CSC 541-FALL 2012 HOME WORK 2

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Worst-Case Method
The worst case bound can be taken as the product of n and the maximum cost incurred in any operation (i.e., 20). Using this approach, we get 20n as an upper bound on the contract cost. The upper bound is correct because the actual cost for n does not exceed 20n.

Aggregate Method
To use the aggregate method for amortized complexity, we determine an upper bound on the sum of the costs for the first n. Hence, Upper Bound On Sum of Actual Costs is

20*(n/12) + 10*(3(n/12) + 5*(4(n/12)) + 5*(4(n/12))
= 5 n (3/2)
= 7.5n

So, the amortized cost for each operation is 7.5

Accounting Method
In order to obtain the best upper bound on the sum of the actual costs, we must set the amortized cost to be the smallest number for which P(n) - P(0) >= 0 is satisfied for all n.

So we start by assigning an amortized cost (obtained by making an educated guess) to each of the different operation types and then proceed to show that this assignment of amortized costs satisfies Equation P(n) - P(0) >= 0 .

The amortized cost for each operation is set to 8. The table below shows the actual costs, the amortized costs, and the potential function value (assuming P(0) = 0) for the first 14 operations of the contract.

Operation	A	B	C	A	B	C	A	B	C	A	B	D	A	B
Actual Cost	5	5	10	5	5	10	5	5	10	5	5	20	5	5
Amortized Cost	8	8	8	8	8	8	8	8	8	8	8	8	8	8
P()	3	6	4	7	10	8	11	14	12	15	18	6	9	12

Hence by the accounting method the amortized cost is determined as 8n.

Potential Method

Since we have determined an amortized cost using the above methods, we ca use this to develop a potential function that satisfies P(n) - P(0) >= 0, and then use the potential function and the actual operation costs (or an upper bound on these actual costs) to verify the amortized costs.
P(0) = 0
P(n) = 0 for n mod 12 = 0
P(n) = 2.5 for n mod 12 = 1 or 3
P(n) = 5 for n mod 12 = 2, 4 or 6

P(n) = 7.5 for n mod 12 = 5, 7 or 9
P(n) = 10 for n mod 12 = 8, or 10
P(n) = 12.5 for n mod 12 = 11

Operation	A	B	C	A	B	C	A	B	C	A	B	D	A	B
Actual Cost	5	5	10	5	5	10	5	5	10	5	5	20	5	5
Amortized Cost	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
P()	2.5	5	2.5	5	7.5	5	7.5	10	7.5	10	12.5	0	2.5	5

So,
amortized(i) = actual(i) + P(i) - P(i-1)

Therefore,
amortized(1) = actual(1) + P(1) - P(0) = 5 + 2.5 - 0 = 7.5,
amortized(2) = actual(2) + P(2) - P(1) = 5 + 5 – 2.5 = 7.5,
amortized(3) = actual(3) + P(3) - P(2) = 10 + 2.5 - 5 = 7.5,
and so on.

Therefore, the amortized cost for each operation is 7.5. So, the actual cost for n operations is at most 7.5n.