CS 721: Advanced Algorithms & Analysis

Homework 6, Fall 2018, Total 190 points

Assigned on: Tuesday, 11/27/2018
Due on: Tuesday, 12/04/2018

1. (15 points) Let $G = (V, E)$ be a flow network on which (A, B) is a minimum cut between source vertex s and sink vertex t. Suppose capacity of each edge of the network is increased by 1. Must (A, B) necessarily remain a minimum cut? If your answer is "yes" prove it. If your answer is "no", provide a counter example.

2. (15 points) Let $G = (V, E)$ be a flow network where each edge has identical capacity c. How will you compute max flow of this network? What is the running time?

3. (10 points) Let $G = (V, E)$ be a flow network carrying maximum flow. Let (A, B) be any cut of this network between source vertex s and sink vertex t. Let $(a, b) \in E$ be an edge of this cut carrying the maximum flow. Suppose this edge is removed to get a new flow network $G' = (V, E')$, where $E' = E \setminus \{(a, b)\}$. Will the maximum flow of the resulting graph G' be strictly less than the maximum flow of G? If your answer is "yes", prove it. If your answer is no, provide a counter example.

4. (30 points) Suppose at any instant in the Wichita area there are k non-moving cell phones and p bases stations, where each cell phone can be directly connected to a base station (without going through other cell phones or base stations) which is not more than d distance away. Locations of all base stations and cell phones are given to you. Each base station can handle at most b cell-phones. Provide an efficient algorithm to assign cell-phones to base stations and analyze its running time.

   Hint: Can you use bipartite matching? Explain how will you convert the given problem to a flow problem.

5. (30 points) Several families are planning a shared car trip to Disneyland. To minimize the possibility of any quarrels, they want to assign individuals to cars so that no two members of a family are in the same car. Suppose there are m families and n cars, where each family has at most 5 members and each car can accommodate at most four persons. Explain how to formulate this problem as a network flow problem. Describe an efficient $O(m^2 n)$ algorithm to assign individuals to cars so that no two members of a family are in the same car.

6. (15 points) Suppose Dr. Smart Joe wins Nobel prize for proving that 3-COLOR problem (which is NP-complete) is solvable in the wort case in $O(n^5)$ time, where n denotes the number of vertices in the graph. Now answer whether the following statements are true or false by providing proper justification.

   (a) All NP-complete problems are solvable in polynomial time.

   (b) All problems in NP, even those that are not NP-complete, are solvable in polynomial time.

   (c) All NP-complete problems are solvable in $O(n^5)$ time.

7. (25 points) Suppose you have been hired as a new manager by Koch industries and your responsibility is to develop and lead a new algorithmic research lab. As a first order of business you need to hire employees.You have a fat file filled with CVs of potential hires. Each candidate has expertise on a list of topics. As a policy of Koch industries, you can

not hire two employees with overlapping expertise. Your job is to hire the largest number of employees from your file such that no two employees have overlapping expertise. Show that this is an NP-complete problem.

Hint: First show that the decision version of the problem is an instance of INDEPENDENCE-SET problem. Show that this problem is in NP. Next polynomially reduce it from CLIQUE to argue that this is an NP-complete problem.

8. (25 points) 4-SAT is similar to 3-SAT, except that each clause contains exactly 4 distinct literals. Show that 4-SAT is NP-Complete by reducing it from 3-SAT.

9. (25 points) Given a set of J jobs of different lengths (time to complete each job), a number of machines m, and a number t, the JOB-SCHED problem is: *Can the jobs in J be assigned to m machines such that all jobs are finished within t units of time, where each machine can work in parallel?*

   Prove that JOB-SCHED is NP-complete by reducing it from PARTITION problem, where PARTITION problem is defined as follows: *Given a set S of integers, can S be split into two subsets whose sums are the same?*.

Submision:

- All texts and diagrams must be electronically produced.
- Your name and page number should appear on each page.
- Entire assignment should be a single PDF file.
- the PDF file should be named in the format HW06_Lastname_Firstname.pdf, for example HW06_Sinha_Kaushik.pdf.
- Submit the pdf file on blackboard.
- **This homework assignment is due at 5:00 pm on the due date.**