3. Calculate the eigenvectors using any of the reduction techniques such as Gauss elimination

The Fadeev-Leverrier Method

The Fadeev-Leverrier method evaluate the coefficients p_i, $i = 1, 2, ..., n$, of the characteristic polynomial

$$\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - .... - p_n = 0$$

The method consists of generating a sequence of matrices A_i that can be employed to determine the p_i values. The process is as follows:

$$A_1 = \mathbf{A}$$
$$p_1 = t_r A_1 \tag{14.11}$$

Remaining values ($i = 2, 3, ..., n$) are evaluated from the recursive equations:

$$A_i = \mathbf{A}(A_{i-1} - p_{i-1}I)$$
$$p_i = \frac{t_r A_i}{i} \tag{14.12}$$

where $t_r A_i$ is the *trace* of the matrix A_i. Remember, the trace is the sum of the diagonal elements of the matrix.

Example 14.4

Determine the coefficients of the characteristic polynomial of the system

$$(-1 - \lambda)x_1 = 0$$
$$x_1 + (-2 - \lambda)x_2 + 3x_3 = 0$$
$$2x_2 + (-3 - \lambda)x_3 = 0$$

using the Fadeev-Leverrier method.

The given system is a third-order one and, therefore, the characteristic polynomial takes the form

$$\lambda^3 - p_1\lambda^2 - p_2\lambda - p_3 = 0$$

The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix}$$

By using the Eq. (14.11),

$$A_1 = \mathbf{A}$$
$$p_1 = t_r A_1 = -6$$

By using the Eq. (14.12),

$$A_2 = \mathbf{A}(A_1 - p_1 I)$$