Example 14.3

Find the eigenvectors of the following system:

$$8x_1 - 4x_2 = \lambda x_1$$
$$2x_1 + 2x_2 = \lambda x_2$$

The characteristic equation of the given system is

$$\begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

That is

$$(8-\lambda)(2-\lambda) + 8 = 0$$

or

$$\lambda^2 - 10\lambda + 8 = 0$$

The roots are

$$\lambda_1 = 6$$
$$\lambda_2 = 4$$

For $\lambda = \lambda_1 = 6$, we get

$$2x_1 - 4x_2 = 0$$
$$2x_1 - 4x_2 = 0$$

Therefore $x_1 = 2x_2$ and the corresponding eigenvector is

$$X_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Similarly, for $\lambda = \lambda_2 = 4$, we get $x_1 = x_2$ and the eigenvector is

$$X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The process of finding the eigenvalues and eigenvectors of large matrices is complex and involves a multistep procedure. There are several methods available and a discussion on all these methods will be beyond the scope of this book. We consider, in the next two sections, the following two methods:

1. Polynomial method
2. Power method

14.5 POLYNOMIAL METHOD

The polynomial method consists of the following three steps:

1. Determine the coefficients p_i of the characteristic polynomial using the Fadeev-Leverrier method
2. Evaluate the roots (eigenvalues) of the characteristic polynomial using any of the root-finding techniques