Assume the following transformation between x and the new variable

$$x = Az + B$$

This must satisfy the following conditions:

At

$$x = a, \quad z = -1 \quad \text{and } x = b, \quad z = 1$$

That is

$$B - A = a$$
$$A + B = b$$

Then

$$A = \frac{b-a}{2} \quad \text{and} \quad B = \frac{a+b}{2}$$

Therefore

$$x = \frac{b-a}{2} z + \frac{a+b}{2}$$

$$dx = \frac{b-a}{2} dz$$

This implies that

$$C = \frac{b-a}{2}$$

Then the integral becomes

$$\frac{b-a}{2} \int_{-1}^{1} g(z)\, dz$$

The Gaussian formula for this integration is

$$\frac{b-a}{2} \int_{-1}^{1} g(z)\, dz = \frac{(b-a)}{2} \sum_{i=1}^{n} w_i g(z_i)$$

where w_i and z_i are the weights and quadrature points for the integration domain (–1, 1)

Example 12.9

Compute the integral

$$I = \int_{-2}^{2} e^{-x/2} dx$$

using Gaussian two-point formula.

$n = 2$ and therefore