Newton Forward difference Formula

$x_i = x_0 + ih \quad \forall i$

$\Delta f(x) = f(x+h) - f(x)$, h – length of the equispaced interval.

$\Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$.

Δ – Forward difference operator

Forward difference formula –

$$f(x) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \cdots + \frac{s(s-1)(s-2)\cdots(s-\overline{n-1})}{n!}\Delta^n f_0$$

$s = \frac{x - x_0}{h}$.

Problem If $f(x)$ is given as

x	0	1	2	3	4
f	1	7	23	55	109

Find $f(0.5)$ and $f(1.5)$ using Newton forward difference

Solution Forward difference table:

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0	1				
		6			
1	7		10		
		16		6	
2	23		16		0
		32		6	
3	55		22		
		54			
4	109				

$$\therefore f(0.5) = 1 + 0.5 \times 6 + \frac{0.5(0.5-1)}{2!} \times 10 + \frac{0.5(0.5-1)(0.5-2)}{3!} \times 6$$

$$= 3.125.$$