

This is the familiar two-point forward difference formula.

Now, let us consider the quadratic approximation. Here, we need to use three points. Thus,

$$p_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + R_2$$

Then

$$p_2'(x) = a_1 + a_2[(x - x_0) + (x - x_1)] + \frac{dR_2}{dx}$$

Thus, we obtain

$$f'(x) = a_1 + a_2[(x - x_0) + (x - x_1)] \quad (11.16)$$

Let $x_0 = x$, $x_1 = x + h$, $x_2 = x + 2h$, Then

$$a_1 = \frac{f(x+h) - f(x)}{h}$$

$$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$= \frac{f(x+2h) - 2f(x+h) + f(x)}{2h^2}$$

Substituting for a_1 and a_2 in Eq. (11.16) and after simplification, we get

$$f'(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} \quad (11.17)$$

This is a three-point forward difference formula. We can obtain a three-point backward difference formula by replacing h by $-h$ in Eq. (11.17). Therefore, the three-point backward difference formula is given by

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h} \quad (11.18)$$

Similarly, we can obtain the three-point central difference formula by letting $x_0 = x-h$, $x_1 = x$, $x_2 = x+h$ in Eq. (11.16). Thus,

$$a_1 = \frac{f(x) - f(x-h)}{h}$$

$$a_2 = \frac{f(x+h) - 2f(x) + f(x-h)}{2h^2}$$