12.7 ROMBERG INTEGRATION

It is clear from the discussions we had so far that the accuracy of numerical integration process can be improved in two ways:

1. By increasing the number of subintervals (i.e. by decreasing h)—this decreases the magnitude of error terms. Here, the order of the method is fixed.
2. By using higher-order methods—this eliminates the lower-order error terms. Here, the order of the method is varied and, therefore this method is known as *variable-order approach*.

The variable-order method can be implemented using Richardson's extrapolation technique discussed in the previous chapter. As we know this technique involves combining two estimates of a given order to obtain a third estimate of higher order. The method that incorporates this process (i.e. Richardson's extrapolation) to the trapezoidal rule is called *Romberg integration*.

According to the Euler-Maclaurin formula, the error expansion for trapezoidal rule approximation to a definite integral is of the form

$$\int_a^b f(x)\mathrm{d}x - T(h) = a_2h^2 + a_4h^4 + a_6h^6 + \cdots$$