Higher-order Deliver.

We can also obtain approximations to higher-order derivatives using the can also obtain approximations, we derive here the formula a second or the complete that the complete the comple √Higher-order Derivatives We can also obtain approximations to ingle the derive here the formula for Taylor's expansion. To illustrate this, we derive here the formula for f"(x). We know that

also expansion. To indefer that
$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + R_1$$

and

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + R_2$$

Adding these two expansions gives

se two expansions gives
$$f(x+h) + f(x-h) = 2f(x) + h^2f''(x) + R_1 + R_2$$

Therefore

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} - \frac{(R_1 + R_2)}{h^2}$$

Thus, the approximation to second derivative is

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

The truncation error is

$$\begin{split} E_t(h) &= -\frac{R_1 + R_2}{h^2} \\ &= -\frac{1}{h^2} \frac{h^4}{4!} \left(f^{(4)} \left(\theta_1 \right) + f^{(4)} \left(\theta_2 \right) \right) \\ &= -\frac{h^2}{12} f^{(4)} \left(\theta \right) \end{split}$$

The error is of order h^2 .

Similarly, we can obtain other higher-order derivatives with the error order h^3 and h^4 rors of order h^3 and h^4 .

Example 11.4

Find ann

(11.14)