

We know that

$$y_0 = y(0) = 0$$

$$y_1 = y(0.25)$$

$$y_2 = y(0.5)$$

$$y_3 = y(0.75)$$

$$y_4 = y(1) = 0$$

$$h = 0.25$$

$$y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = e^{x^2}$$

$$i = 1, x = 0.25$$

$$y'' = \frac{y_2 - 2y_1 + y_0}{0.0625} = e^{(0.25)^2} = 1.0645$$

$$y_2 - 2y_1 + y_0 = 0.0665$$

$$i = 2, x = 0.50$$

$$y'' = \frac{y_3 - 2y_2 + y_1}{0.0625} = e^{(0.5)^2} = 1.2840$$

$$y_3 - 2y_2 + y_1 = 0.0803$$

$$i = 3, x = 0.75$$

$$y'' = \frac{y_4 - 2y_3 + y_2}{0.0625} = e^{(0.75)^2} = 1.7551$$

$$y_4 - 2y_3 + y_2 = 0.1097$$

Letting $y_0 = 0$ and $y_4 = 0$, we have the following system of three equations

$$-2y_1 + y_2 = 0.0665$$

$$y_1 - 2y_2 + y_3 = 0.0803$$

$$y_2 - 2y_3 = 0.1097$$

Solution of these equations results in

$$y_1 = y(0.25) = -0.1175$$

$$y_2 = y(0.50) = -0.1684$$

$$y_3 = y(0.75) = -0.1391$$

The major steps of the process

are given in Algorithm 14