An alternative is to use the line which is parallel to the tangent at the point $(x_{i+1}, y(x_{i+1}))$ to extrapolate from y_i to y_{i+1} as shown in Fig. 13.2. That is

$$y_{i+1} = y_i + m_2 h$$

where m_2 is the slope at $(x_{i+1}, y(x_{i+1}))$. Note that the estimate appears to be overestimated.

A third approach is to use a line whose slope is the average of the slopes at the end points of the interval. Then

$$y_{i+1} = y_i + \frac{m_1 + m_2}{2} h \tag{13.20}$$

As shown in Fig. 13.2, this gives a better approximation to y_{i+1}. This approach is known as *Heun's method.*

The formula for implementing Heun's method can be constructed easily. Given the equation

$$y'(x) = f(x, y)$$

we can obtain

$$m_1 = y'(x_i) \quad = f(x_i, y_i)$$

$$m_2 = y'(x_{i+1}) = f(x_{i+1}, y_{i+1})$$

and therefore

$$m = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1})}{2}$$

Equation (13.20) becomes

$$\boxed{y_{i+1} = y_i + \frac{h}{2}\,[f(x_i, y_i) + f(x_{i+1}, y_{i+1})]} \tag{13.21}$$

Note that the term y_{i+1} appears on both sides of Eq. (13.21) and, therefore, y_{i+1} cannot be evaluated until the value of y_{i+1} inside the function $f(x_{i+1}, y_{i+1})$ is available. This value can be predicted using the Euler's formula as

$$\boxed{y_{i+1} = y_i + h \times f(x_i, y_i)} \tag{13.22}$$

Then, Heun's formula becomes

$$\boxed{y_{i+1} = y_i + \frac{h}{2}[f(x_i, y_i) + f(x_{i+2}, y^e_{i+1})]} \tag{13.23}$$

Equation (13.23) is an improved version of Euler's method. Since it attempts to correct the values of y_{i+1} using the predicted value of y_{i+1} (by Euler's method), it is classified as a *one-step predictor-corrector*