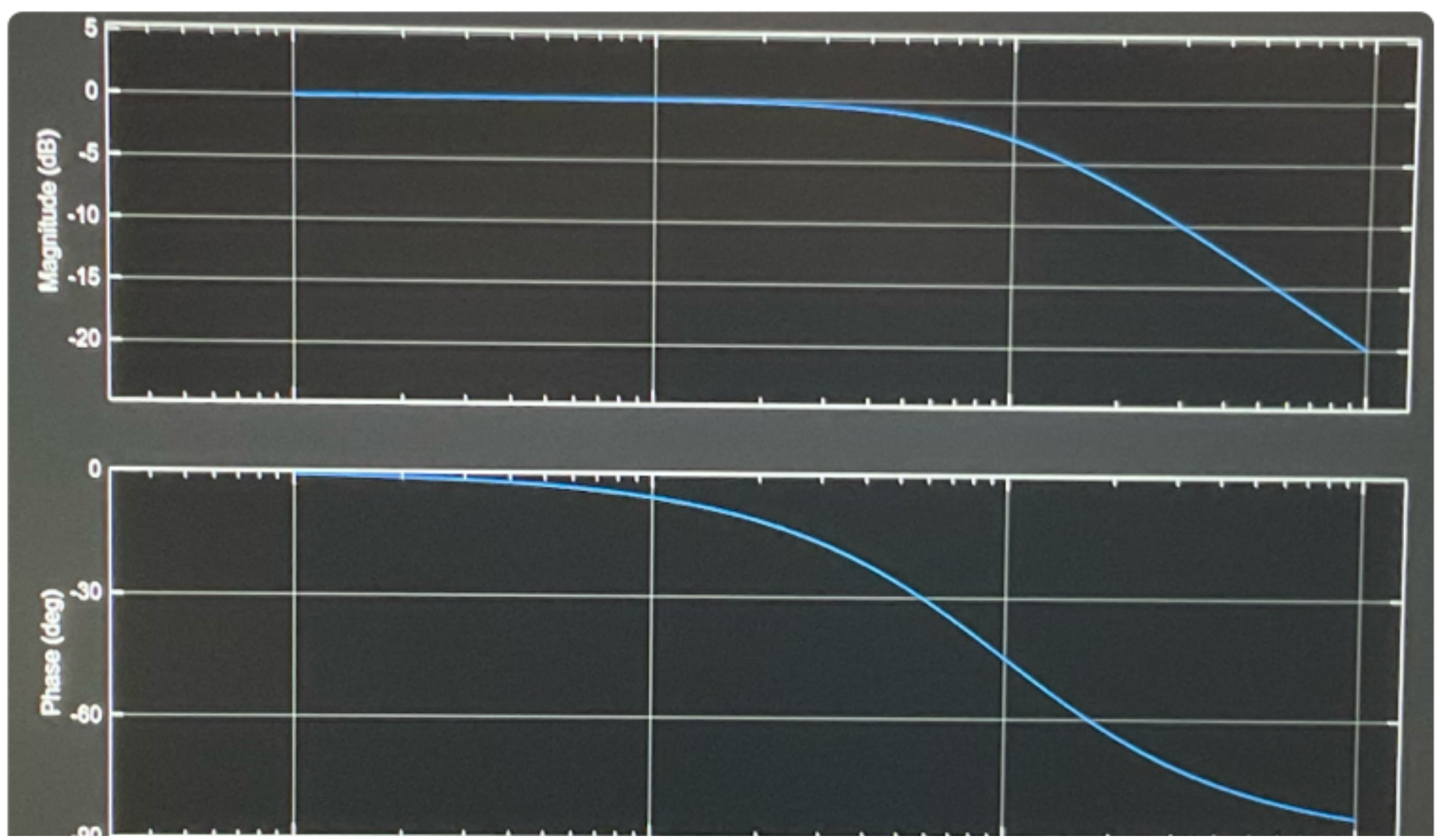
2.

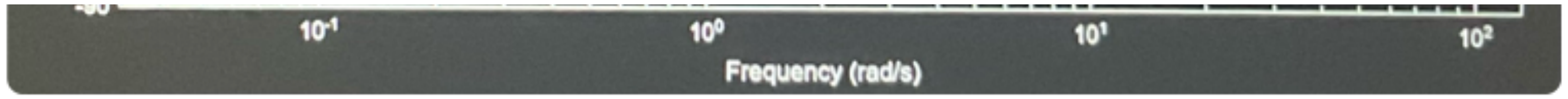




Cut of $\omega \Rightarrow |G_1(j\omega)| = \frac{1}{\sqrt{2}} = \frac{10}{\sqrt{\omega^2+100}} \Longrightarrow \omega^2 + 100 = 200 \Longrightarrow \omega_{cutoff} = 10$

3.

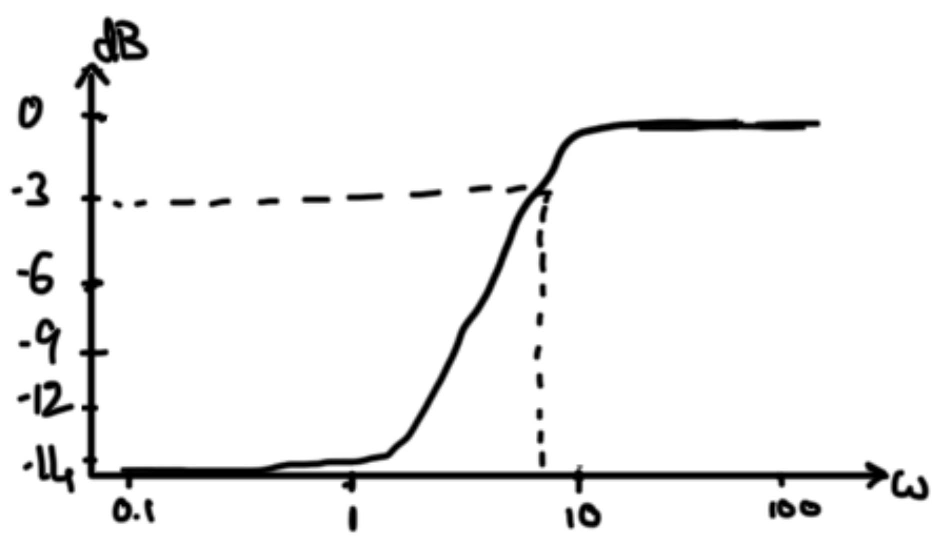


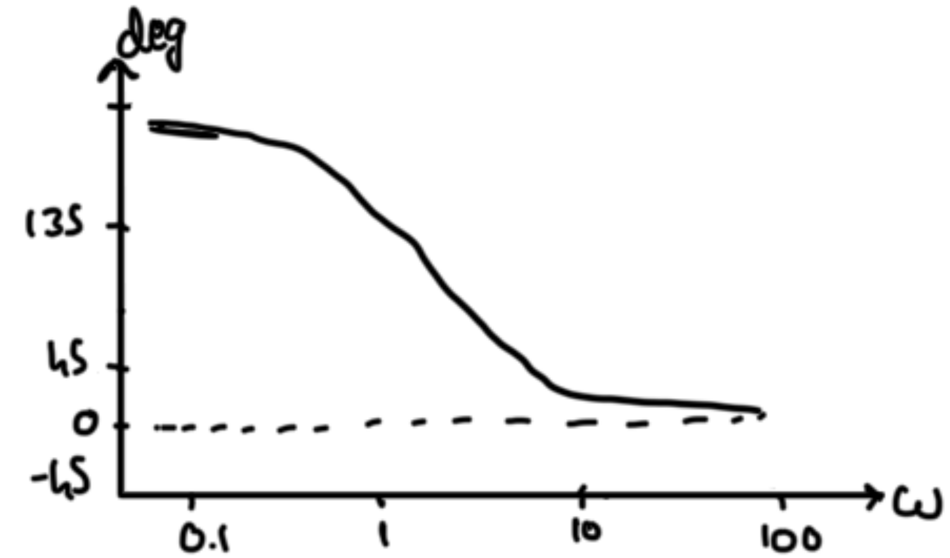


1.2 $G_2(s) = \dfrac{s-2}{s+10}$

1. zeros $= 2$, poles $= -10$, $G_2(0) = -1/5$

2. $|G_2(j\omega)| = \sqrt{\dfrac{4+\omega^2}{100+\omega^2}} = \dfrac{1}{\sqrt{2}} \implies 8 + 2\omega^2 = 100 + \omega^2 \implies \omega_{cutoff} = \sqrt{92}$ $\qquad \dfrac{12\omega}{\omega^2-20}$





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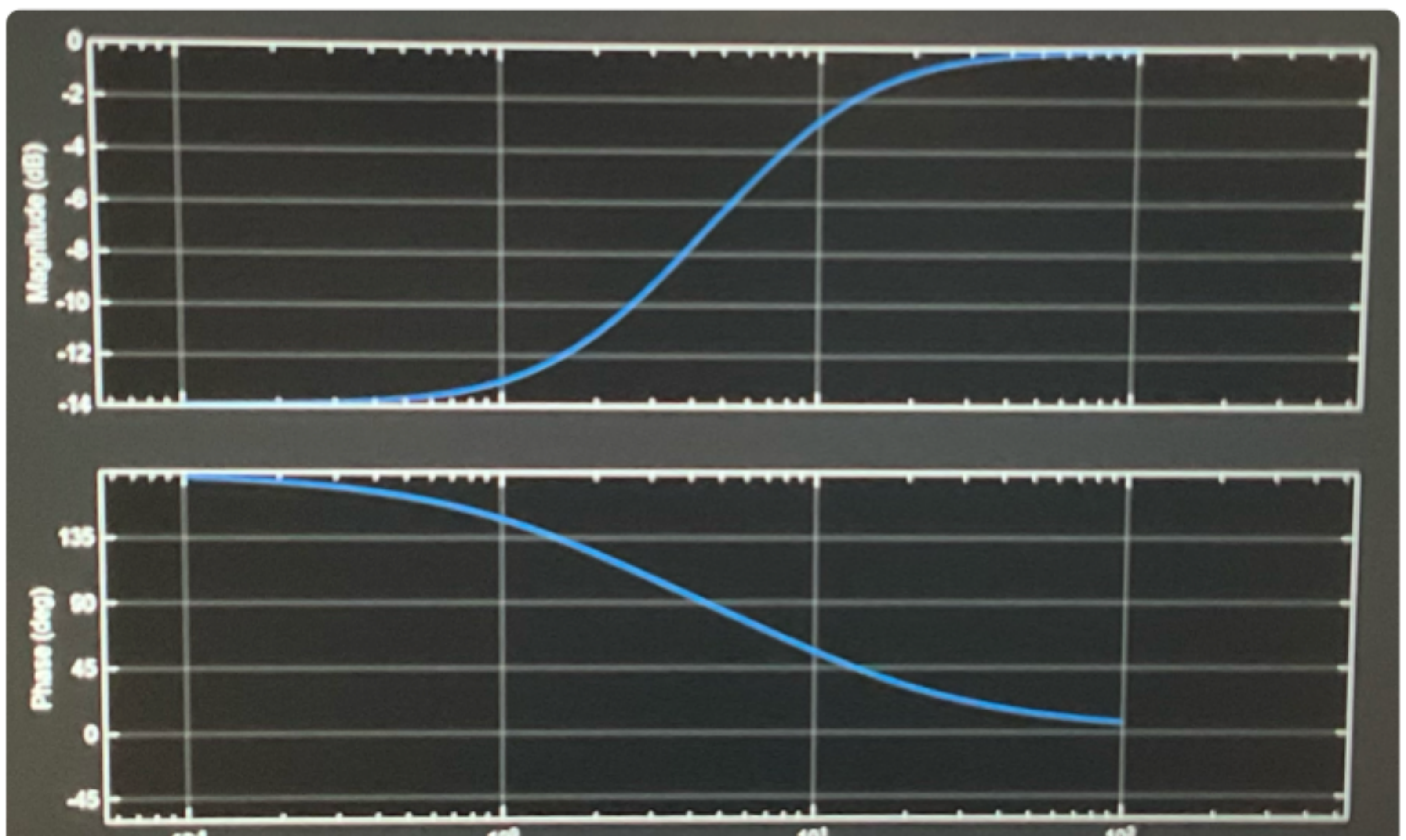
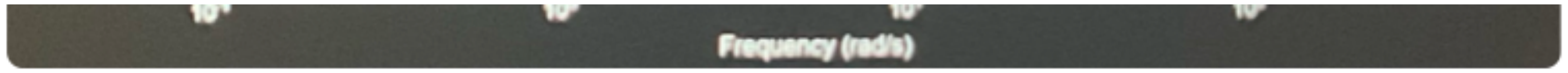


4. let numerator have a $(s+a)$ term
right plane root $\Rightarrow a<0$
left plane root $\Rightarrow a>0$

at $s = j\omega \Rightarrow a + j\omega$
$\angle(a+j\omega) = \tan^{-1}(\omega/a)$

Since $\omega > 0$:
if $a<0$, $(a+j\omega)$ will give a negative phase contribution;
else if $a>0$, it will

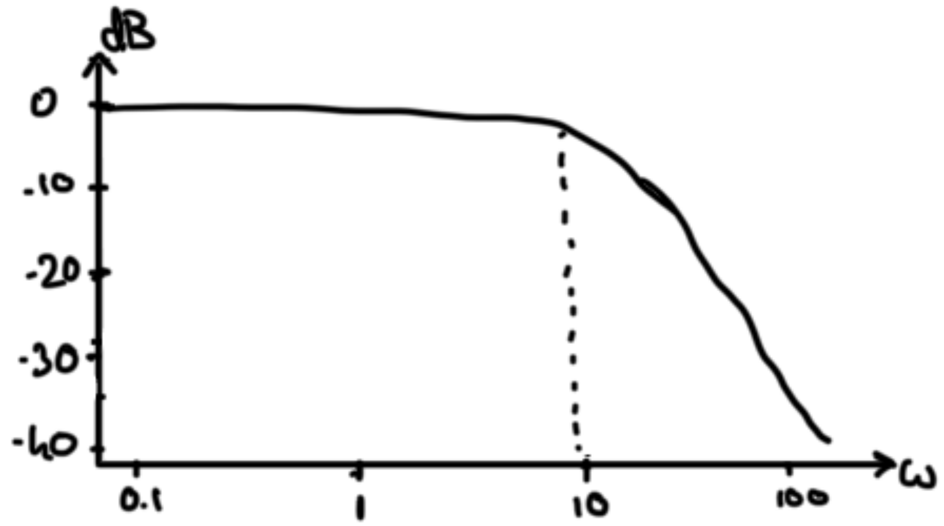


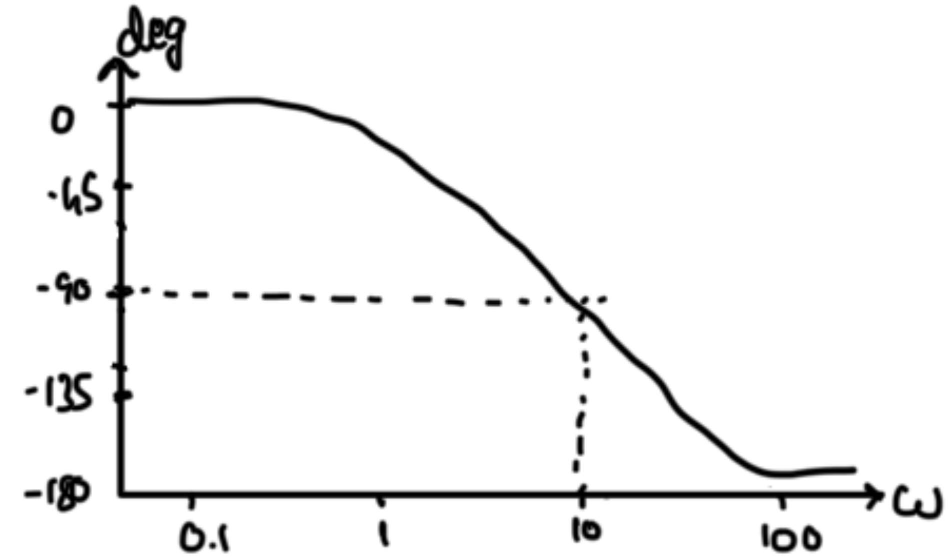
give a positive phase contribution;

1.3. $G_3(s) = \dfrac{100}{s^2+10s+100}$

1. $s^2+10s+100 = 0 \Longrightarrow s = -10\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}j\right)$ = poles

2.





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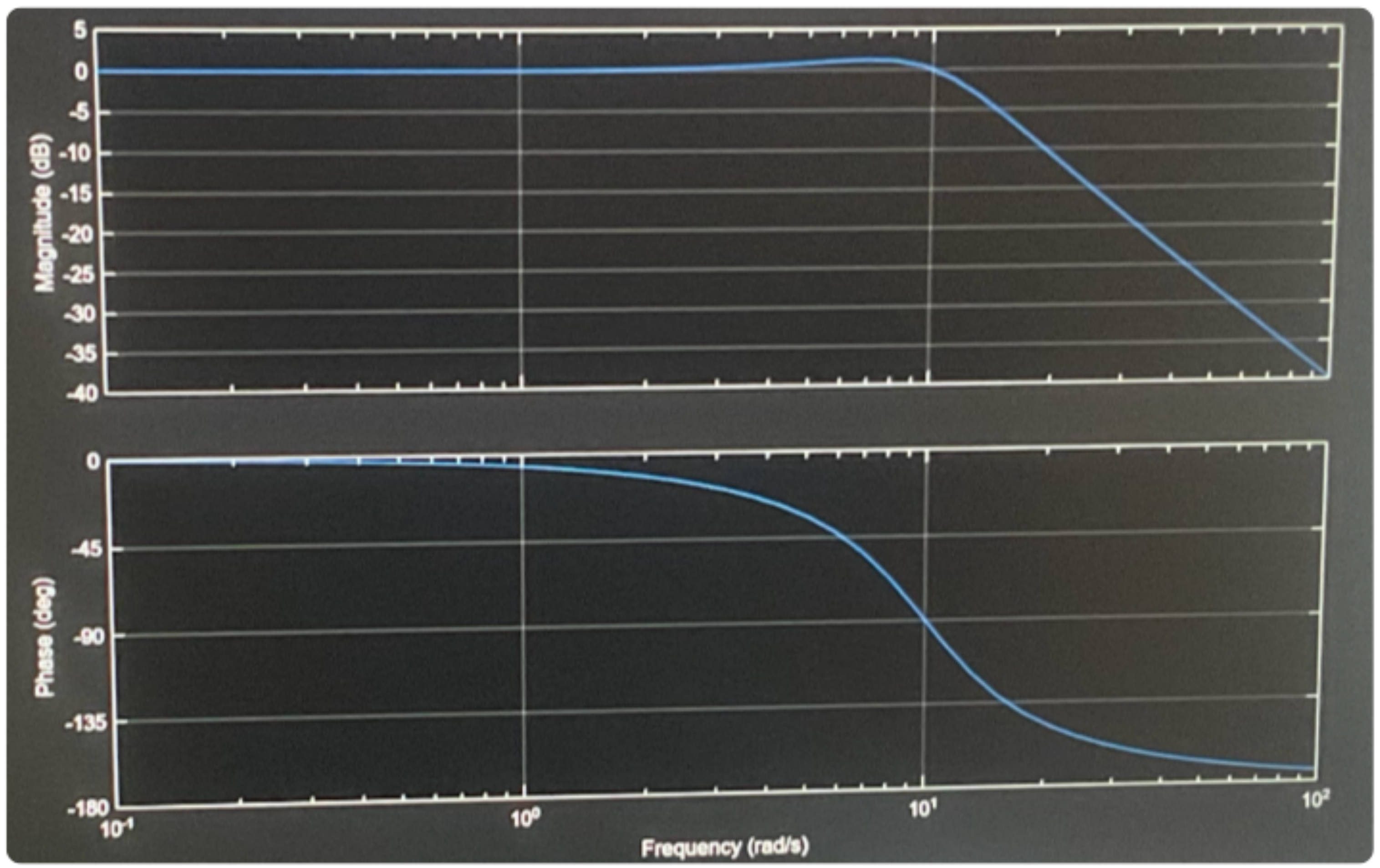


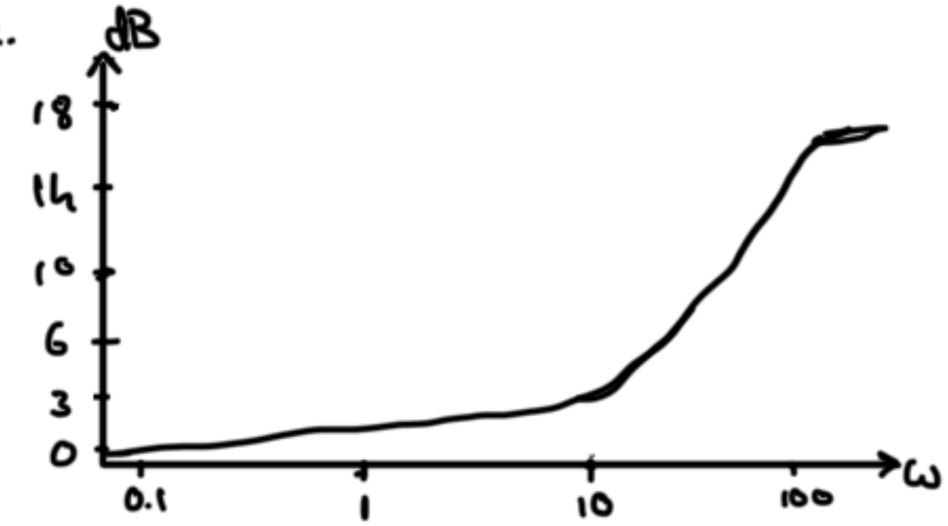
$G_3(10j) = \frac{1}{j}$

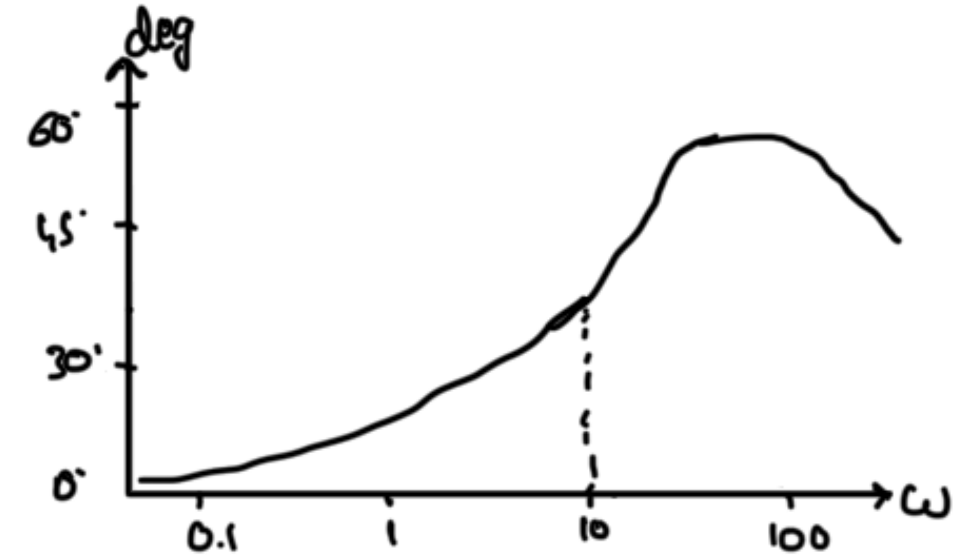
$\Rightarrow \angle G_3(10j) = -90^\circ$

1.4. $G_4(s) = \dfrac{0.1s+1}{0.01s+1} = \dfrac{10s+100}{s+100}$

1. zeros = -10 , poles = -100

2.





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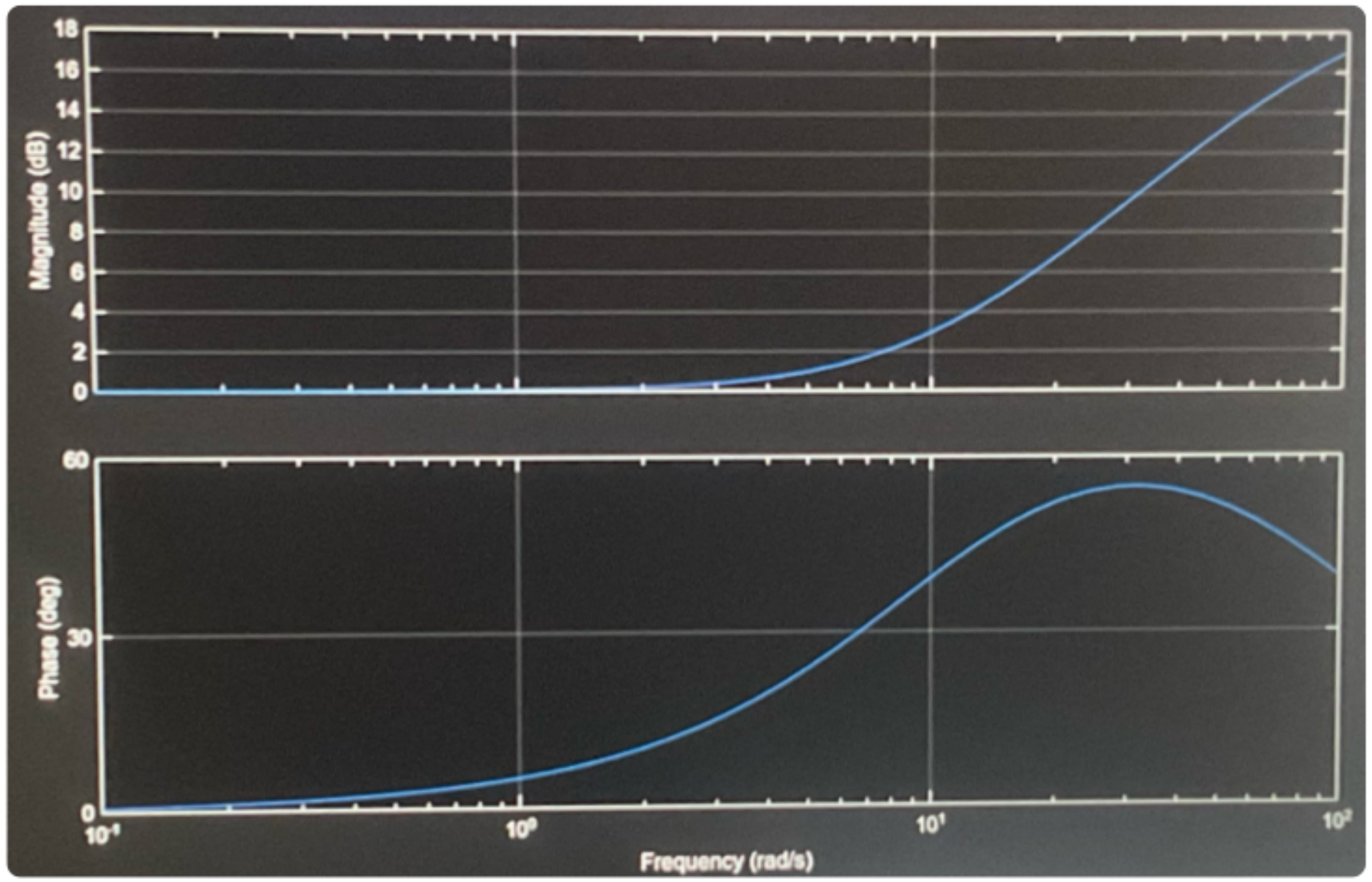


4. $G_4(j\omega) = 10\left(\frac{j\omega+10}{j\omega+100}\right)$

$\angle G_4 = \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{100}$

for $10 < \omega < 100$

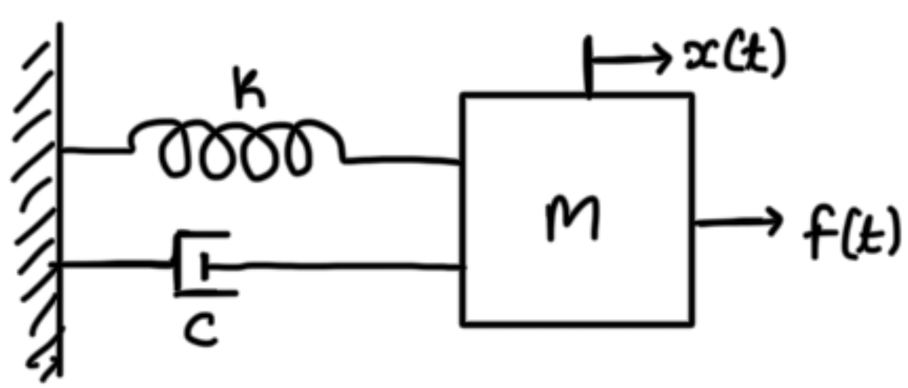
$0 > \tan^{-1}\frac{\omega}{10} > \frac{\pi}{4}$

$0 > \tan^{-1}\frac{\omega}{100} < \frac{\pi}{4}$

hence $\angle G_4 > 0$

∴ adds positive phase

2.



input force : f(t)

output displacement : x(t)

B.1

1. $M\dfrac{d^2x}{dt^2} = f(t) - b\dfrac{dx}{dt} - cx(t)$ where b is damping constant

2. Taking Laplace : $(Ms^2 + bs + c)\,X(s) = F(s)$

3. $G(s) = \dfrac{X(s)}{F(s)} = \dfrac{1}{Ms^2 + bs + C}$

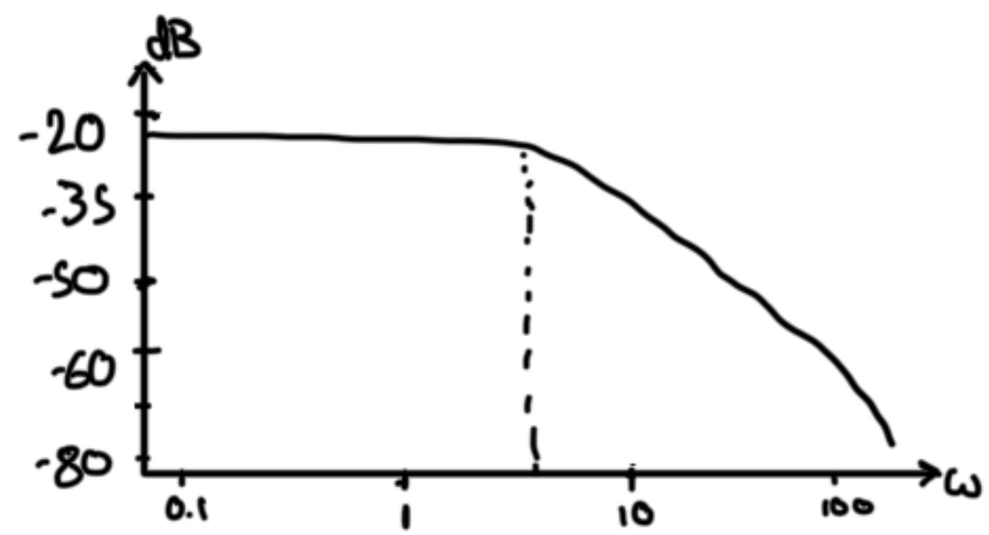
B.2

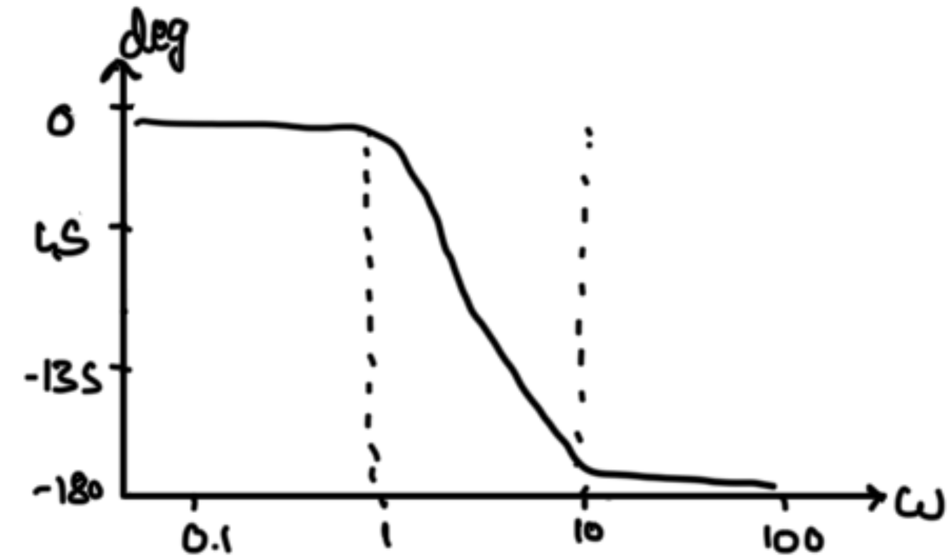
1. $M = 1,\quad b = 4,\quad c = 16$ / all SI units

$G(s) = \dfrac{1}{s^2 + 4s + 16}$

2. $s^2 + 4s + 16 = 0 \implies s = -2 \pm \sqrt{4 - 16} = -2(1 \pm j\sqrt{3})$ = poles

3.





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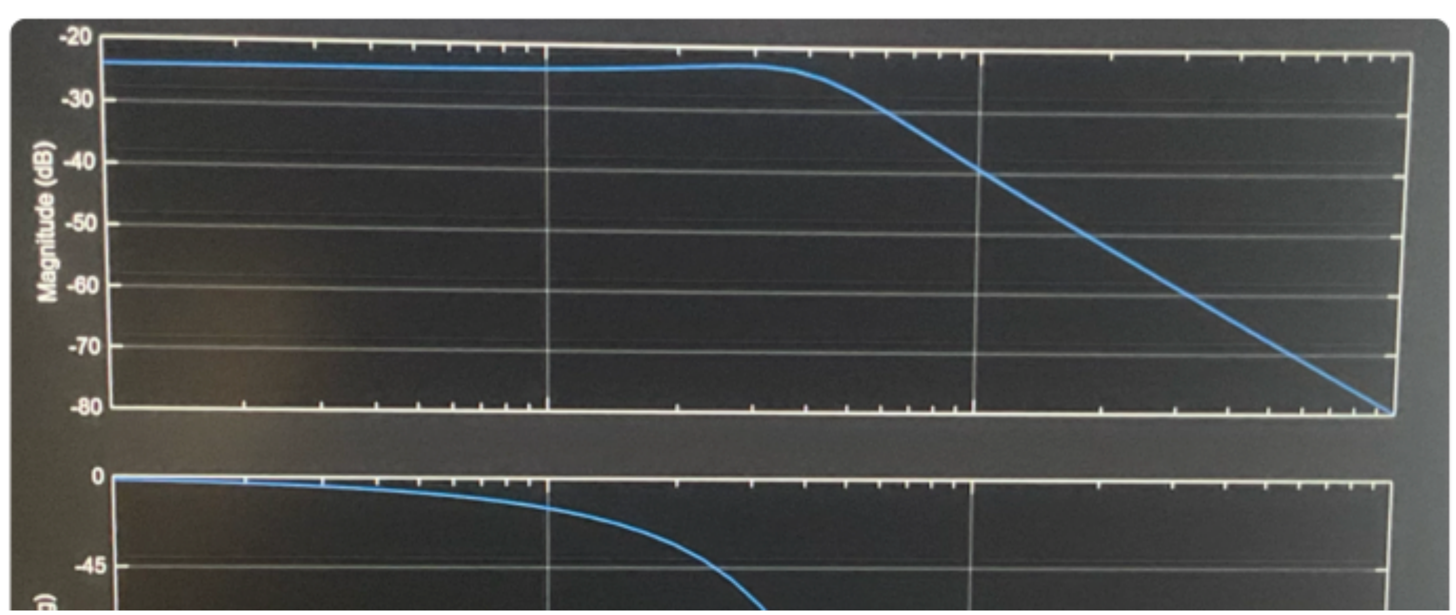

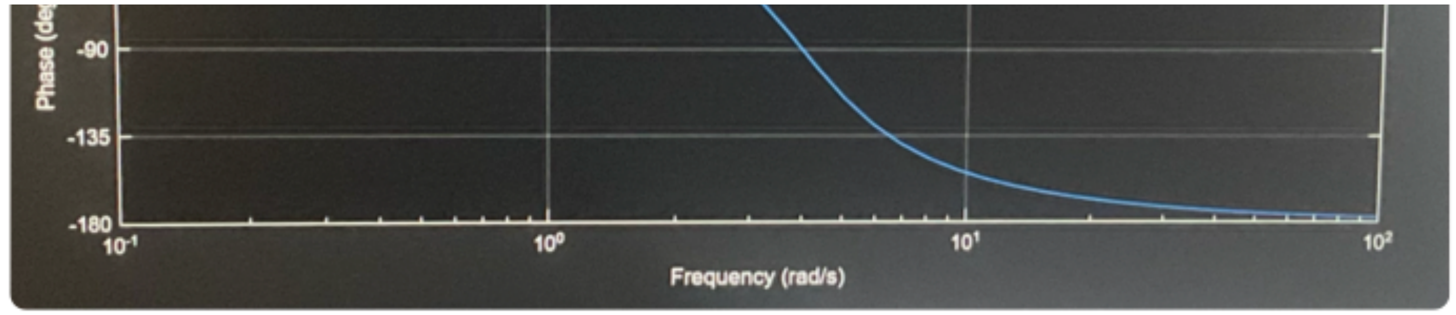

PROCEDURE TO PLOT BODE on MATLAB:

for ex: $G(s) = \dfrac{as+b}{ps^2+qs+r}$

```
num = [a b];
den = [p q r];

G = tf(num, den);
bode(g, {0.1,100});    % angular frequency 0.1 to 100
grid on;
```

$\boxed{\triangleright \text{ Run}}$