$$= 2hy_0 + 2h^2 y_0' + \frac{4}{3}h^3 y_0'' + \frac{2}{3}h^4 y_0''' + \ldots$$ (2)

Subtracting (2)-(1)

$$E = \left[2hy_0 + 2h^2 y_0' + \frac{4}{3}h^3 y_0'' + \frac{2}{3}h^4 y_0''' + \ldots\right] - \frac{h}{3}\left[y_0 + 4\left(y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \ldots\right) + \left(y_0 + 2hy_0' + \frac{4h^2}{2!}y_0'' + \ldots\right)\right]$$

Simplifying, we get $E = -\frac{h^5}{90}y^{iv}{}_0$ Error in the interval $[x_0, x_2]$. Neglecting terms of $h^6, h^7 \ldots$

Similarly, error in the interval $[x_2, x_4] = -\frac{h^5}{90}y^{iv}{}_2$

Hence total principal error $E = -\frac{h^5}{90}\left[y^{iv}{}_0 + y^{iv}{}_2 + y^{iv}{}_4 + \ldots + y^{iv}{}_{2(n-1)}\right]$

Let y^{iv} (£) be the maximum of $\left|y^{iv}{}_0\right|, \left|y^{iv}{}_2\right|, \ldots \left|y^{iv}{}_{2(n-1)}\right|$.

Then we have $E < -\frac{h^5}{90}y^{iv}(\pounds) = -\frac{(b-a)h^4}{180}y^{iv}(\pounds)$

Hence the error of Simpson's $\frac{1}{3}$ *rule* is *of order* h^4.

Algorithm 4.2: Algorithm for Simpson's $\frac{1}{3}$rd rule

```
1. Define f(x)
2. Enter the values of upper and lower limit of a,b
3. Enter the number of steps, N
4. Ns=2
5. h=[(a-b)/N]/Ns
6. sum=0
7. do
   {
          sum=sum+h/3*(((func(a)+4*(func(a+h))+func(a+2*h))));
          a=a+Ns*h;
   }while (a<b);
8. print Sum
```