where

$$h_i = (b - a)/2^i$$
$$x_k = a + kh_i$$

Equation (12.27) is known as *recursive trapezoidal rule.*

Example 12.7

Compute Romberg estimate R_{22} for

$$\int_1^2 1/x \, dx$$

First we apply the basic trapezoidal rule to obtain $R(0,0)$

$$R(0, 0) = \frac{h}{2}[f(a) + f(b)]$$

$$= \frac{2-1}{2}(1 + 1/2) = 0.75$$

Now, we obtain $R(1, 0)$ and $R(2, 0)$ using equation (12.27)

$$R(1, 0) = \frac{R(0,0)}{2} + h_1 f(x_1)$$

$$= \frac{0.75}{2} + \frac{1}{2} \times \frac{1}{1.5} = 0.7083333$$

$$R(2, 0) = \frac{R(1,0)}{2} + h_2[f(x_1) + f(x_3)]$$

$$= \frac{0.7083333}{2} + \frac{1}{4}[f(1.25) + f(1.75)]$$

$$= 0.6970237$$

Now, Romberg approximations can be obtained using Eq. (12.26).

$$R(1, 1) = \frac{4R(1,0) - R(0,0)}{3}$$

$$= \frac{4(0.7083333) - 0.75}{3} = 0.6944444$$

$$R(2, 1) = \frac{4R(2,0) - R(1,0)}{3}$$

$$= \frac{4(0.6970237) - 0.7083333}{3} = 0.6932538$$

$$R(2, 2) = \frac{16R(2,1) - R(1,1)}{15}$$