4/08/20

Bisection Method:-

To find roots of a polynomial, transcendal eq

Algo:- For any continious function $f(x)$

Step1:- For two points say a and b such that $a<b$ and $f(a) * f(b) < 0$

Step2:- Find midpoint $c = \frac{a+b}{2}$

Step3: If c is the root of $f(x)$ ie $f(c)=0$

else, $f(c) * f(b) < 0$, let $a = c$

else $f(c) * f(a) < 0$, let $b = c$

Step4:- Repeat step 2, step 3, step 4 until $f(c) = 0$

Q. Determine the root of given eqn $x^2 - 3 = 0$

let $f(x) = x^2 - 3$

let $a = 1$, $b = 2$

Iteration	a	b	c	f(a)	f(b)
1	1	2	1.5	-2	1
2	1.5	2	1.75	-0.75	1
3	1.5	1.75	1.625	-0.75	0.0625
4	1.625	1.75	1.6875	-0.3594	0.0625
5	1.6875	1.75	1.7188	-0.1523	0.0625
6	1.7188	1.75	1.7344	-0.0457	0.0625
7	1.7188	1.7344	1.7266	-0.0457	0.0081

3. $x^3 - x - 3 = 0$

$f(x) = x^3 - x - 3$

let $a = 1$, $b = 2$

Iteration	a	b	c	f(a)	f(b)
1	1	2	1.5	-1	3
2	1.5	2	1.75	-1.125	3
3	1.5	1.75	1.625	-1.125	0.6094
4	1.625	1.75	1.6875	-0.3398	0.6094
5	1.625	1.6875	1.6562	-0.3398	0.1179
6	1.6562	1.6875	1.6719	-0.1132	0.1179
7	1.6562	1.6719	1.6641	-0.1132	0.0014

So at the 7th iteration, we get the interval [1.6562, 1.6719]

∴ 1.6719 is the approximate solution.

1. ~~$2^x - x - 2 = 0$~~

~~$f(x) = 2^x - x - 2$~~

~~let $a = 1$, $b = 2$~~

$\sin x - 2x + 1$

$f(x) = \sin x - 2x + 1$

let $a = 0$, $b = 1$

Iteration	a	b	c	f(a)	f(b)
1	0	1	0.5	(+1)	-0.158
2	0.5	1	0.75	0.4794	-0.158
3	0.75	1	0.875	0.1816	-0.158
4	0.875	1	0.9375	0.0175	-0.158
5	0.875	0.9375	0.9062	0.0175	-0.068
6	0.875	0.9062	0.8906	0.0175	-0.02
7	0.875	0.8906	0.8828	0.0175	-0.00

So at the 7th iteration we get the interval
[0.875, 0.8906]

Approximate solution is 0.8906

$2^x - x - 2 = 0$

$f(x) = 2^x - x - 2$

Let a = 1, b = 2.

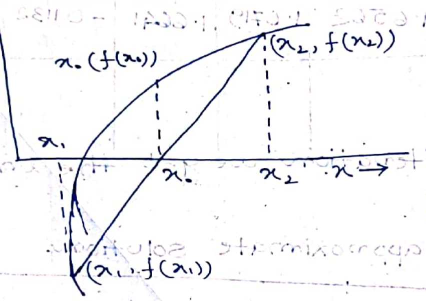
$f(a) = 2^1 - 1 - 2 = (-1)$

$f(b) = 2^2 - 2 - 2 = 0$

∴ Sollution = 2

* 5/08/20

Regula Falsi Method (False position method)



Algorithm:-

Step1:- Define function f(x)

Step2:- Input ⓐ lower and upper guesses a and b

ⓑ tolerable error e.

Step3:- If f(a) * f(b) > 0 print "incorrect initial guesse
 goto step 2
 endif

Step4:- Do $c = b - \frac{(f(b) * (b-a))}{f(b) - f(a)}$

 if f(a)*f(c) < 0 ; b = c

 else a = c

while (fabs (f(c)) > e)

Step5:- print root c

Qn:- Find the root of the solution $x^3 + 3x - 5$

$f(x) = x^3 + 3x - 5$ let a = 1, b = 2

Iteration	a	b	c	f(a)	f(b)
1	1	2	1.1	−1	9
2	1.1	2	~~0.864~~ 1.1354	−0.369	9
3	1.1354	2	~~0.8522~~ 1.1478	−0.1301	9
4	~~0.8522~~ 1.1478	2		−0.0443	9
5	1.1519	2			9
6	1.1534	2			9
7	1.1539	2			9
8	1.1540	2			9
9	1.1541	2			9
10	1.1541	2			9

Qn:- $f(x) = 3x^2 + 6x - 45 = x^2 + 2x - 15$.

Iteration	a	b	c	f(a)	f(
[illegible]	2	4	2.875	−7	9
2	2.875	4	2.9859	−0.9844	9
3	2.9859	4	2.9984	−0.1125	[illegible]
4	2.9984	4	2.9998	−0.0125	
5	2.9998	4	2.998	−0.0014	[illegible]
6					
7					

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3. $f(x) = \cancel{3x^2 + 6x - 45} = 0$

$\cancel{x^2 + 2x - 15}$

1. $f(x) = x - a^x$

let $a = 0$, $b = 1$

Iteration	a	b	c	f(a)	f(b)
1	0	1	0.5	-1	1
2	0	0.5	~~0.5 - √a~~ 0.3333 - $a^{0.3333}$	-1	0.5 - √a
3	0	0.3333		-1	
4					
5					
6					
7					

(ii) $f(x) = \sin x - x + 2$ [x in Radium]

Iteration	a	b	c	f(a)	f(b)
1	2	3	2.51426	0.90930	-0.85888
2	2.51426	3	2.55218	0.07273	-0.85888
3	2.55218	3	2.5541	0.00369	-0.85898
4					
5					
6					
7					

Secand Method :-

Drawback :-

i) The secand iterative formula needs the previous two iterates x_1 and x_2 to estimating the new one.

ii) It has slower rate of convergemce.

· The convergemce of secant method is reffered to as super linear convergemee.

① $4x^2 - 2x - 6 = 0$, $2x^2 - x - 3$

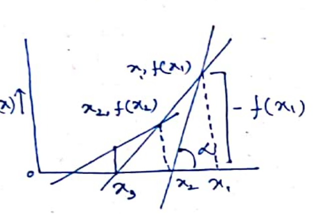
$f(x) = 4x^2 - 2x - 6$

It	a	b	c	f(a)	f(b)
1	2	3	1.6667	3	12
2	3	1.6667	1.5600	12	0.88
3	1.6667	1.5600	1.5036	0.8890	0.3
4	1.5600	1.5036	1.5000	0.3072	0.0
5	~~1.5036~~	~~1.5000~~		~~0.0180~~	

(2) $x^2 - 5x + 6$

It	a	b	c	f(a)	f(b)
1	4	5	3.5	2	6 0.75
2	5	3.5	3.2857	6	0.3673
3	3.5	3.2857	3.0800	0.75	0.0864
4	~~3.2857~~	3.0800	3.0167	0.3673	
5	3.0800	3.0167		0.0864	

Newton-Raphson method:-



Consider a graph of $f(x)$ as shown in the fig. Let us assume that x_1 is an approximate root of $f(x)=0$. Draw a tangent at the curve $f(x)$ at $x = x_1$ as shown in the fig. The point of intersection of the tangent with the x axis gives the second approximation to the root. Let the point of intersection be x_2. The slope of the tangent is given by

$$\tan\alpha = \frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

where $f'(x_1)$ is the slope of $f(x)$ at $x = x_1$. Solving for x_2 we obtain

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

This is called Newton Raphson Formula.

The next approximation would be $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

In general $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

The process will be terminated when the diff between two successive values is within a prescribed result.

(1) $f(x) = x^3 - x - 3$

$f'(x) = 3x^2 - 1$

$x = 2.$

1st

$x_2 = 1.7272$

2ND

$x_3 = 1.6737$

3RD

$x_4 = 1.6717$

$x_5 = 1.6717$

(3) $x \tan x - 1$

$f'(x) = \tan x + x \sec^2 x$

$x = 1$

1st

$x_2 = 1 - \frac{(-0.9825)}{} = 1.9653$

$= 3.55 \times 10^{-4}$

2ND

$x_3 = -0.9325$

(2) $f(x) = x^3 - 3x - 2$

$f'(x) = 3x^2 - 3.$

The Newton-Raphson method approximates the curve of $f(x)$ by tangents. Complications will arise if the derivative $f'(x_n)$ is zero. In such cases, a new initial value for x must be chosen to continue the procedure.

~~$x_3 = 1.67167$~~

- Convergence of Newton Raphson Method:-

Let x_n be an estimate of a root of function $f(x)$. If x_n and x_{n+1} are close to each other, then using Taylor series expansion we can state

$$f(x_{n+1}) = f(x_n) + f'(x_n)(x_{n+1} - x_n) + \frac{f''(R)}{2}(x_{n+1} - x_n)^2 \quad \text{--- (I)}$$

where R lies somewhere in the interval x_n to x_{n+1} and third and higher have been dropped.

Let us assume that the exact root of $f(x)$ is x_r; Then $x_{n+1} = x_r$ then $x_{n+1} = 0$ and substitute this values in eq (I) we get

$$0 = f(x_n) + f'(x_n)(x_r - x_n) + \frac{f''(R)}{2}(x_r - x_n)^2 \quad \text{--- (II)}$$

We know that the Newton's iterative formula is given by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Derive Newton Raphson formulla using the Taylor series expansion:-

Assume, that x_n is an estimate of a root of the function $f(x)$. consider small interval h such that

$$h = x_{n+1} - x_n$$

We can express $f(x_{n+1})$ using Taylor series expan as follows:-

$$f(x_{n+1}) = f(x_n) + f'(x_n) h + f''(x_n) \frac{h^2}{2!} + \cdots$$

If we neglect the terms containing the second order and higher derivatives, we get.

$$f(x_{n+1}) = f(x_n) + f'(x_n) h$$

If x_{n+1} is a root of $f(x)$ then

$$f(x_{n+1}) = 0 = f(x_n) + f'(x_n) h$$

Then

$$h = \frac{f(x_n)}{f'(x_n)} = x_{n+1} - x_n$$

Therefore

$$x_{n+1} = x_1 - \frac{f(x_n)}{f'(x_n)}$$