2. $x_1 = \frac{5+2x_2}{3}$; $x_2 = \frac{x_1+x_3}{2}$; $x_3 = 2x_2 - 1$

$x_1^{(1)} = 1.7$ $x_1^{(2)} = 3.4$ $x_1^{(3)} = 3.1$ $x_1^{(4)} = 3.8$

$x_2^{(1)} = 0.9$ $x_2^{(2)} = 2.1$ $x_2^{(3)} = 3.2$ $x_2^{(4)} = 4.6$

$x_3^{(1)} = 0.8$ $x_3^{(2)} = 3.2$ $x_3^{(3)} = 5.4$ $x_3^{(4)} = 8.2$

NUMERICAL INTEGRATION

- A definite integral is a form: $I = \int_a^b f(x)dx$ which can be treated as the area under the curve $y = f(x)$ enclosed within limit $x = a, y = b$
- Could be use a better alternative approach that uses simple arithmatic calculation to compute the area that can be easily implemented in computer. This approach is known with the concept of continious summation to find the area.
- There is a set of method known as Newton course rule in which sampling points are equally spaced.
- We will discuss a method Romberg integration designed to improve estimate Newton course formula.

Newton Course Method:-

i) It is the most popular and widely used numerical integration ~~it forms the basis~~ formula. It forms the basis for a number of numbers. This is known as Newton - Cotes Method.

ii) The derivation of Newton- Course formula is based on polynomial interpolation - And nth degree polynomial that interpolate the values of $f(x)$ at $n+1$ evenly spaced point can be used to replace integrant $f(x)$ of the integral $I = \int_a^b f(x)dx$ and the resultant formula is called $(n+1)$ point Newton-Course Formula.

iii) If the limits of integration a, b are in the set of the interpolation x_i $(i = 0,1,\ldots n)$ the formula is called closed form.

iv) If the limits of integration a,b.

Simpson R:

$$\int_{a=x_0}^{b=x_n} y\,d(x) = \frac{h}{2}\left[y_0 + 2(y_1+y_2+\cdots+y_{n-1}) + y_n\right]$$

$\int_0^1 (4x - 3x^2)dx$ by taking 10 subinterval by trapizoidal

$f(x) = 4x - 3x^2$

$1 - 0 = 10h$ $[b - a = nh]$

$h = 0.1$

$x_0 = 0$; $x_1 = 0.1$; $x_2 = 0.2$; 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1

x_1

$\int_0^1 (4x - 3x^2)dx = \frac{0.1}{2}\left[0 + 2(\quad) + 1\right]$

$\int_0^1 (4x - 3x^2)dx$

$4\left[\frac{x^2}{2}\right]_0^1 - 3\left[\frac{x^3}{3}\right]_0^1$

$4 \times \frac{1}{2} - 3 \times \frac{1}{3}$

$2 - 1$

$= 1$

Absolute error

|True - Approximate|

$= 1 - 0$

$= 0.005$

Relative error.

Compare value of π. $\int_0^1 \frac{dx}{1+x^2}$ using trapizoidal rule with 10 subinterval. Absolute, Relative, percentage error

$h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	1	0.9901	0.9615	0.9175	0.8621	0.8	0.7353	0.6712	0.6098	0.5525	0.5

$$\int_0^1 \frac{dx}{1+x^2} = \frac{0.1}{2}\left[1+0.5+2(12.4446)\right]$$

$$= 0.78498$$

Now from calculas:-

$$\int_0^1 \frac{dx}{1+x^2} \quad \int_0^1 \tan^{-1}x = \tan^{-1} 1 = \pi/4.$$

$$\therefore \frac{\pi}{4} = 0.78498$$

$$\pi = 3.13992 \text{ (Approx)}$$

Absolute error $= |\text{True} - \text{Absolute}|$

$$= |3.14159 - 3.13992|$$

$$= 1.67 \times 10^{-3}$$

Relative error $= 5.315 \times 10^{-4}$

Percentage $= 0.053\%$

3. HW

(1) Calculate $\int_{-1}^{1} e^x dx$ using composite trapizoidal rule ~~to~~ 2 number, 4 number sub interval

(2) $\int_0^1 \sqrt{1-x^3}\, dx$ taking $n = 5$

(3) $\int_1^2 \frac{e^x}{x} dx$; $n = 4$

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$F_1, F_2 \ldots F_n$ are tabulated value $x_1, x_1+h, \ldots x_1+2h$

For i =

$$sum = \frac{f_i + f_{n+1}}{2}$$

iii) for j = 2 to n

$sum = sum + f(j)$

end for

iv) $sum = sum + 2f(j)$

v) Final Result =

vi) write Integral

vii) Stop

Simpson's 1/3 RD Formula (Composite):—

~~putting n = 2 in 41 and takin~~

N: Is true for even number of subinterval or odd number of points.

$nh = b - a$

Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using simpson 1/3 RD rule $h = \frac{1}{4}$

x	0	0.25	0.5	0.75	1.0
$f(x)$	1	0.94	0.8	0.64	0.5

$$\int \frac{dx}{1+x^2} = \left(1 + 0.5 + 4(0.94 + 0.64) + 2 \times 0.8\right) \frac{1}{3} \cdot \frac{1}{4}$$

$= 0.785$

2. Evalute $\int_1^2 e^{-\frac{x}{2}} dx$, using taking 4 subintervals

$4h = 1$

$h = 0.25$

x	1	1.25	1.50	1.75	2.0
$f(x)$	0.60653	0.53526	0.47237	0.41686	0.36788

$\frac{0.25}{3}[(0.60653+0.36788)+4(0.53526+0.41686)+2\times 0.47237]$

$= 0.4773025$

1. $\int_0^1 \frac{dx}{1+x}$ by simpson composite rule taking 11 coordinate. find value of $\log_e 2$ upto 5 significant

$n = 11 - 1 = 10$

$1 - 0 = 10h$

$h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	1	0.9091	0.8333	0.7692	0.7143	0.6667	0.625	0.5882	0.5556	[illegible]	0.5

$\int_0^1 \frac{dx}{1+x} = \frac{0.1}{2}(1+0.4762+4(\text{———})+2(\text{———}))$

$= \cancel{0.6440}\ \cancel{0.6434}$

$= 0.69315$

from calculus we have $\int_0^1 \frac{dx}{1+x} = \log x]_0^1$

$= \log 1 - \log 0$

$= \log[1+x]_0^1$

$= \log_e[1+1] - \cancel{\log[1+0]}$

$= \log 2 - \log$

So $\log_e 2 = 0.69315$

2. $\int_0^2 (2x^3+3x^2)dx$ ~~sim~~ taking 6 ordinates. Comment on the result obtained comparing them with exact value.

$n = 6 - 1 = 5$

$2 - 0 = 5h$

$h = \frac{2}{5} = 0.4$

x	0	0.4	0.8	1.2	1.6	2.0
$f(x)$	0	0.608	2.944	7.776	15.872	28

$\int_0^2 (2x^3+3x^2)dx$

$\frac{0.4}{3}[0+28+4(\overset{odd}{\text{———}})+2(\overset{even}{\text{———}})]$

$= 13.2224$

$\int_0^2 (2x^3+3x^2)dx$

$\frac{2}{4}\frac{x^4}{4}]_0^2 + \frac{3}{3}x^3]_0^2$

$\frac{1}{2}\times 16 + 8$

$= 16$

Absolute error $= 16 - 13.2224 = 2.7776$

H.W

$\int_{0.2}^{1.5} e^{-x^2}dx$ using 4 subdiv

$\int_0^1 e^x dx$ by choosing step size 0.1

3. $\int_1^3 \frac{dx}{x}$ 4 strips

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2. $\int_0^1 e^x dx$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f(x)$	1	1.1052	1.2214	1.3499	1.4918	1.6487	1.8822	2.0138	2.2255	2.4596	2.[illegible]

$$\int_0^1 e^x dx = \frac{0.1}{3}\left[1 + 2.7183 + 34.3088 + 13.6418\right]$$

$= 1.7223$

3. $\int_1^3 \frac{dx}{x}$

x	1	1.5	2	2.5	3
$f(x)$	1	0.6667	0.5	0.4	0.3333

$3 - 1 = 4h$

$\Rightarrow h = \frac{1}{2} = 0.5$

$$\int_1^3 \frac{dx}{x} = \frac{0.5}{3}\left[1 + 0.3333 + 4.2868 + 1\right]$$

$= 1.1$

1. $\int_{0.2}^{1.5} e^{-x^2}$

x	0.2	0.525	0.85	1.175	1.5
$f(x)$	0.9608	0.7591	0.4855	0.2514	0.1[illegible]

$1.5 - 0.2 = 4h$

$1.3 = 4h$

$h = 0.325$

$$\int_{0.2}^{1.5} e^{-x} = \frac{0.325}{3}\left[0.9608 + 0.1054 + 4.042 + 0.971\right]$$

$= 0.65858$

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