Question 02:

a) Running Gale - shapley Algorithm on the given list

Men - optimality / women - pessimality (where men proposes first)

As per the list, victor proposes to Bertha. And, Bertha is unpaired so she has to accept the proposal. $\boxed{\text{Victor} \rightarrow \text{Bertha.}}$

→ Next, Wyatt will propose to propose according to his priority list.

Wyatt proposes to Bertha. Bertha unpairs with victor and forms a pair with wyatt as his priority is higher than victor in her list. $\boxed{\text{Wyatt} \rightarrow \text{Bertha}}$

→ Now, Victor is unpaired and he proposes to the next person in his priority list. Victor proposes to Amy. As, Amy is unpaired she has to accept the proposal.

$\boxed{\text{Victor} \rightarrow \text{Amy}}$

→ Next, Xavier starts proposing according to his priority list. proposes to Bertha. And, as per Bertha's priority list, Bertha unpairs with Wyatt and forms a pair with Xavier. $\boxed{\text{Xavier} \rightarrow \text{Bertha}}$

Now Wyatt is unpaired, so starts proposing to form a pair. And this process continues till there is no man left unpaired.

Wyatt → Amy (X) couldnot form a pair as Amy prefers Victor over Wyatt.

$\boxed{\text{Wyatt} \rightarrow \text{Diane } (\checkmark)}$: Diane is unpaired so has to accept the proposal.

Next, Yancey → Amy (X) : Amy prefers Victor over Yancey.

Yancey → Diane (X) : Diane prefers Wyatt over Yancey.

$\boxed{\text{Yancey} \rightarrow \text{clare } (\checkmark)}$: clare is unpaired so has to accept the proposal.

Next, Zeus → Bertha (X) : Bertha prefers Xavier over Zeus.

Zeus → Diana (X) : Diane prefers Wyatt over Zeus.

$\boxed{\text{Zeus} \rightarrow \text{Amy } (\checkmark)}$: Amy prefers Zeus over Victor. so breaks the pair and forms a pair with Zeus.

Now, Victor is unpaired and he will start proposing from where he stopped in his priority list.

→ Victor will propose to Diane, who is next in his priority list.

Victor → Diane (X) : Diane prefers Wyatt over Victor.

$\boxed{\text{Victor} \rightarrow \text{Erika} (\checkmark)}$: Erika should accept the proposal as she is unpaired.

→ As there are no unpaired men left, Algorithm terminates.

Gives the result, A stable matching. And, there are no unstable pairs.

Result : Men optimality : Stable matching

$\boxed{\text{Xavier} \rightarrow \text{Bertha, Wyatt} \rightarrow \text{Diane, Yancey – Clare, Zeus - Amy, Victor} \rightarrow \text{Erika.}}$

b) Modifying Gale - Shapley Algorithm such that women will propose instead of men.

As per the priority list table, Amy starts proposing according to priority list.

→ Amy → Zeus (✓) : Zeus accepts the proposal as he is unpaired.

Bertha → Xavier (✓) : Xavier accepts the proposal as he is unpaired.

Clare → Wyatt (✓) : Wyatt accepts as he is unpaired.

Diane → Wyatt (✓) : Wyatt unpairs with Clare as he prefer Diane over Clare.

Clare is unpaired, so she will propose again.

Clare → Xavier (X) : Xavier prefers Bertha over Clare.

Clare → Yancey (✓) : Yancey accepts as he is unpaired.

Erika → Yancey (X) : Yancey prefers Clare over Erika.

Erika → Wyatt (X) : Wyatt prefers Diane over Erika.

Erika → Zeus (X) : Zeus prefers Amy over Erika.

Erika → Xavier (X) : Xavier prefers Bertha over Erika.

Erika → Victor (✓) : Victor accepts as he is unpaired.

As there are no unpaired women left, Algorithm terminates.

Result : Gives us a stable matching with no unstable pairs.

Women optimality :

$\boxed{\text{Amy} \rightarrow \text{Zeus, Bertha} \rightarrow \text{Xavier, Diane} \rightarrow \text{Wyatt, Clare} \rightarrow \text{Yancey, Erika} \rightarrow \text{Victor}}$

c) Comparing the matches produced by both men-optimality and women-optimality approaches, both gives us the same result.

Gale-shapley algorithm, gives us the unique stable matching no matter what approach we choose.

3) Given functions, $f_1(n) = n^{2.5}$, $f_2(n) = \sqrt{2n}$, $f_3(n) = n+10$

$f_4(n) = 10^n$, $f_5(n) = 100^n$, $f_6(n) = n^2 \log n$.

Polynomials grow slower compared to the exponential functions when 'n' increased. So, $f_4(n)$ & $f_5(n)$ will be growing faster compared to the other functions.

For exponential functions, they can be ordered based on the bases.

So, $f_5(n) > f_4(n)$.

And, in polynomial functions $f_1(n) = n^{2.5}$, $f_2(n) = \sqrt{2n}$

$f_3(n) = n+10$, $f_6(n) = n^2 \log n$.

Functions with highest exponent value grows faster compared to other functions.

$f_1(n) = n^{2.5}$, $f_2(n) = \sqrt{2n} = \sqrt{2} \cdot (n^{1/2})$, $f_3 = (n)^1 + 10$, $f_6 = n^2 \log n$.

$f_2(n) < f_3(n) < f_1(n)$ based on the exponent value.

$f_6(n) = n^2 \log n = O(n^3)$

$n^2 \log n \leq c \cdot (n^3)$

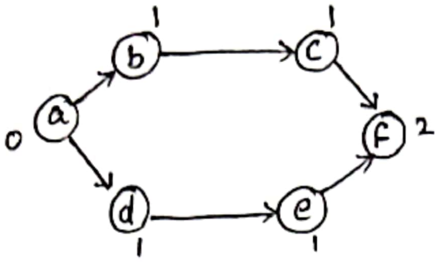
grows faster than n^2 and also logarithmic functions are slower when compared to polynomial functions.

$\Rightarrow n^2 \log n < n^{2.5} \Rightarrow f_6(n) < f_1(n)$.

Arranging the given functions in ascending order of growth rate,

$$\boxed{f_2(n) < f_3(n) < f_6(n) < f_1(n) < f_4(n) < f_5(n)}$$

4) For Topological Ordering, Starting node will be the node with Zero incoming edges. and the last node will be the node with max incoming edges.



a : 0 incoming edges

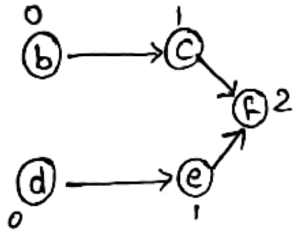
f : 2 incoming edges.

Vertices : {a, b, c, d, e, f}

edges : {ab, bc, ad, de, cf, ef}

In topological order, edge uv : node u should be present before node 'v'.

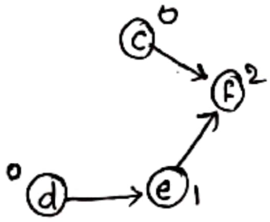
Starting with node 'a': remove node a and related edges. and calculate new degree.



now, there are 2 possibillities, a-b & a-d

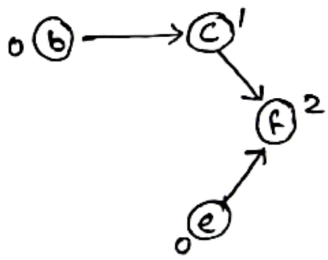
a-b

remove node b.



a-d

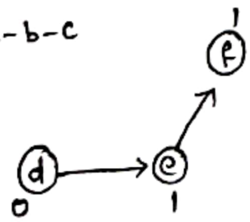
remove node d.



Again, we have 2 options for each.

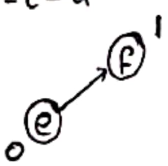
a-b-c and a-b-d, a-d-b and a-d-e

a-b-c

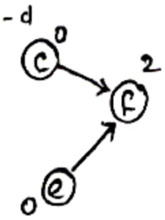


next remove node d,

a-b-c-d



a-b-d



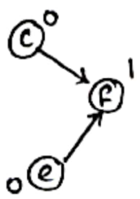
we have 2 options for a-b-d

a-b-d-c

and

a-b-d-e

a-d-b



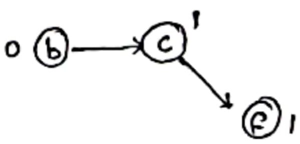
we have 2 options for a-d-b.

a-d-b-c

and

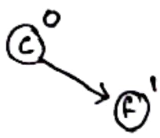
a-d-b-e

a-d-e

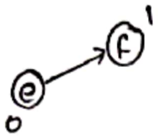


remove node b,

a-d-e-b



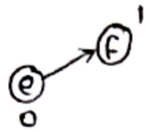
For a-b-c-d



remove node e,

$\boxed{a-b-c-d-e-f}$

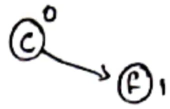
For a-b-d-c



remove node e,

$\boxed{a-b-d-c-e-f}$

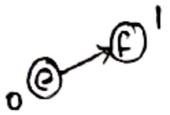
For a-b-d-e



remove node c,

$\boxed{a-b-d-e-c-f}$

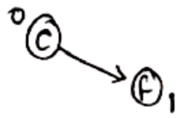
For a-d-b-c



remove node e,

$\boxed{a-d-b-c-e-f}$

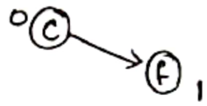
For a-d-e-b



remove node c,

$\boxed{a-d-e-b-c-f}$

For a-d-b-e



remove node c,

$\boxed{a-d-b-e-c-f}$

Total number of possible Topological orders are <u>6</u>.

→ Topological orders : {a, b, c, d, e, f}, {a, b, d, c, e, f}, {a, b, d, e, c, f}

{a, d, b, c, e, f}, {a, d, e, b, c, f} and {a, d, b, e, c, f}