13.9 ACCURACY OF MULTISTEP METHODS

We know that for each differential equation, there is an optimum step size h. If h is too large, accuracy diminishes and if it is too small, round-off errors would dominate and reduce the accuracy.

By computing the predicted and corrected values of y_{i+1}, we can estimate the size and sign of the error. Let us denote the predicted value by y^p_{i+1}. Similarly, denote the truncation error in predicted value by E_{tp} and corrected value by E_{tc}. Then, we have,

$$E_{tp} = y - y^p_{i+1}$$

$$E_{tc} = y - y^c_{i+1}$$

where y denotes the exact value of $y(x_{i+1})$. The difference between the error is

$$E_{tp} - E_{tc} = y^c_{i+1} - y^p_{i+1} \tag{13.47}$$

A large difference indicates that the step size is too large. In such cases, we must reduce the size of h.

Milne-Simpson Method

Both the Milne and Simpson formulae are of order h^4 and their error terms are of order h^5.

The truncation error in Milne's formula is

$$E_{tp} = \frac{28}{90} y^{(5)}(\theta_1) h^5$$

The truncation error in Simpson's formula is

$$E_{tc} = -\frac{1}{90} y^{(5)}(\theta_2) h^5$$

If we assume that $y^{(5)}(\theta_1) = y^{(5)}(\theta_2)$ then

$$\frac{E_{tp}}{E_{tc}} = -28$$

or,

$$E_{tp} = -28E_{tc}$$

Substituting this in Eq. (13.47) we obtain,

$$E_{tc} = -\frac{y^c_{i+1} - y^p_{i+1}}{29}$$