12.4 SIMPSON'S 1/3 RULE

Another popular method is Simpson's 1/3 rule. Here, the function $f(x)$ is approximated by a second-order polynomial $p_2(x)$ which passes through three sampling points as shown in Fig. 12.4. The three points include the end points a and b and a midpoint between them, i.e., $x_0 = a$, $x_2 = b$ and $x_1 = (a + b)/2$. The width of the segments h is given by

$$h = \frac{b - a}{2}$$

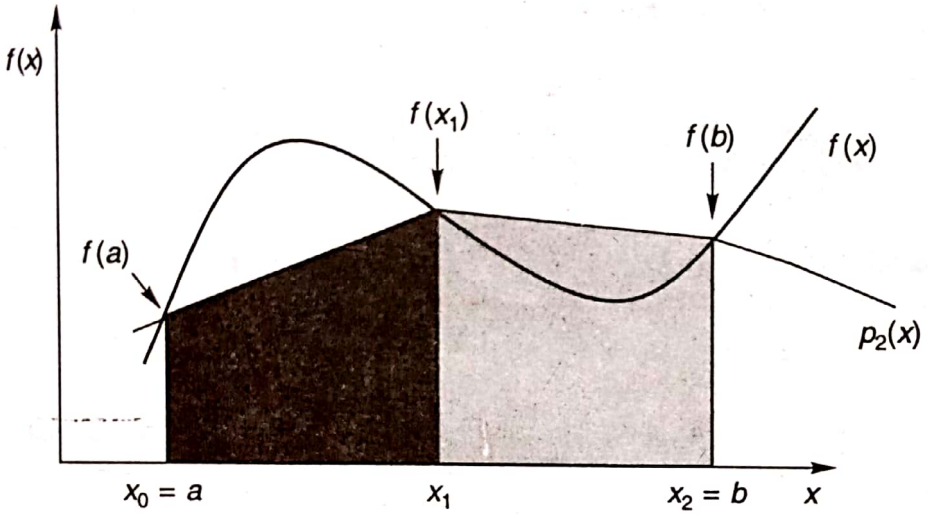

Fig. 12.4 Representation of Simpson's Three-point rule

The integral for Simpson's 1/3 rule is obtained by integrating the first three terms of equation (12.5), i.e.,

$$I_{s1} = \int_a^b p_2(x)\,dx = \int_a^b (T_0 + T_1 + T_2)\,dx$$

$$= \int_a^b T_0\,dx + \int_a^b T_1\,dx + \int_a^b T_2\,dx$$

$$= I_{s11} + I_{s12} + I_{s13}$$

where

$$I_{s11} = \int_a^b f_0\,dx$$

$$I_{s12} = \int_a^b \Delta f_0\, s\,dx$$

$$I_{s13} = \int_a^b \frac{\Delta^2 f_0}{2}\, s(s-1)\,dx$$