Putting $n = 2$ in the formula 4.1 and taking the curve through $(x_0, y_0), (x_1, y_1)$ and $(x_2, y_2)$ as a polynomial of degree two so, that differences of order higher than two vanish, we get,

$$\int_{x_0}^{x_0+2h} f(x)\,dx = 2h\left[y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0\right]$$

$$= 2h\left[y_0 + (y_1 - y_0) + \frac{1}{6}(y_2 - 2y_1 + y_0)\right]$$

$$= \frac{h}{3}[y_0 + 4y_1 + y_2]$$

Similarly, $\int_{x_0+2h}^{x_0+4h} f(x)\,dx = \frac{h}{3}[y_2 + 4y_3 + y_4]$

... ... ... ... ... ...

$$\int_{x_0+(n-2)h}^{x_0+2h} f(x)\,dx = \frac{h}{3}[y_{n-2} + 4y_{n-1} + y_n]$$

Adding the above integrals, we get

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \dots\dots + y_{n-1}) + 2(y_2 + y_4 + \dots\dots + y_{n-2})\right]$$

which is known as Simpson's $\frac{1}{3}rd$ rule.

When we use this formula, the interval$(x_0, x_n)$must be divided into an even number of subintervals of width $h$.

### 4.6.1 Error in Simpson's $\frac{1}{3}$rd Rule

Suppose the subinterval is 2; then from Simpson's $\frac{1}{3}rd$ rule is

$$\int_{x_0}^{x_2} f(x) = \frac{h}{3}[y_0 + 4y_1 + y_2] = \frac{h}{3}[y_0 + 4\times(y_0 + h) + (y_0 + 2h)] \quad (1)$$

Assuming that $f(x) = F'(x)$

$\therefore \int_{x_0}^{x_2} F(x) = F(x_2) - F(x_0) = F(x_0 + 2h) - F(x_0)$

From Taylor Series expansion's we have,

$$= F(x_0) + 2hF'(x_0) + \frac{(2h)^2}{2!}F''(x_0) + \frac{(3h)^3}{3!}F'''(x_0) + \dots - F(x_0)$$