$$= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} - \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 1 & 4 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 & 0 \\ -3 & -2 & 3 \\ 2 & 2 & -3 \end{bmatrix}$$

$$p_2 = \frac{t_r A_2}{2} = -5$$

imilarly,

$$A_3 = \mathbf{A}(A_2 - p_2 I)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \left\{ \begin{bmatrix} -5 & 0 & 0 \\ -3 & -2 & 3 \\ 2 & 2 & -3 \end{bmatrix} - \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$p_3 = \frac{t_r A_3}{3} = 0$$

Therefore, the characteristic polynomial is

$$\lambda^3 + 6\lambda^2 + 5\lambda = 0$$

or

$$\lambda(\lambda^2 + 6\lambda + 5) = 0$$

Evaluating the Eigenvalues

Let us consider the characteristic polynomial obtained in Example 14.4.

$$\lambda(\lambda^2 + 6\lambda + 5) = 0$$

One of the roots is $\lambda_1 = 0$. The other two roots can be obtained using the familiar quadratic formula as