proportionally with decrease in h. There is no roundoff error.

Central Difference Quotient

Note that Eq. (11.3) was obtained using the linear approximation to $f(x)$. This would give large truncation errors if the functions were of higher order. In such cases, we can reduce truncation errors for a given h by using a quadratic approximation, rather than a linear one. This can be achieved by taking another term in Taylor's expansion, i.e.,

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(\theta_1) \tag{11.6}$$

Similarly,

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(\theta_1) \tag{11.7}$$

Subtracting Eq. (11.7) from Eq. (11.6), we obtain

$$f(x+h) - f(x-h) = 2hf'(x) + \frac{h^3}{3!} [f'''(\theta_1) + f'''(\theta_2)] \tag{11.8}$$

Thus, we have

$$\boxed{f'(x) = \frac{f(x+h) - f(x-h)}{2h}} \tag{11.9}$$

with the truncation error of

$$E_t(h) = -\frac{h^2}{12} [f'''(\theta_1) + f'''(\theta_2)] = -\frac{h^2}{6} f'''(\theta)$$

which is of order h^2. Equation (11.9) is called the second-order *central difference quotient*. Note that this is the average of the forward difference quotient and the backward difference quotient. This is also known as *three-point formula*. The distinction between the two-point and three-point formulae is illustrated in Fig. 11.1(a) and Fig. 11.1(b). Note that the approximation is better in the case of three-point formula.