

The entire process of repeated use of Richardson's extrapolation technique can be represented in general form as

$$T(h/2^i, j) = \frac{4^j T(h/2^i, j-1) - T(h/2^{i-1}, j-1)}{4^j - 1}$$

where  $i = 0, 1, 2 \dots$  denotes the depth of division and  $j \leq i$  denotes the level of improvement.

We can further simplify the notation of Eq. (12.25) by defining

$$R_{ij} = T(h/2^i, j)$$

Thus, we have

$$R_{ij} = \frac{4^j R_{i, j-1} - R_{i-1, j-1}}{4^j - 1}$$

Equation (12.26) is known as *Romberg integration formula*. Note that this equation, when expanded, will form a lower-diagonal matrix. The elements of the matrix **R** are computed row by row in the order indicated in Fig. 12.5. The circled numbers indicate the order of computations and the arrows indicate the dependencies of elements. An element at the head end depends on the element at the tail end.

