$$m_2 = \frac{2 \times 4.34}{1.5} = 4.8$$

$$y(1.5) = 3.1 + \frac{0.25}{2}(4.96 + 5.79) = 4.44$$

Iteration 3

$$m_1 = \frac{2 \times 4.44}{1.5} = 5.92$$

$$y_e(1.75) = 4.44 + 0.25(5.92) = 5.92$$

$$m_2 = \frac{2 \times 5.92}{1.75} = 6.77$$

$$y(1.75) = 4.44 + \frac{0.25}{2}(5.92 + 6.77) = 6.03$$

Iteration 4

$$m_1 = \frac{2 \times 6.03}{1.75} = 6.89$$

$$y_e(2.0) = 6.03 + 0.25(6.89) = 7.75$$

$$m_2 = \frac{2 \times 7.75}{2} = 7.75$$

$$y(2.0) = 6.03 + \frac{0.25}{2}(6.89 + 7.75) = 7.86$$

Exact solution of the equation

$$y'(x) = 2y/x \text{ with } y(1) = 2$$

is obtained as

$$y(x) = 2x^2$$

The exact values of $y(x)$ and the estimated values by both the metho are tabulated below.

x	$y(x)$		
	Euler's method	Heun's method	Analytical
1.00	2.00	2.00	2.00
1.25	3.00	3.10	3.125
1.50	4.20	4.44	4.50
1.75	5.60	6.03	6.125
2.00	7.20	7.86	8.00

All estimated values are accurate to two decimal places. It is clear th Heun's method provides better results compared to Euler's method.

Error Analysis

It can be easily shown that He... order metho