Table 12.2 *(Contd.)*

n	i	w_i	z_i
6	1	0.17132	−0.93247
	2	0.36076	−0.66121
	3	0.46791	−0.23862
	4	0.46791	0.23862
	5	0.36076	0.66121
	6	0.17132	0.93247

Example 12.10

Use Gauss-Legendre three-point formula to evaluate

$$\int_2^4 (x^4 + 1)\,dx$$

Given $n = 3$, $a = 2$, and $b = 2$. Hence

$$I_g = \frac{b-a}{2} \sum_{i=1}^{3} w_i g(z_i)$$

$$= w_1 g(z_1) + w_2 g(z_2) + w_3 g(z_3)$$

$$x = \frac{(b-a)}{2} z + \frac{b+a}{2} = z + 3$$

Therefore,

$$g(z) = (z+3)^4 + 1$$

For $n = 3$, we have

$$w_1 = 0.55556 \qquad z_1 = -0.77460$$

$$w_2 = 0.88889 \qquad z_2 = 0.0$$

$$w_3 = 0.55556 \qquad z_3 = 0.77460$$

Then

$$I_g = 0.55556\,[(-0.77460 + 3)^4 + 1]$$
$$+0.88889\,[(0+3)^4 + 1]$$
$$+0.55556[(0.77460 + 3)^4 + 1]$$
$$= 14.18140 + 72.88898 + 113.33105$$
$$= 200.40143$$

We can verify the answer with analytical solution which is 200.4 [illegible] that three-point Gauss formula should give exact answer for a [illegible] order polynomial. The difference in the answer is due to round[illegible] Roundoff error can be minimised by increasing the precision of Gauss parameters.