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Sunday, 7 December 2025 3:50 PM

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \; ; \; \boxed{\text{Rank}(A) = \underline{1}} \;\; \because \text{ Only 1 linearly independent row.}$$

$\text{Null}(A) = 4 - \text{Rank}(A)$ [Rank-Nullity theorem]

$\Rightarrow \boxed{\text{Null}(A) = 3} \Rightarrow$ An eigenvalue $\lambda = 0$ has multiplicity of 3.

$\text{Tr}(A) = 4$ & sum of eigen values $= \text{Tr}(A)$

$\therefore \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 4$ [Let $\lambda_1 = \lambda_2 = \lambda_3 = 0$]

$\Rightarrow \boxed{\lambda_4 = 4}$

$\therefore$ Rank$(A) = \underline{1}$

$\therefore$ Eigen values $= 4, 0, 0, 0$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow[\text{Transformations}]{\text{Row}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \therefore \underline{\text{Rank}(C) = 2}$$

$\because \text{Rank}(C) = 2 \Rightarrow \text{Null}(A) = 4 - 2$

$\Rightarrow \boxed{\text{Null}(A) = 2} \rightarrow$ Eigenvalue $\lambda = 0$ has multiplicity of 2

$\text{Tr}(C) = 4 =$ sum of eigen values.

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 4$ [Let $\lambda_3 = \lambda_4 = 0$]

$\Rightarrow \underline{\lambda_1 + \lambda_2 = 4}$

Now, $$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} \lambda_1 + \lambda_3 \\ \lambda_2 + \lambda_4 \\ \lambda_1 + \lambda_3 \\ \lambda_2 + \lambda_4 \end{bmatrix}$$ } Disjoint systems

$R_3 \leftrightarrow R_2$
$C_3 \leftrightarrow C_2$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Identical blocks
$\Rightarrow$ identical eigen values

$\lambda s = 2, 0$

$\lambda s = 2, 0$

$\lambda s = 2, 2, 0, 0 \Rightarrow \boxed{\lambda_1 = \lambda_2 = 2}$

Rank(C) = 2
Eigen values = 2, 2, 0, 0