11.3 DIFFERENTIATING TABULATED FUNCTIONS

Suppose that we are given a set of data points (x_i, f_i), $i = 0, 1, \ldots n$ wh correspond to the values of an unknown function $f(x)$ and we wish estimate the derivatives at these points. Assume that the points equally spaced with a step size of h.

When function values are available in tabulated form, we n approximate this function by an interpolation polynomial $p(x)$ discus in Chapter 9 and then differentiate $p(x)$. We will use here Newto divided difference interpolation polynomial.

Let us first consider the linear equation

$$p_1(x) = a_0 + a_1 (x - x_0) + R_1$$

where R_1 is the remainder term used for estimation. Upon different tion of this formula, we obtain

$$p_1'(x) = a_1 + \frac{dR_1}{dx}$$

Then the approximate derivative of the function $f(x)$ is given by

$$f'(x) = p_1'(x) = a_1$$

We know that

$$a_1 = f[x_0, x_1]$$

$$= \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

On substituting

$$h = x_1 - x_0$$

$$x_1 = x + h$$

$$x_0 = x$$

we get

$$\boxed{f'(x) = \frac{f(x+h) - f(x)}{h}} \qquad (11.15$$