→ Intro to GRE Math. Basics

* Try not to use calc.
* Math logic & number sense
* Usually, simple logic
* Mental math
* On screen calculator
* Estimate if answers are spread apart

→ Dividing by 5 = Multiply by 2 then divide by 10.

→ $(x5)^2 = \underline{x \times (x+1)}\ \underline{25}$ Ex. $25^2 = \underline{6}\ \underline{25}$

→ $n^2 = (n+1)^2 - n - (n+1)$
$= (n-1)^2 + n + (n-1)$

Math Question types

→ MCQs
→ Multiple Answers
→ Numeric Entry
→ Quantitative Comparision. (QC) (7-9 Qs)
45s to 2min

Example Q

Quantity A

$\frac{3}{7} + \frac{2}{5}$

Quantity B

$\frac{13}{27} + \frac{41}{97}$

Approximate

$\frac{13}{27} > \frac{13}{28} > \left(\frac{12}{28} = \frac{3}{7}\right)$

$\frac{41}{97} > \frac{41}{100} > \left(\frac{40}{100} = \frac{2}{5}\right)$

$\therefore$ B > A

32.8% of 5929 | 41.6% of 5041

✗ [30% of 6000, 1800] < [40% of 5000, 2000]

✓ [33.3^- % of 6000^- $= 2000^-$] < [40^+ % of 5000^- $= 2000^+$]

Usually, no long calcs in any Qs.

$\frac{35}{8}$ | $\frac{13}{3}$

$\frac{35}{8} - 4$ | $\frac{13}{3} - 4$

$\frac{3}{8}$ | $\frac{1}{3}$

$\frac{9}{24} > \frac{8}{24}$

$x > 3$

	Quantity A	Quantity B
	$x+10$	$3x+3$
-3	$x+7$	$3x$
$-x$	7	$2x$
$/2$	3.5	x

$\boxed{\text{Undetermined}}$ coz x can be decimal or integer.

$\frac{14}{11}$		$\frac{9}{7}$
7×14	$<$	9×11

Add / Subtract same number from A & B.
Multiply / Divide by same +ve number.

Geometry

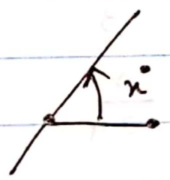
Lines & Angles

→ Line ——— or ⟷

→ Lines are straight but not horizontal or vertical

→ Line segment A———B

→ Angle occurs between line segments &/or lines.



→ Straight angle 180°

→ Right angle 90°

→ ⊥ → perpendicular lines.

→ Do not assume lines are perpendicular.

→ Two shapes are congruent if they have same shape + size.

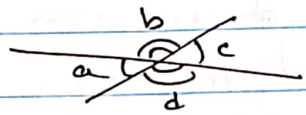
→ A bisector cuts something into congruent pieces.

→ A line can be a perpendicular bisector. Every point on a perpendicular bisector is equidist from two end points of the segment it is bisecting.

→ $x^\circ + y^\circ = 180^\circ$ supplementary

→ $x^\circ + y^\circ = 90^\circ$ complementary

→ Pairs of opp angles are congruent.

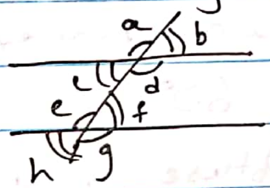


$a = c$ Vertical angles.

$b = d$. $b + c = 180$, $a + d = 180^\circ$

→ Parallel lines. Not parallel if not mentioned.

→ Line cutting across two parallel lines → Transversal.

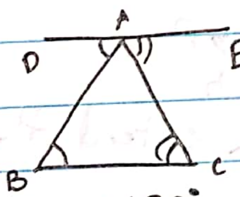
→ 

$a = d = e = g$ $a + b = 180^\circ$

$b = c = f = h$.

Triangles

→ Sum of angles $= 180^\circ$ (Sides & vertices)

→ 

DE ∥ BC

Proof that sum is 180°.

→ Acute $< 90^\circ$, Obtuse $> 90^\circ$, Right $= 90^\circ$

→ BC is opp to angle A.

→ Largest angle is opp the longest side.

→ Smallest angle is opp the shortest side

→ $a + b > c$ sum of sides inequality. greater than

→ $P - Q <$ third side $< P + Q$ P & Q are sides of a Δ.

Assumptions & Estimations

→ Straight line looks straight, it is straight

→ ⊥, ||, size, lengths → do NOT assume.

→ Looks like a square but is not.

→ Do NOT assume things.

→ Trust only of info given

Geometry Strategies

→ Try to find bigger shapes from smaller ones & smaller ones from bigger ones.

Triangles

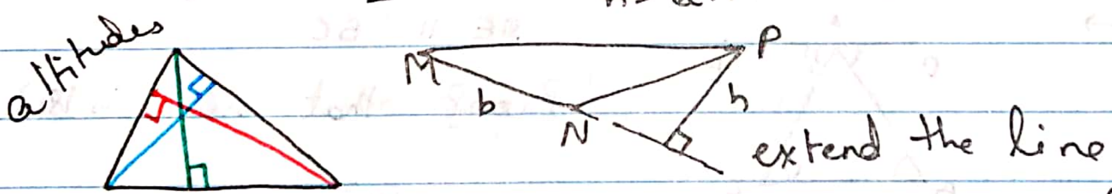
→ Isosceles – 2 or more sides equal   2 angles are equal

→ Equilateral – 3 sides equal.   60°

→ One angle can be right or obtuse in an isosceles Δ.

→ Equilateral Δ is also isosceles Δ.

→ Area of Δ = $\frac{1}{2}bh$ h = altitude

→ 

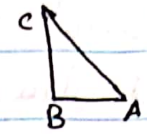
→ Median – vertex to mid point of opp side. (doesn't divide angle).

→ Angle bisector – divides angle but not side.

→ All are same in an isosceles Δ.

Altitude = Median = perpendicular bisector = angle bisector.

→ 2 legs & hypotenuse in a rt Δ.



$AB^2 + BC^2 = AC^2$

3, 4, 5
~~6, 8, 10~~
5, 12, 13
8, 15, 17
7, 24, 25

→ Do NOT square big nos. Divide sides by GCF, find side, multiply by GCF.

→ Ratio of angles = Ratio of sides

Similar triangles

→ Same shape, diff size, same angles, diff sides.

→ Angles same. Sides are proportional. (scale factor k).

→ Sides scale factor k → Area scale factor k^2.

→

Right Triangles

→ Isosceles right Δ. $(45°-45°-90°)$ $(1-1-\sqrt{2})$

→ Δ formed by the angle-$\perp$-side-bisector in an equilateral Δ.

→ Area of equi Δ $= \frac{\sqrt{3}a^2}{4}$

Quadrilaterals

→ Square, Rectangle, trapezoid, parallelogram, rhombus.

→ Sum of 4 angles → 360°.

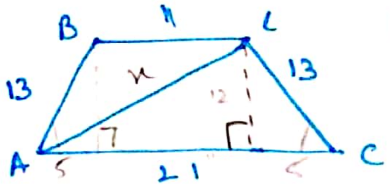
→ Diagonals of a llgm bisect eachother.

→ Big four → opp sides equal, opp angles equal, opp sides ll, diagonals bisect. If one is true, all are true.

→ Diagonals of rhombus are $\perp$. (not part of big 4)

→ Diagonals of rectangle are equal (not part of big 4)

→ Diagonals of square are equal & $\perp$.



$\angle A = \angle C$

$x^2 = 12^2 + 16^2$

$x = 20$

3, 4, 5

$120 + \frac{1}{2} \times 5 \times 12$

$+ \frac{1}{2} \times 12 \times 9$

→ Symmetrical trapezoids - Non || sides are equal.

→ To find diagonal, figure out pyth theorem.

Area of Quadrilaterals

→ Area usually base × height.

→ Trapezoid $\left(\frac{b_1 + b_2}{2}\right) h$ or divide into 2Δs & 1 rect.

→ Area of rhombus = b × h.

Polygons

→ Diagonal - line connecting two non-adjacent vertices.

→ Sum of angles = 180 × no. of Δs it can be divided into.

→ n sided polygon has (n-3) diagonals from one vertex and (n-2) triangles. ∴ Sum of angles is $(n-2) \times 180°$.

Regular polygons

→ Equal sides & angles. Equilateral and Equi angular.

Circles

→ Diameter is the longest chord.

→ Circumference $= 2\pi r = \pi d$. ($\pi = 3.14$) (3 or $22/7$)

→ Part of circumference = arc.

→ Area $= \pi r^2$

→ Chord - point connecting two points on the circumference.

→ Angle with vertex at the center is the central angle.

→ Diameter divides circle into two halves.

→ Size of central angle = Size of arc.

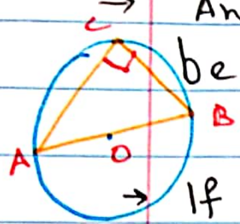
→ Equal central angles ⇒ equal arcs

→ Equal chords ⇒ equal arcs.

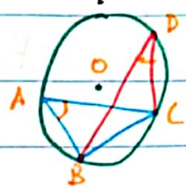
→ Angle with vertex on the angle is inscribed angle.

→ Measure of inscribed angle is half the measure of the arc it intercepts.

→ An inscribed angle that intercepts a semicircle has to be a right angle. ($\angle ACB = 90°$)



→ If two inscribed angles intercept the same arc/chord, the angles are equal. $\angle BAC = \angle BDC$



→ Radius is $\perp$ to any tangent

→ $\dfrac{\text{arclength}}{2\pi r} = \dfrac{\text{angle}}{360°} = \dfrac{\text{area of sector}}{\pi r^2}$

* Make sure to read which arc/sector is being asked.

Volume & Surface area

→ Shape	Vol	Surface Area
Cube	s^3	$6s^2$
Cuboid	hwd	$2hw + 2hd + 2wd$
Cylinder	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$

→ ~~Cut~~ Shapes have face & space diagonals.

$(\text{Space diagonal})^2 = h^2 + w^2 + d^2$.

→ Spheres. Vol = $\frac{4}{3}\pi r^3$

→ Sphere occupies $\frac{2}{3}$ of a cylinder's volume.

Scale factor & Scaling

→ Scaling factor for similar shapes. Imp for areas & volumes.

→ If scaling factor = k, area is usually scaled by k^2.
& volume by k^3.

Units of Measurement

→ One unit in Q but another in ans.

1 ft = 12 inc   1 m = 100 cm

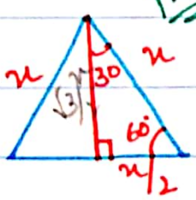
Test gives conversions.

* Ratio of angles & sides in a Δ.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

→ Special right Δs.

Imp	Angles	Sides	formulae
	45, 45, 90	$(1, 1, \sqrt{2})$	hyp = $\sqrt{2}$ leg
	30, 60, 90	$(1, \sqrt{3}, 2)$	long leg = $\sqrt{3}$ short leg hyp = 2 short leg

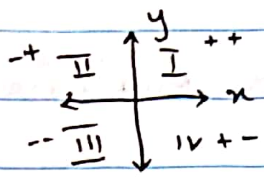


$x = 2$

$1, \sqrt{3}, 2$

A Δ has the max area when it is a right Δ.

Coordinate Geometry



4 quadrants

Two points are enough to determine a line.

Horizontal line $\rightarrow y = k$ x-axis is $y = 0$

Vertical line $\longrightarrow x = k'$ y-axis is $x = 0$

→ Slope – Angle of a line with the axes.

Geometry – Numeric, Geometric, Algebraic aspects

Slope = rise over run ~~Rise horizontal dist, Run vertical~~ dist

Rise – Vertical Separation, Run – Horizontal separation

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

→ If slope is a/b then point on the same line from (x_1, y_1) would be

- right kb then up ka or
- left kb then down "ka"

Ex. Slope = $5/3$ Point: $(2,-1)$ find points (a,b) on the line where $|a|, |b| <= 10$.

right 3, up 5 $\rightarrow (2,-1) \rightarrow (5,4) \rightarrow (8,9)$

left 3, down 5 $\rightarrow (2,-1) \rightarrow (-1,-6)$

→ Slopes of $\perp$ lines would be m & $-\dfrac{1}{m}$

→ Slope $= 1 \Rightarrow$ line makes $45°$ angle.

→ Intercept - point at which a line crosses an axis. To find intercepts, plug in $x=0$ & $y=0$. For $y=0$, you get x-intercept & vice versa.

→ Good way of getting equation for complicated graph is by using the intercepts.

→ Slope - intercept form   +ve slope ??

$$\boxed{y = mx + c}$$ $m \rightarrow$ slope, $c \rightarrow$ y-intercept

→ vertical lines have undefined slope.

→ Horizonal lines have zero slope.

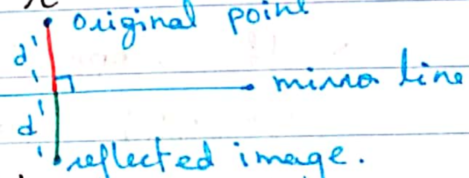
→ Dist $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

→ Or you can use slope triangle.

→ eq of circle with center (a, b) is

$$(a-x)^2 + (b-y)^2 = r^2.$$

→ Mirror line



Every point on the mirror line is equi dist from the original & reflected image.

→ Reflection over x-axis

x - coordinates are same, y - coordinates change signs.

Illy for reflection over y-axis

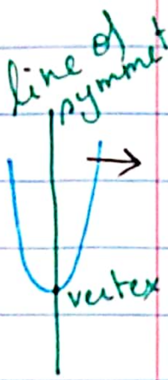
→ Reflection over $x=y$ line Swap the coordinates

→ Reflection over $y=-x$ line Swap coordinates & signs.

→ Mirror line is always the $\perp$ bisector of the line connecting original & reflected point.

→

Line	Reflected x	Reflected y	(for x, y)
x-axis	x	$-y$	
y-axis	$-x$	y	
$y = x$	y	x	
$y = -x$	$-y$	$-x$	



→ Quadratic graphs - parabola $y = ax^2 + bx + c$

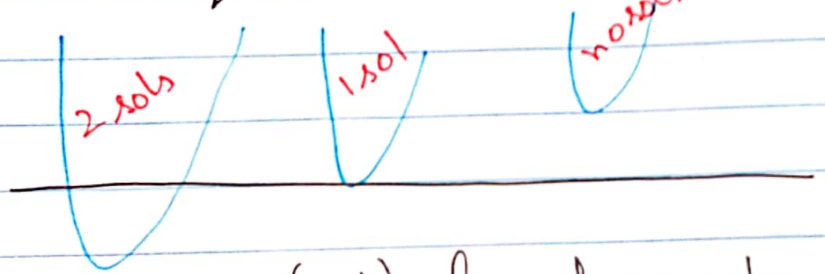
$a > 0$ → upward parbola

$a < 0$ → downward parabola.

$|a| > 1$ → skinny

$|a| < 1$ → wide

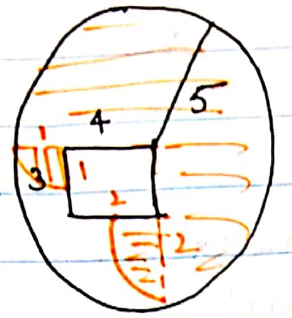
→ Quadratic eq can have, 2, 1 or 0 sols.



→ If vertex is (a, b) line of symmetry is $x = a$.

Imp Qn.

Cow tied to corner of shed



$$\frac{3}{4}\pi r^2 + \frac{1}{4}\pi (r')^2 + \frac{1}{4}\pi (r'')^2$$

$$\frac{3}{4}\pi \times 5^2 + \frac{1}{4}\pi 2^2 + \frac{1}{4}\pi 1^2$$

$$\frac{80}{4}\pi = 20\pi.$$

Percentages & Ratios

$\frac{8}{33} = \frac{24}{99} > \frac{24}{100} \cancel{x} = 24\%$ Approximation tricks

37% of x = $3(10\%$ of $x) + 7(1\%$ of $x)$

Increase & Decrease of %s.

$Y \uparrow$ by 30%. $\quad Y + \frac{30}{100}Y$.

$Y \downarrow$ by 30%. $\quad Y - \frac{30}{100}Y$

Multiplier

new price = multiplier × old price.

Multiplier of $P\% \uparrow$ = $1 + P\%$ as decimal

Multiplier of $P\% \downarrow$ = $1 - P\%$ as decimal.

$(\text{Multiplier} - 1) \times 100 = \% \uparrow$ or $\downarrow$.

Sequential Percent Changes

→ Same % ↓ then ↑ or ↑ then ↓ doesn't put you in the starting position.

→ $\uparrow x\%$ & $\downarrow y\%$. NEVER add or subtract %s.

Suppose $\uparrow 30\%$ & $\downarrow 40\%$ $\quad 1.3 \times 0.6 = 0.78 \Rightarrow 22\% \downarrow$

∴ Price was $22\% \downarrow$ than original price.

Scanned with CamScanner

Simple & Compound Interest

Simple interest applied only on ~~that~~ Principal & not on interest.

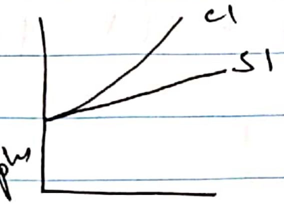
$x\%$ interest on P.

CI > SI if more than 1 yr.

Simple Interest

1000\$ 5%

1050 → 1100 → 1150



graphs

graph for SI

y - years P → principal,

r → multiplier A → Amount.

$A = Pr^y$. $r = \left(1 + \frac{I}{100}\right)$ I % interest.

Compounding period - How many times an year is the interest applied. quarterly - 4 times

monthly - 12 times , daily - 365 time.

Suppose bank pays 5% annual interest compounding quarterly, it means the bank pays $5\%/4 = 1.25\%$ each quarter.

n → no. of compounding periods in a year

$\therefore \quad r = \left(1 + \frac{I}{100n}\right)$ $A = P r^{ny}$

n ↑, A ↑

Ratios

$x : y : z$ is A to B to C rato to make D.
Then ratio of A to made D is $x : x+y+z$.

Ratio with diff num & deno units is a rate.

Arithmetic & Fractions

Number → ANY real no. NOT JUST +ve integers

Natural 1,2,3,4,5... Integer ... -2, -1, 0, 1, 2 ...

Whole 0, 1, 2, 3 ... Rational ... -1.5, -0.2, 0, 1.5 ...

Irrational $\pi, \sqrt{2}, \sqrt{3} \ldots$

All of these together are real nos.

BODMAS

Rounding 0-4 round ↓ 5-9 round ↑

7.384 → 7 7.499 → 7

Do not do ~~double rounding~~ DO NOT

6.48 → 6.5 → 7 NO!!

6.48 → 6. ✓✓✓

Golden ratio $\frac{1+\sqrt{5}}{2}$

$n/0 = \infty$ ✗ NOT part of test

$\frac{b}{a}$ is reciprocal of $\frac{a}{b}$

If $\frac{a}{b} > 1$ then $\frac{a+n}{b+n}$ is closer to 1 than $\frac{a}{b}$

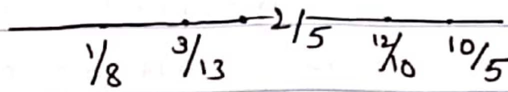
& $\frac{a}{b} > \frac{a+n}{b+n}$

If $a/b < 1$ then $\frac{a+n}{b+n}$ is closer to 1 than a/b

& $\frac{a+n}{b+n} > a/b$.

Similarly if you add n to the numerator & m to the denomenator, the result will move closer to $\frac{n}{m}$.

Ex $\frac{1}{8}$. $\frac{1+2}{8+5} = \frac{3}{13}$



$\frac{10}{5}$ $\frac{10+2}{5+5} = \frac{12}{10}$

If you subtract, effect will be opposite.

Improper faction $\frac{a}{b}$ if $a > b$

Mixed fraction $3\frac{3}{4} = \frac{15}{4}$

always < 1

Conversion between both is imp.

Cross multiplication.

"of" → multiply "is" - equals.

Statistics

Single number as a representitive of the set as a whole or "measures of center". Mean, Median, Mode.

→ Mean (Avg) = $\frac{\text{sum of N entries}}{N}$

→ Median - middle no. on an ordered list

If $N = 2n+1$, Median = $n+1^{th}$ number.

If $N = 2n$, Median = avg of $(n \,\&\, n+1)^{th}$ nos.

→ Mode - most frequently occuring number (may no exist or maybe more than 1)
(least imp)

→ Mean is heavily influenced by outliers while median isn't influenced at all.

Ex {2, 4, 6, 8}

→ In an evenly spaced list, mean = median.

Symmetrical lists also have mean = median.

Ex. $\underbrace{4, 8}_{4}, \underbrace{8, 13}_{5}, \underbrace{13, 23}_{10}, \underbrace{23, 25}_{2}, \underbrace{25, 27}_{2}, \underbrace{27, 37}_{10}, \underbrace{37, 42}_{5}, \underbrace{42, 46}_{4}$

More ↑ the outliers, ↑ the mean.

→ Range = max - min (measure of spread).

→ Standard deviation. (better measure of spread)

→ ~~SD~~ Deviation from mean

→ SD ≥ 0 , never -ve.

→ If all nos. on a list are the same dist from the mean, that dist is the SD.

→ SD is avg deviation from the mean.

→ Set with most values towards extremes have higher SD than a set with values equal or closer to the mean.

→ SD doesn't change if you add or subtract the same no. from each value in the set.

→ SD deals only with the spacing ~~with~~ between nos, not their position on the no. line.

→ If every value in the set is multiplied by K, SD is also multiplied by K.

→ SD ↑ as value of outliers ↑s

→ If new additions to set are far away from the mean, then SD of new set will be larger.

→ Even if mean doesn't change, SD will.

→ If new numbers added are mean ± SD, then SD won't change

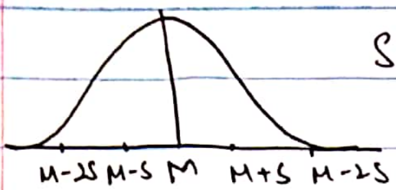
→ If new number are closer to mean than SD, then new SD will ↓.

→ If ~~two~~ new members added are equal to the mean, then that will give max ↓ in SD.

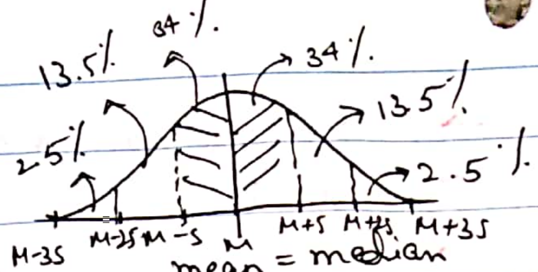
→ $SD = \sqrt{\dfrac{\Sigma (x_i - \mu)^2}{N}}$ SD^2 = varience

→ 

Normal distribution | Gaussian curve / bell curve



Symmetrical



mean = median
for normal distributio

→ Quartiles - The 3 numbers that would divide a list into 4 smaller lists

- $Q1$ → Bottom 25% (median of lower list)
- Q_2 Median → 50% (divides list into 50%)
- $Q3$ → Separates upper 25% from lower 75%.
  → median of upper list
- $Q4$ Max

Ex $\{3, 3, 4, 4, 4, 9, 11, 13, 14, 15, 15, 17, 17\}$

Median $= 11$

lower list $= \{3, 3, 4, 4, 9\}$ (exclude 11)

$Q1 = 4$

upper list $= \{13, 14, 15, 15, 17, 17\}$ (exclude 11)

$Q_3 = 15$.

$Q_4 = 17$.

Ex. 1203 people take GMAT. Q1 of scores = 510. only one person got 510. How many people got >510?

601 + 1 + 601

1 Median 1203

— 300 — 1 — 300 —

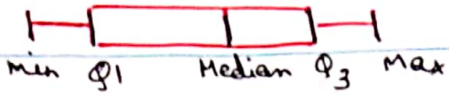
1 Q_1 601

1 median score.

People above Q_1 = 300 + 1 + 601 = 902

Boxplots

Imp

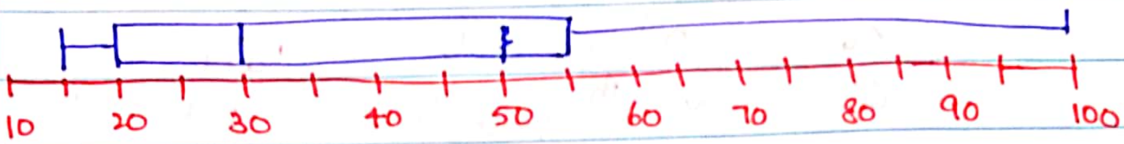


Distances vary between all 5 points coz no guarantee that median is avg of max & min.

Suppose above box plot ~~shows scores. It shows that very few people got in the range~~

Suppose above plot is scores. Bottom 25% got scores in a close range. Nex 25% got spread out scores.

Ex



Scores of 2800 students. What is the mean?

(A) M < 30 (B) M = 30 (C) M > 30 (D) Can't say.

$Q_1 = 20$ Min = 15 Median = 30 $Q_3 = 55$ Max = 100

Clearly, there are more scores in the range of 30 to 100 than 15 to 30. The mean is dragged further away from the median if the outliers are more. ∴ (C) M > 30.

→ The box in the box plot represents the middle 50% of the population. Size of box i.e $Q_3 - Q_1$ is called Interquartile range or IQR. It reflects the spread of the most typical values.

Quartiles & boxplots apply ONLY for large population.

Applying on a small range will give inaccurate results. %s won't be the same.

Percentiles

Percentile ≠ Quartile.

Ex - If a score is in the 40% percentile, it means the score is larger than 40% of the distribution.

→ If a score is ~~p~~ the p^{th} percentile of a distribution, it is larger than p% of the scores in the distribution

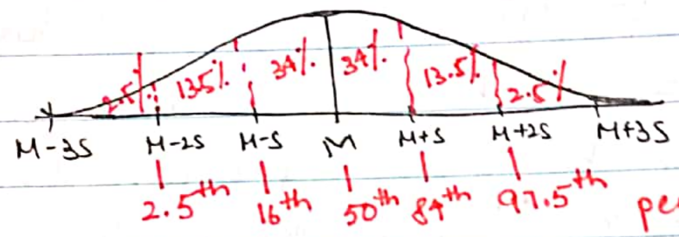
→ Percentiles apply only for large ~~distribution~~ population

→ Highest score is 99% percentiles. There is no 100^{th} percentile.

Min	$= 0^{th}$ percentile
Q_1	$\approx 25^{th}$ percentile
Median	$\approx 50^{th}$ percentile
Q_3	$\approx 75^{th}$ percentile
Max	$= 99^{th}$ percentile.

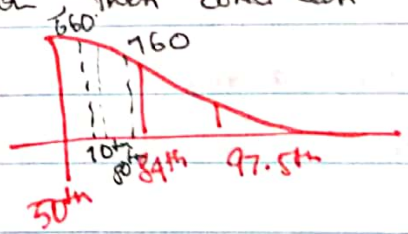
larger the population, more accurate these equations.

→ For normal distribution



Halfway between percentiles is NOT same as halfway between scores.

Ex If 660 is 70^{th} percentile & 760 is 80^{th} percentile of a normal distribution, then what can be said about the 75^{th} percentile?



Check

Data Interpretation

1 set per section. 1 graph - 3 to 4 Qs. can be any type of Qs except Quantitative Comparision.

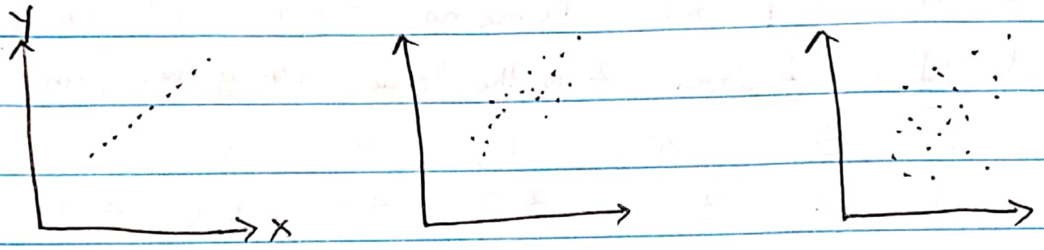
→ Read acompaying text

* All graphs are drawn to scale

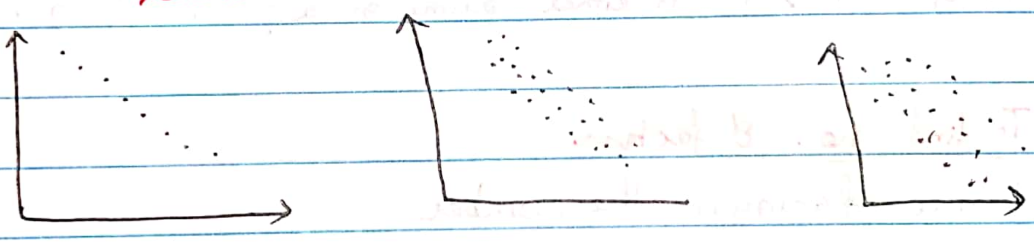
→ Check axes

→ Check numbers on graphs.

→ Check units on the graph & in the question



Strong +ve correlation → Weak +ve correlation.



Strong -ve correlation → weak -ve correlation.

Uncorrelated.

→ Do not be intimidated by diff types of graphs. Take a breath & look at them. They'll be easy to read.

Integer Properties

$A * B = C$ A & B are factors of C

Every integer has atleast two factors - 1 & itself

C is divisible by A if A is a factor, C is a multiple of A.

Divisibility by 2, 3, 4, 5, 6, 9

* In the test, nothing can be assumed to be an integer.

Prime nos.

1 isn't a prime. Prime no. - no with only 1 & itself as factors. 2 is the lowest & only even prime.

2, 3, 5, 7, 11, 13, 17, 19

23 29 31 37 41 43 47 53 59.

Every $N > 1$ is either prime or a product of primes.

To find no. of factors

1. Prime factorize the number.
2. Get all the powers of each of the prime nos.
3. Add 1 to each power.
4. Multiply all the powers to get the no. of factors.

Ex $8400 = 84 \times 100 = 4 \times 21 \times 5^2 \times 2^2 = 2^4 \times 3 \times 5^2 \times 7$ (1)

(2) $\{4, 1, 2, 1\} \rightarrow \{5, 2, 3, 2\}$ (3) $\rightarrow 60$ (4) factors.

→ To find odd prime factors, consider powers of odd primes $\{1, 2, 1\} \rightarrow \{2, 3, 2\} \rightarrow 12$ Odd factors

→ Even factors = All - odd.

→ Exponents of prime factors of a square are even.

ex $12^2 = 144 = 12 \times 12 = 4^2 \times 3^2 = 2^4 \times 3^2$

→ Squares are only integers with odd no. of factors.

Greatest Common Factor / Divisor

→ Greatest no. on common factor list.

→ To find GCF of 2 nos, find their prime factors. Then find highest power of each prime factor in common. Then multiply.

Ex GCF 360, 800 $360 = 2^3 \times 3^2 \times 5$

2 common → 3 $800 = 2^5 \times 5^2$

3 common → 0 5 common → 1 $\therefore GCF = 2^3 \times 5 = 40$

Least Common Multiple

To find LCM of M & N with GCF G.

Let $N = G \times A$, $M = G \times B$ $LCM = A \times B \times G = \frac{N \times M}{G}$

→ Zero is an even no. Evens & odds are +ve & -ve and are integers.

Imp. Q. $(P^2 + QR)$ is an even integer. P is an odd integer which is true (a) either Q or R is odd integer (b) both Q & R are odd integers (c) either Q or R is even (d) both Q & R are even (e) No solid conclusion.

E is an answer. $P^2 + QR$ → Even ⇒ P^2 & QR are odd.

Q R are not necessarily integers. Q & R can be 3 & 5 or 3 & 5/3. ∴ No conclusion.

→ Dividend = Divisor × Quotient + Reminder

$\boxed{\text{Imp Q}}$ If divisor > dividend, the integer quotient = 0 & the remainder = dividend.

→ Ex. Smallest +ve int divisible by 12 with a reminder of 5 is 5, not 17.

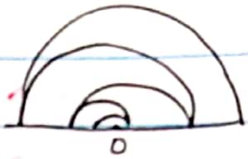
Powers & Roots

→ · $1^n = 1$ · $a^0 = 1$ for $a \neq 0$ · $(-ve)^{2n} = +ve$ · $(-ve)^{2n+1} = -ve$

→ $n, a > 1$, $a^n < a^{n+1} < a^{n+2} \dots$ (n & a +ve)

→ $n > 1$, $a < 1$ $a^n > a^{n+1} > a^{n+2} \dots$

→ $n > 1$ $a < -1$ $a^{2n} > a^{2n+1}$



Rules

- $(a^m)(a^n) = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $a^{-m} = \left(\dfrac{1}{a}\right)^m$ $(m>0)$
- $a^{-m} > a^{-m-1}$ $(m>0)$
- $a^n b^n = (ab)^n$
- $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$
- $a^s = a^t \Rightarrow s = t$.

→ $\sqrt{2} = 1.414$ $\sqrt{3} = 1.732$ $\sqrt{5} = 2.236$ $\sqrt{6} = 2.449$

$\sqrt{7} = 2.645$

* $x^2 = 4 \Rightarrow x = \pm 2$

But consider +ve roots mostly if not an equation

→ $\sqrt{2} > 1$ but $k^2 = 2$ doesn't imply $k > 1$

→ $\sqrt{-x}$ $x > 0$ imaginary

→ $A < B < C \Rightarrow \sqrt{A} < \sqrt{B} < \sqrt{C}$

→ $b > 1$ $\sqrt{b} < b$

→ $0 < b < 1$ $\sqrt{b} > b$

→ $(+ve)^3 = +ve$ $(-ve)^3 = -ve$ ∴ $\sqrt[3]{-ve}$ exists but $\sqrt{-ve}$ doesn't

→ $\sqrt[n]{0} = 0$ $\sqrt[n]{1} = 1$

→ $A < B < C \Rightarrow \sqrt[n]{A} < \sqrt[n]{B} < \sqrt[n]{C}$

→ $b > 1$, $n > m \Rightarrow 1 < \sqrt[n]{b} < \sqrt[m]{b} < b$

→ $0 < b < 1$, $n > m$ $\Rightarrow$ $0 < b < \sqrt[n]{b} < \sqrt[m]{b} < 1$

→ $\sqrt[2n]{-ve}$ imaginary $\sqrt[2n+1]{-ve}$ exists.

→ $\sqrt{x} = x^{1/2}$

→ $\sqrt{PQ} = \sqrt{P}\sqrt{Q}$ $\sqrt{\frac{P}{Q}} = \frac{\sqrt{P}}{\sqrt{Q}}$ $\sqrt{P} + \sqrt{Q} \neq \sqrt{P+Q}$

→ To remove roots in an eq, square both sides.

→ $b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$

→ $b^{1/n} = \sqrt[n]{b}$

→ To solve exponential eqs, equate the bases.

$(\sqrt{x})(\sqrt{x}) = x$ $\sqrt[n]{x}\sqrt[n]{x} = x^{2/n}$

→ If there is a ~~rd~~ radical in the denominator rationalize it

ex $\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Imp

→ $(a+b)^2 = a^2+b^2 + 2ab$

→ $(a-b)^2 = a^2+b^2 - 2ab$

→ $(a+b)(a-b) = a^2 - b^2$

Permutations & Combinations

→ If task N can be done in n ways & part 1 of task N can be done in n_1 ways, part 2 in n_2 ways, etc. then.

$$n = n_1 \cdot n_2 \cdot n_3 \ldots$$

This is the Fundamental Counting Principle.

→ $n!$ (n factorial) $= n \times (n-1) \times (n-2) \ldots \times 1$

→ In a collection of n items with more than one set of identical items, (1 group of b identical items, 2nd group of c identical items, 3rd group of e identical items, etc. then total no. of arrangements N would be

$$\boxed{N = \frac{n!}{b!\,c!\,d!}}$$

$b+c+d$ not necessarily $= n$.

→ Eliminating repetitions (ex. handshakes) When a method of counting counts an item more than once, divide the total no. of possibilities by the times each item was counted.

Ex. no. of handshakes in a room with N people

$$\frac{N(N-1)}{2}$$

Eliminate repetitions when order doesn't matter.

→ ^nC_r 'n choose r' choose a group of r from a group of n $^nC_r = {}^nC_{n-r}$

→ $$^nC_r = \frac{n!}{r!\,(n-r)!} = {}^nC_{n-r}$$

→ ^nC_r when order of group doesn't matter.

Algebra

→ $ax^2+bx+c \rightarrow ax^2+(p+q)x+c$

$p+q=b \qquad pq=ac$

Works usually if $a=1$

→ When solving for two eqs, if x & y both get eliminated, then x & y have infinite solution. Usually happens when an eq is a multiple of the other.

Ex $2x-y=5$ & $2y-4x=-10$

→ When you land up at an untrue statement while solving eqs, implies eqs have no soln. Usually when two same eqs are equal to diff things.

Ex $x-2y=5$ & $3x-6y=8$.

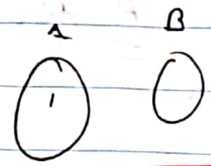
→ Absolute value eqs have two sols. * Recheck the sols by substituting.

Ex. $|x|=5 \Rightarrow x=\pm 5$

→ In Inequalites $x<5 \neq x\leq 4$. unless it is specified that x is an integer

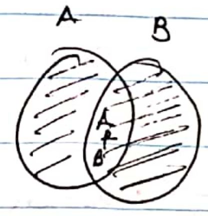
Probability

$0 \leq P \leq 1$



→ Mutually exclusive / Disjoint events – A & B $\boxed{\text{cannot}}$ happen together [P(A and B) = 0)

$P(A \text{ or } B) = P(A) + P(B)$



→ Non mutually exclusive events A & B

$P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$

→ Independent events – Outcome of A has no effect on outcome of B.

$P(A \& B) = P(A) * P(B)$.

→ If independent, then events are not mutually exclusive

→ Conditional Prob. $P(A|B)$.

$$P(A \& B) = P(B) * P(A|B)$$
$$= P(A) * P(B|A)$$

→ Binomial probability (very hard Qs).

p = prob of success on one trial

n = no. of trials

r = no. of successes

$$\boxed{P = ({}^{n}C_{r})(p^{r})(1-p)^{n-r}}$$

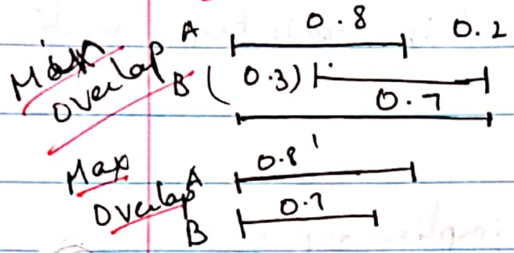
→ P(at least one success) = 1 − P(zero successes)

→ Strategies to answer Qs.

1) Use Instincts
2) Complement rule ("at least 1" rule)
3) Overlap.

Ex $P(A) = 0.8$ } $P(B) = 0.7$

$P(A \text{ or } B) = 1$ $\qquad$ $P(A \,\&\, B) = 0.5$



$P(A \text{ or } B) = 0.8$ $\qquad$ $P(A \,\&\, B) = 0.7.$

Ranges of $P(A \,\&\, B)$ & $P(A \text{ or } B)$.

TIPS

→ For avg speed of total trip, DO NOT take just avg of speeds.
Suppose trip is split into two parts.

$T_1 = \frac{D_1}{S_1}$ $\quad T_2 = \frac{D_2}{S_2}$ $\quad S_{avg} = \frac{D_1 + D_2}{T_1 + T_2}$

→ With problems involving multiple trips, each trip has its own $D = ST$ eq.

→ Objects going in opp direction implies gap is decreasing at speed $S_1 + S_2$

→ Objects going in same direction implies gap is -
- ~~dec~~ increasing at $|S_1 - S_2|$ if slower obj is behind
- decreasing at $|S_1 - S_2|$ if faster obj is behind.

→ Mixture Question.

Solute + Solvent → solution.

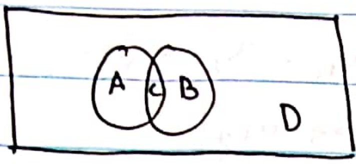
Solute ↓ dissolved chemical

Solvent ↓ chemical in which ~~water~~ solute is dissolved in.

→ Concentration → strength of solution or how much solute is dissolved ~~in~~ in a given quantity of water. (always expressed as %)

$$\text{Concentration} = \frac{\text{amount of solute}}{\text{total amount of solution}} \times 100$$

→ Venn Diagrams



$A + B + C + D = \text{total}$

→ Double Matrix method

	type I A	type I B	Totals
type II A	a	b	e
type II B	c	d	f
Totals	g	h	i

$a + b = e$ $\quad a + c = g \quad e + f = i$

$c + d = f$ $\quad b + d = h \quad g + h = i$

Ex. 300 employees. 120 females. 200 advanced degrees. Rest only college degree. 80 males with college degree. How many females with adv degree?

	M	F	Tot
Adv	100	$\boxed{100}$ Ans	200
Clg	80	20	100
Tot	180	120	300

→ Suppose you have to find out consecutive nos. Let the middle no. be 'a' & then ±1 the prev & next nos.

Ex. $a-2,\ a-1,\ a,\ a+1,\ a+2.$

→ n^{th} term of sequence $a \rightarrow a_n$

→ • In an arithmetic seq, $\boxed{a_n = a_1 + (n-1)d}$

$a_1 \rightarrow 1^{st}$ term, $d \rightarrow$ common diff.

• Each no. in an arithmetic seq give remainder a_1 when divided by d.

→ Sum of first n +ve integers $= \frac{N(N+1)}{2}$

→ Atleast One of 'n' consecutive nos. will be divisible by 'n'

→ Sum of a set of n consecutive integers will be divisible by n if n is odd.

QC STRATEGIES

→ If you pick numbers to substitute, make sure to consider all cases as answer can be (D) as well.

Remember, 'number' DOES NOT imply integer.

→ DO NOT trust pictures. Only use facts given in text

→ Try to write nos as $X-1$ or $X+1$ to find factors.

Ex. 899 factors $899 = 900 - 1$

$30^2 - 1^2 = (30+1)(30-1)$

$\therefore$ Factors 1, 29, 31, 899.

→ QCs are generally quick comparision. No lengthy calcs.
Approximation usually works.

ALL THE BEST !!!