$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h} \tag{14.5}$$

$$y_i'' = \frac{y_{i+1} - 2y_i - y_{i-1}}{h^2} \tag{14.6}$$

These are second-order equations and the accuracy of estimates can be improved by using higher-order equations.

The given interval (a, b) is divided into n subintervals, each of width h. Then

$$x_i = x_0 + ih = a + ih$$
$$y_i = y(x_i) = y(a + ih)$$
$$y_0 = y(a)$$
$$y_n = y(a + nh) = y(b)$$

This is illustrated in Fig. 14.2. The difference equation is written for each of the internal points $i = 1, 2, ..., n - 1$. If the *DE* is linear, this will result with $(n - 1)$ unknowns $y_1, y_2, ..., y_{n-1}$. We can solve for these unknowns using any of the elimination methods.

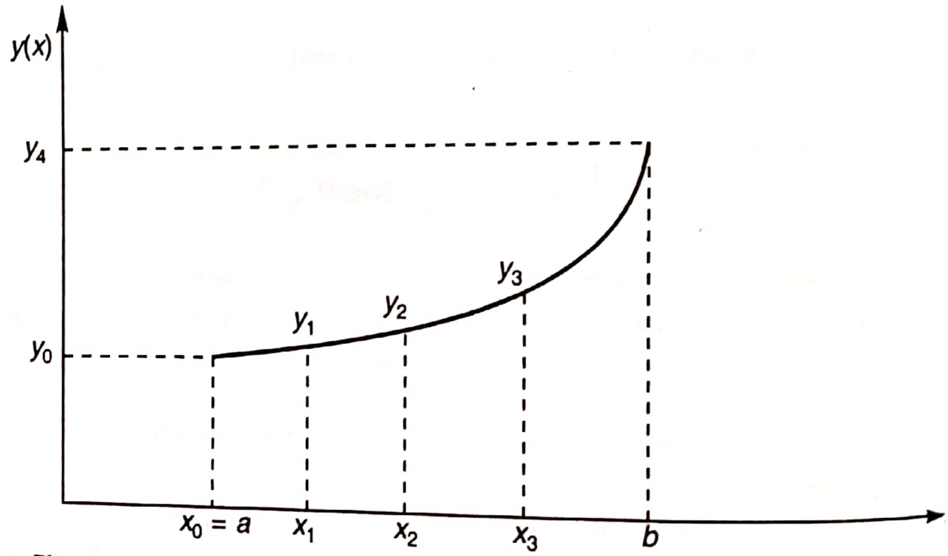

Fig. 14.2 Solution of DE by finite difference method

Note that smaller the size of h, more the subintervals and, therefore, more are the equations to be solved. However, a smaller h yields better estimates.

Example 14.2

Given the equation

$$\frac{d^2y}{dx^2} = e^{x^2} \quad \text{with} \quad y(0) = 0, \quad y(1) = 0$$

estimate the values of $y(x)$ at $x = 0.25, 0.5$ and 0.75.