

CS 721: Advanced Algorithms & Analysis

Homework 3, Fall 2018, Total 40 points

Assigned on: Tuesday, 10/02/2018

Due on: Tuesday, 10/09/2018

1. (10 points) Let $G = (V, E)$ be an undirected connected graph, where all edge weights are positive and equal. Describe an algorithm (no need to provide pseudo-code) that finds MST of G and is asymptotically more efficient than Prim's and Kruskal's algorithm. What is the running time of your algorithm?
2. (15 points) Suppose all edge weights in a graph are integers in the range 1 to $|V|$. How fast can you make Kruskal's algorithm run? What if the edge weights are integers in the range 1 to W for some constant W ?

Hint: You can use the fact that counting sort can sort n integers in the range 0 to k in $\Theta(n + k)$ time.

3. (15 points) Let T be an MST of a graph $G = (V, E)$. Suppose we decrease the weight of one of the edges of T . Show that T (with decreased edge weight) is still an MST for G . More formally, let T be a minimum spanning tree for G with edge weights given by a weight function w . Choose one edge $(x, y) \in T$ and a positive number k and define a new weight function w' by

$$w'(u, v) = \begin{cases} w(u, v), & \text{if } (u, v) \neq (x, y) \\ w(x, y) - k, & \text{if } (u, v) = (x, y) \end{cases}$$

Show that T is also an MST for G with edge weights given by w' .

Hint: First, find out how $w(T)$ and $w'(T)$ are related (their values differs in a single edge). In particular, which one is larger among these two? Next, consider any other spanning tree T' (different from T). You need to show that $w'(T) < w'(T')$, that is even with the new edge weights w' , T is MST and T' is not. To show this, you need to consider two cases, either the edge $(x, y) \in T'$, that is the edge (x, y) whose weight differs under new edge weight w' is a member of T or $(x, y) \notin T'$. Show that in both these case, $w'(T) \leq w'(T')$.

Submission:

- All texts and diagrams must be electronically produced.
- Your name and page number should appear on each page.
- Entire assignment should be a single PDF file.
- the PDF file should be named in the format HW03_Lastname.Firstname.pdf, for example HW03_Sinha_Kaushik.pdf.
- Submit the pdf file on blackboard.
- **This homework assignment is due at 7:00 pm on due date since we will discuss the solution in class on the due date.**