CS721 Homework - 3

Question 1. (40 points) Suppose you are managing construction of billboards on the 21st St. N. which runs from east to west on a straight line. There are n possible sites for billboard construction given in the array x[1,...,n], where $0 \leq x[1] < x[2] < \cdots < x[n]$ specifies the distance of each possible billboard location from the west side end of 21st St. N. There is also an array p[1,...n] containing the payment information, i.e., if you place a billboard at location x[j] you receive payment p[j].

Restrictions imposed by the Sedgwick county requires that any pair of billboard must be more than 3 miles apart. You would like place the billboard at a subset of sites so as to maximize your revenue, subject to Sedgwick county's placement restriction. For example, if n = 4, x = [3,4,8,9] and p = [5,6,5,1] then optimal solution will place billboards at x[2] and x[3] with revenue p[2] + p[3] = 6 + 5 = 11.

Suppose you are also given the array prev[1,...,n] where prev[j] stores the location index of the previous billboard site (to the west of x[j]) that satisfies Sedgwick county's billboard restriction. If no such location exists the prev[j] = 0, otherwise, prev[j] = $\max\{i : i < j \text{ and } x[i] < x[j] - 3\}$. Let R(j) denote the optimal revenue obtained by placing billboards at a subset of locations x[1],x[2],...,x[j] satisfying Sedgwick county's restriction. Then our goal is to find R(n). First, write R(j) in terms of solution of smaller subproblem (optimal substructure) and give a dynamic programming solution to find R(n) that takes input x[1,...,n],p[1,...,n] and prev[1,...,n]. What is the running time of your solution? For convenience assume prev[1] = 0 and R(0) = 0

Question 2. (30 points) Answer the following.

(a) Suppose you insert a node with key value 36 in the red-black tree shown on page 310 of your textbook (Figure 13.1). If the inserted node is colored red, will the resulting tree be a red black tree? What if it is colored black?

(b) Explain why the longest path from a node x in a red black tree to a descendant leaf has length at most twice that of the shortest path from node x to a descendant leaf.

(c) What is the largest possible number of internal nodes in a red-black tree with blackheight k? What is the smallest possible number?

(d) Describe a red-black tree on n keys that realizes the largest possible ratio of red internal nodes to black internal nodes. What is this ratio? What tree has the smallest possible ratio, and what is the ratio?

Question 3. (30 points) Suppose T is a binary search tree with m nodes, A is a an array with n elements, and a procedure P employs the Tree insert operation of Page 294 of your textbook repeatedly to insert all elements of A into T, one after another. What is the best case and worst case running time of this operation? Express your answer in Big-Oh or Θ notation in terms of m and n and provide brief justification for your answer.