addition to the truncation error introduced by the methods themselves. Therefore, we also discuss the errors and ways to minimise them.

11.2 DIFFERENTIATING CONTINUOUS FUNCTIONS

We discuss here the numerical process of approximating the derivative $f'(x)$ of a function $f(x)$, when the function itself is available.

Forward Difference Quotient

Consider a small increment $\Delta x = h$ in x. According to Taylor's theorem, we have

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(\theta) \tag{11.1}$$

for $x \leq \theta \leq x + h$. By rearranging the terms, we get

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2} f''(\theta) \tag{11.2}$$

Thus, if h is chosen to be sufficiently small, $f'(x)$ can be approximated by

$$\boxed{f'(x) = \frac{f(x+h) - f(x)}{h}} \tag{11.3}$$

with a truncation error of

$$\boxed{E_t(h) = -\frac{h}{2} f''(\theta)} \tag{11.4}$$

Equation (11.3) is called the first order *forward difference quotient*. This is also known as *two-point formula*. The truncation error is in the order of h and can be decreased by decreasing h.

Similarly, we can show that the first-order *backward difference quotient* is

$$\boxed{f'(x) = \frac{f(x) - f(x-h)}{h}} \tag{11.5}$$

Example 11.1

Estimate approximate derivative of $f(x) = x^2$ at $x = 1$, for $h = 0.2, 0.1, 0.05$ and 0.01 using the first-order forward difference formula.

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

Therefore,

$$f'(1) = \frac{f(1+h) - f(1)}{h}$$