



Figure 4.1

by the chord joining the two points.

4.5.4 Error in Trapezoidal Rule

From the calculus we have

$$I = \int_{x_0}^{x_0+h} f(x) dx = F(x_0+h) - F(x_0)$$

$$I_T = \frac{h}{2} [f(x_0) + f(x_0+h)]$$

$$\text{Then error is } E_T = I - I_T = F(x_0+h) - F(x_0) - \left[\frac{h}{2} (f(x_0) + f(x_0+h)) \right]$$

$$= \left[F(x_0) + hF'(x_0) + \frac{h^2}{2} F''(x_0) + \dots \right] - F(x_0) - \frac{h}{2} \left[f(x_0) + f(x_0) + hf'(x_0) + \frac{h^2}{2} f''(x_0) + \dots \right]$$

$$= hf(x_0) + \frac{h^2}{2} f'(x_0) + \frac{h^3}{3!} f''(x_0) + \dots - h \left[f(x_0) + \frac{h}{2} f'(x_0) + \frac{h^2}{4} f''(x_0) + \dots \right] = h^3 \left(\frac{1}{6} - \frac{1}{4} \right) f''(x_0) + \dots$$

$$= -\frac{h^3}{12} f''(x_0)$$

We neglect the three and higher order derivatives.

Then, $E_T = -\frac{h^3}{12} f''(x_0)$, it is the truncating error of order h^3 for a single interval. For the whole interval, i.e. $x=a$ to $x=b$. The error of composite formula is

$$E_T = -\frac{h^3}{12} [f''(x_0) + f''(x_1) + f''(x_2) + \dots + f''(x_{n-1})]$$

where $x_0 = a$, $x_n = x_0 + nh = b$