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CSA-0669

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ANALYTICAL QUESTIONS

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1) Solve the following Recurrence Relations:

a) $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$

$$x(n) = x(n-1) + 5 \quad -(1)$$

$$x(n-1) = x(n-1-1) + 5$$

$$x(n-1) = x(n-2) + 5 \quad -(2)$$

$$x(n-2) = x(n-2-1) + 5$$

$$x(n-2) = x(n-3) + 5 \quad -(3)$$

Sub (3) and (2)

$$x(n-1) = x(n-3) + 5 + 5$$

$$x(n-1) = x(n-3) + 10 \quad -(4)$$

Sub (4) in (1)

$$x(n) = x(n-3) + 5 + 10$$

$$= x(n-3) + \underbrace{15}_{3 \times 5}$$

$$= x(n) = x(n-K) + 5K \quad -(5)$$

$$= n - K = 1$$

$$n - 1 = K$$

Eq (5)

$$= x(n) = x(1) + 5(n-1)$$

$$= x(1) + 5n - 5$$

$$= \boxed{O(n)}$$

b) $x(n) = 3x(n-1)$ for $n>1$, $x(1) = 4$

$$x(n) = 3x(n-1) \quad -(1)$$

$$x(n-1) = 3x(n-1-1)$$

$$x(n-1) = 3x(n-2) \quad -(2)$$

$$x(n-2) = 3x(n-2-1)$$

$$x(n-2) = 3x(n-3) \quad -(3)$$

sub (3) in (2)

$$x(n-1) = 3(3x(n-3))$$

$$x(n-1) = 9x(n-3) \quad -(4)$$

sub (4) in (1)

$$x(n) = 3(9x(n-3))$$

$$x(n) = 27x(n-3)$$

$$\to 3^3$$

$$= 3^k x(n-k) \quad -(5)$$

$$\boxed{k = n-1}$$

$$n-k = 1$$

$$= x(n) = 3^{n-1} x(1) = 3^{n-1} \cdot 4$$

$$= 3^{n-1} \cdot 4 = \boxed{O(3^n)}$$

$x(n) = x(n/2) + n$ for $n > 1$ $(n = 2^k)$

$x(1) = 1$

$x(n) = x(n/2) + n$ — ①

$x(n/2) = x(n/4) + n$ — ②

$x(n/4) = x(n/8) + n$ — ③

Sub ③ in ②

$x(n/2) = x(n/8) + n + n$

$x(n/2) = x(n/8) + 2n$ — ④

Sub ④ in ①

$x(n) = x(n/8) + 2n + n$

$\boxed{x(n) = x(n/8) + 3n}$ — ⑤

$x(n) = (n/2) + n + (n/4) + n \cdots\cdots (n/2^k) + nk$

$x(n) = \frac{n}{2^k} + kn$

$\frac{n}{2^k} = 1$ $\boxed{n = 2^k}$

$\log n = \log 2^k$

$\log n = k \log 2$

$\boxed{\log n = k}$

$\therefore \boxed{O(\log n)}$ //

4) $n(n) = n(n/3) + 1$

$n(n) = n(n/3) + 1$ — ①

$n(n/3) = n(n/9) + 1$ — ②

$n(n/9) = n(n/27) + 1$ ③

Sub ③ in ②

$n(n/3) = n(n/27) + 2$ — ④

Sub ④ in ①

$$\boxed{n(n) = n(n/27) + 3}$$ — ⑤

$$n(n) = n + \frac{n+1}{3} + \frac{n+2}{3} + \frac{n+3}{27} \ldots \frac{n}{3K} + K$$

$$\frac{n}{3K} = 1$$

$$\boxed{n = 3^k}$$

$$\log n = \log 3^k$$

$$\boxed{K = \log n}$$

$$\therefore \boxed{O(\log n)}$$ //

Evaluate the following Recurrences completely

(i) $T(n) = T(n/2) + 1$ where $n = 2K$ for all $K \geq 0$

$$T(n) = aT(n/b) + f(n)$$

$$\therefore f(n) = n^k \log n^P$$

where $K >= 0$ and is a real number.

Case 1 :-

$\log_b a > K$ then $\Theta(n^{\log_b a})$

Case 2 :-

$\log_b a = K$

$P > -1$ $\Theta(n^k \log n^{P+1})$

$P = -1$ $\Theta(n^k \log\log n)$

$P < -1$ $\Theta(n^k)$

Case 3 :-

$\log_b a < K$

$P \geq 0$ $\Theta(n^k \log n^P)$

$P < 0$ $\Theta(n^k)$

Step 1 :-

$$T(n) = T(n/2) + 1$$

$$T(n) = aT(n/b) + f(n)$$

$a = 1$, $b = 2$

$\Rightarrow \log_b a$

$= \log_2 1$

$\rightarrow \boxed{0}$

Step:2:-

$f(n) = 1$ $\quad\quad$ $f(n) = n^k \log_n^p$

$p = 1$ $\quad$ $k = 1$

$1 > 0$

$\log_b^a < k$

Step:3:-

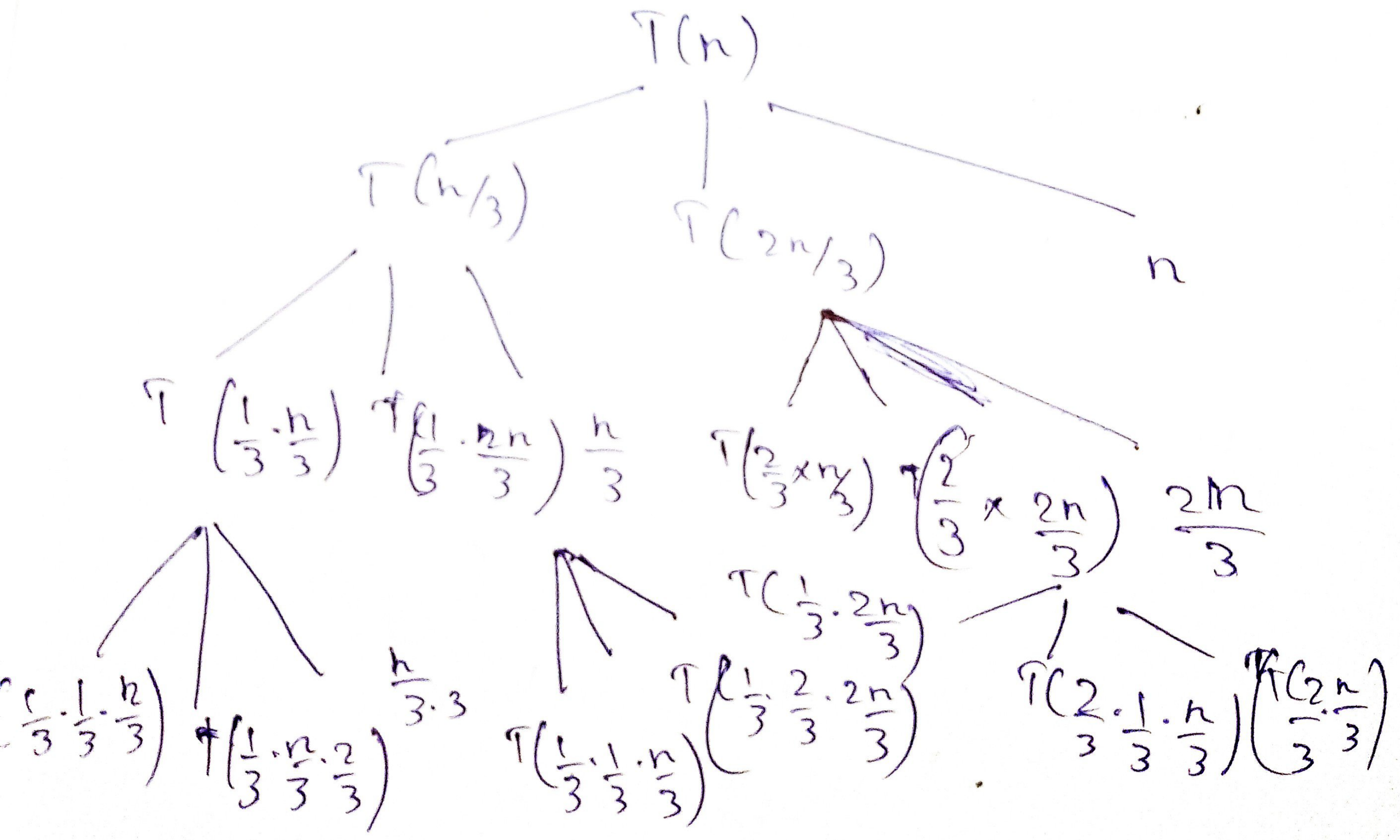
$\Theta(n^k \log_n^p)$

$= \Theta(n^1 \log n^1)$

$= \Theta(n \log n^1)$

$\therefore$ The time $= \boxed{\Theta(\log n)}$

(iii) $T(n) = T(n/3) + T(2n/3) + cn$



Assume $\frac{2^k n}{3^k} = 1$

$\Rightarrow \frac{n}{\left(\frac{3}{2}\right)^k} = 1$

$\Rightarrow n = \left(\frac{3}{2}\right)^k$

$\log n = k \log(3/2)$

$k = \frac{\log n}{\log \frac{3}{2}}$

Time complexity $= O(nk)$

$\boxed{\text{Time Complexity} = \Theta(n \log_{3/2} n)}$

3.) Min 1(A[0 .... n-1])

If :
    n=1
    return A[0]
else :
    temp = Min1 ([A[0 ...... n-2])
        If temp <= A[n-1]
            return temp
    else :
        Return A[n-1]

This algorithm computes minimum element in an Array A of size n.

If [i < n, A < i] is smaller than all elements then A[j]; j = i+1 to n-1, then it returns A[i]. It also returns the leftmost minimal element.

b) Set up a recurrence relation for the algorithm and solve it.

$T(n) = T(n-1) + 1$, when $n > 1$

$T(1) = 0$ (no compare when $n = 1$)

$T(n) = T(1) + (n-1)$

$= 0 + (n-1)$

$= \boxed{n-1}$

$\therefore \boxed{\text{Time} = O(n)}$

Q. Analyse the order of growth.

(i) $f(n) = 2n^2 + 5$, $g(n) = 7n$.

use $\Omega(g(n))$ notation.

Given:

$f(n) = 2n^2 + 5$

$c \cdot g(n) = 7n$

$f(n) \geq c \cdot g(n)$

when $n = 1$,

$f(1) = 2(1)^2 + 5 = 7$

$g(1) = 7$

when $n = 2$,

$f(2) = 2(2)^2 + 5 = 8 + 5 = 13$

$g(2) = 7 \times 2 = 14$

when $n = 3$,

$f(3) = 2(3)^2 + 5$

$\Rightarrow 18 + 5 = 23$

$g(3) = 21$

$n = 1$, $7 = 7$

$n = 2$, $13 = 14$

$n = 3$, $23 = 21$

$n \geq 3$, $f(n) \geq g(n) \cdot c$

$\therefore$ when $n \geq 3$ $f(n)$ is always greater than or equal to $c\,g(n)$

$\boxed{f(n) = \Omega(g(n))}$