Given: $A = uv^T$ [2×2 matrix, Rank$(A) = 1$] $u_{2\times 1}$, $v^T_{1\times 2}$

Now, $Au = (uv^T)u$
$= u.\underbrace{(v^T u)}_{(1\times 2).(2\times 1) = \text{Scalar no.} = K \text{ (Let)}}$

$\Rightarrow Au = Ku$ $\therefore$ u is an eigen vector — proved.

Now, Rank$(A) = 1 \Rightarrow$ Null$(A) = 2-1$
$= 1 \longrightarrow \lambda = 0$ has multiplicity of 1.
$\therefore \lambda_1 = 0$

Now, $\lambda_2 = K$
$= v^T.u$

$\therefore$ $\boxed{\begin{aligned}\lambda_1 &= 0 \\ \lambda_2 &= v^T.u\end{aligned}}$

$A = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} u_1 v_1 & u_1 v_2 \\ u_2 v_1 & u_2 v_2 \end{bmatrix}$

$\therefore$ Tr$(A) = u_1 v_1 + u_2 v_2$ ✓

Now, $\lambda_1 + \lambda_2 = v^T.u = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = v_1 u_1 + v_2 u_2$ ✓

$\therefore$ $\lambda_1 + \lambda_2 = $ Tr(A) $\therefore$ Proved.