With a better heuristic function, A* algorithm will always provide a better and optimal solution compared to BFS Algorithm.

For basic cases where we can reach the goal state with 2-3 moves BFS and A* algorithm will have very similar time but with complex cases which might involve 20-30 or maybe more than 30 moves A* algorithm will always provide a better and optimal solution compared to BFS.

Case 01:

Starting state: 012345687 -> 012345678 (1 Move)

BFS: 1ms A*: 1ms

Case 02:

Starting state: 012485367 -> 012345678 (4 Moves)

BFS: 1ms A*: 1ms

Case 03:

Starting state: 327615480 -> 012345678 (21 Moves)

BFS: 1035ms

A*: 21ms

Case 04:

Starting state: 210385746 -> 012345678 (22 Moves)

BFS: 1248ms

A*: 435ms

Based on different initial states, for very basic cases which involves 1-2 moves BFS and A* will end up to the goal state in similar time but when the moves required to reach the goal state are getting increased A* will give an optimal solution in very less time based on the heuristic function we used compared to BFS.