Given a set of tabulated values of the integrated $f(x)$, to determine the value of $\int_{x_0}^{x_n} f(x)dx$ called numerical integration. We subdivide the interval of integration into a large number of subintervals of equal width h and replace the function tabulated at the points of subdivision by any one of the interpolating polynomials like Newton's–Gregory's, Sterling's Bessel's over each of the subintervals and evaluate the integral. We have several numerical integration formulas' we shall derive here.

4.5.1 Newton-Cote's Formula

Let $I = \int_a^b y dx$, where y takes the values $y_0, y_1, y_2, \ldots\ldots, y_n$ for $x = x_0, x_1, x_2, \ldots\ldots\ldots x_n$.

Let the interval of integration $[a,b]$ be divided into n equal sub-intervals, each of width $h = \dfrac{b-a}{n}$. So, that $x_0 = a,\ x_1 = x_0 + h, x_2 = x_0 + 2h, \ldots\ldots, x_{n=} x_0 + nh = b$

$$\therefore I = \int_{x_0}^{x_0+nh} f(x)\, dx$$

Since any x is given by $x + x_0 = uh$ and $dx = hdu$.

$$I = h\int_0^n f(x_0 + uh)\, du$$

$$= h\int_0^n \left[y_0 + u.\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \ldots \right] du \text{ [By Newton's Forward interpolation]}$$

$$= h\left[u.y_0 + \frac{u^2}{2!}\Delta y_0 + \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right)\Delta^2 y_0 + \ldots\ldots \right]_0^n$$