

# Time Series Modeling

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# Goals

After this lecture, you will be able to:

- Explain key properties of time series data
- Describe, measure, and remove **trend** and **seasonality** from a time series
- Understand the concept of **stationarity**
- Create and interpret **autocorrelation function** (acf) plots
- Understand **ARIMA** models for forecasting
- Create your own time series forecast

# Outline

Properties of time series data

Applications and examples

Descriptive methods for understanding a time series

Forecasting

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# What is a time series?

A time series is a sequence of observations over time.

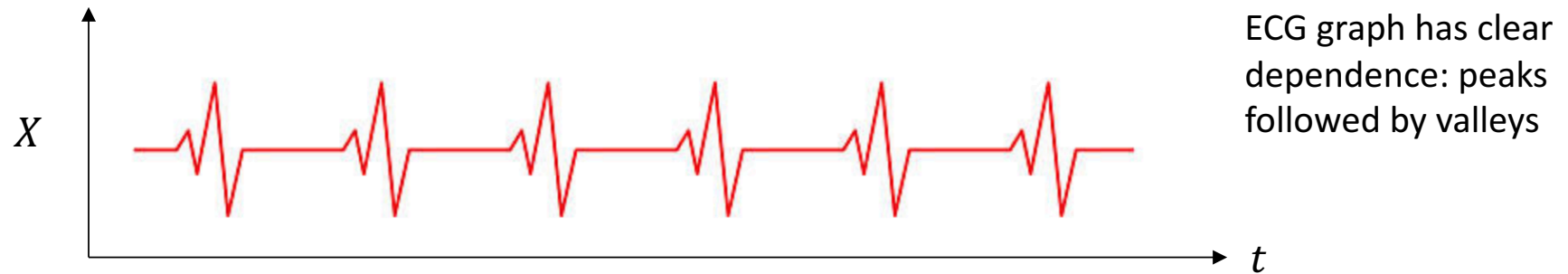


Notation: We have observations  $X_1, \dots, X_n$ , where  $X_t$  denotes the observation at time  $t$

In this lecture, we will consider time series with observations at equally-spaced times (not always the case, e.g. point processes).

# Dependent Observations

Each observation in a time series is **dependent** on all other observations.



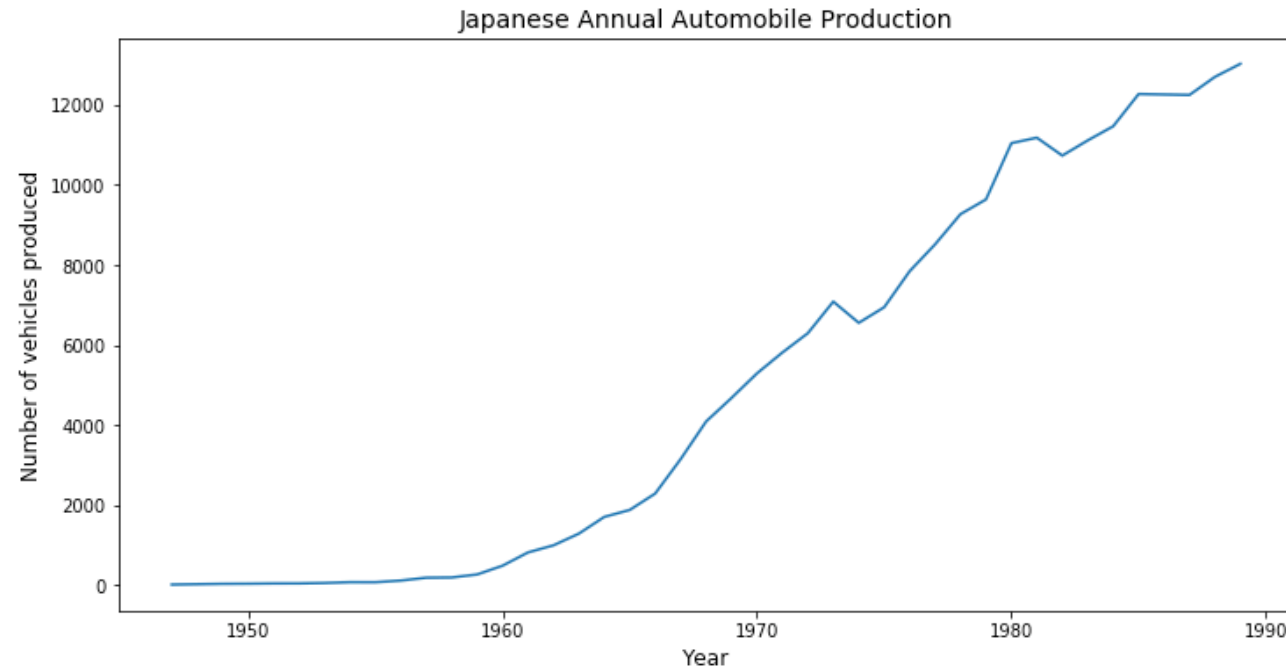
Why is this important? Most statistical models assume that individual observations are independent. But this assumption does not hold for time series data.

Analysis of time series data must take into account the *time order* of the data.

# Trend and Seasonality

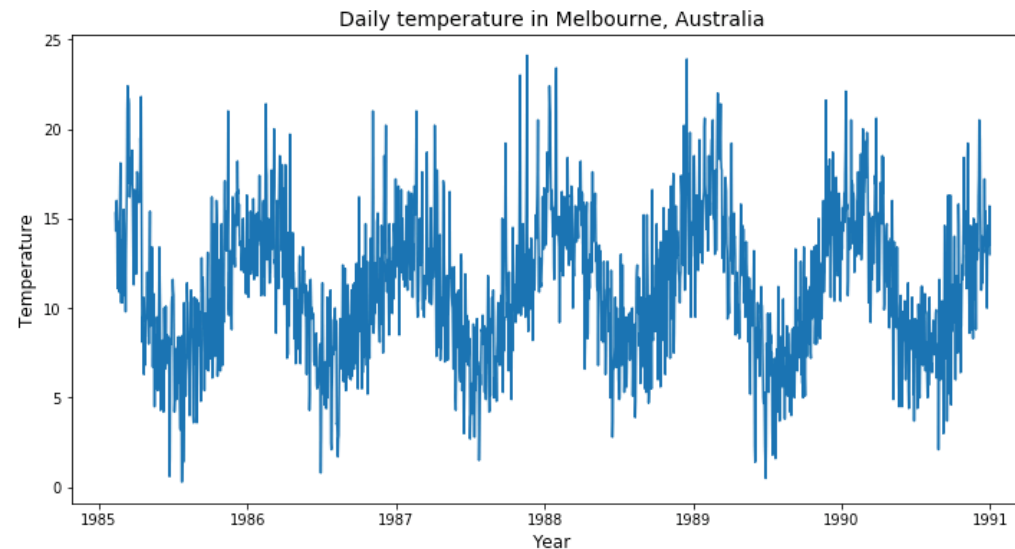
Many time series display trends and seasonal effects.

A **trend** is a change in the long term mean of the series.



# Trend and Seasonality

A **seasonal effect** is a cyclic pattern of a *fixed period* present in the series.



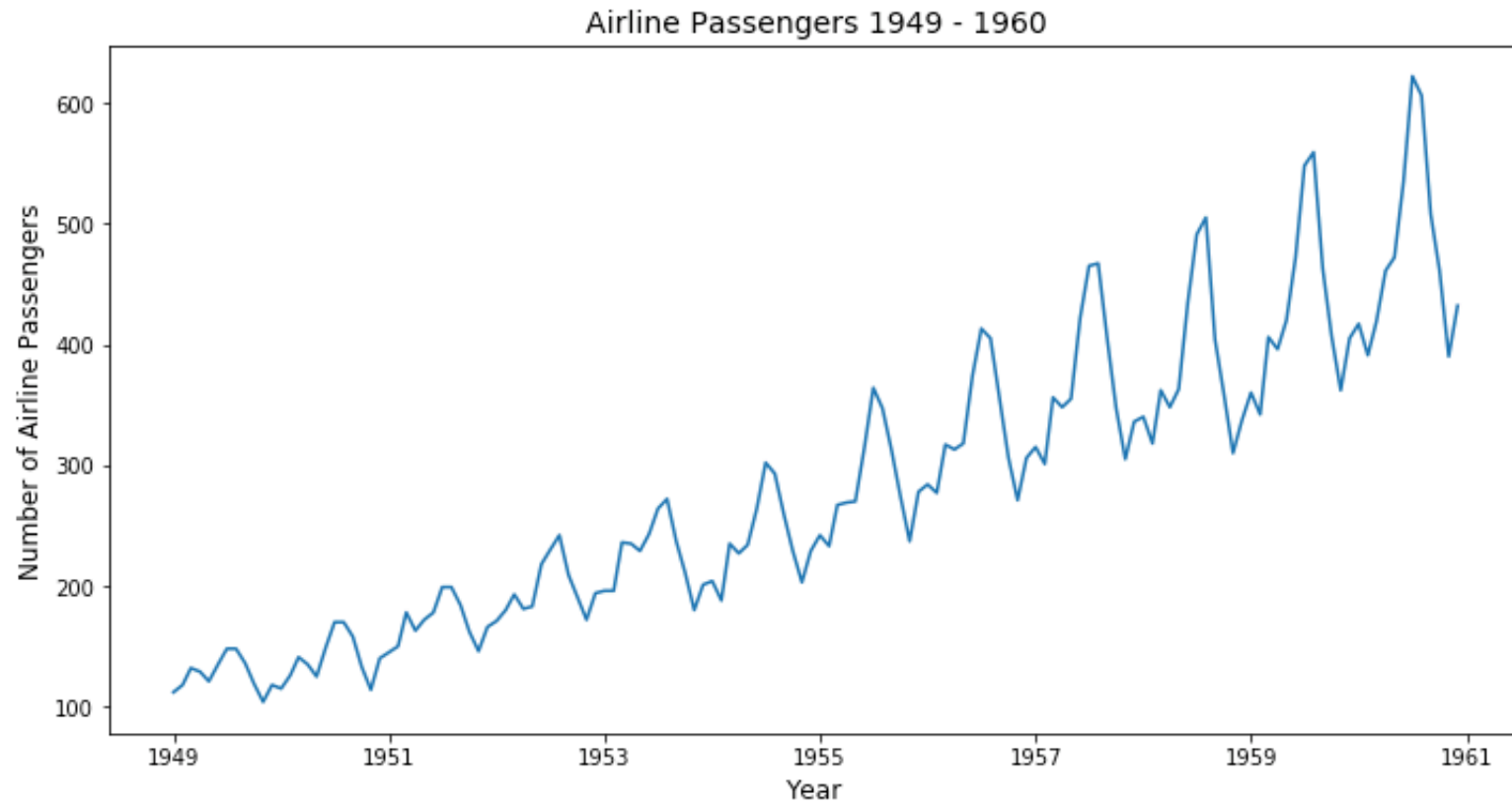
The season (or period) is the length of the cycle (e.g. an annual season).

Seasonal effect can be additive (constant over time) or multiplicative (increasing over time).

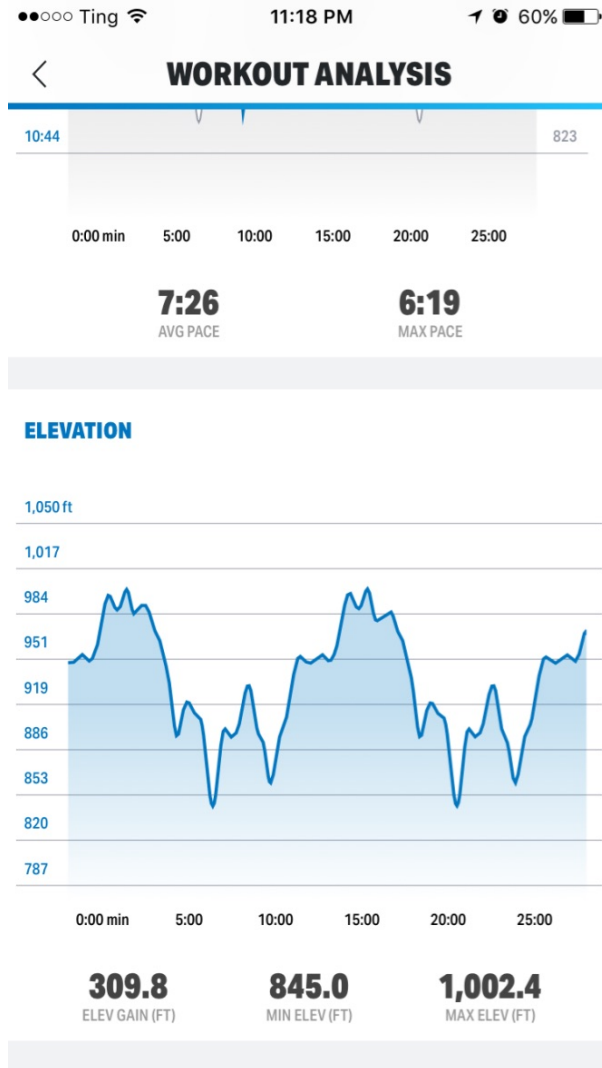


# Trend and Seasonality

A series can have both a trend and a seasonal effect.



# Trend and Seasonality



A fun example: seasonal patterns are quite common.

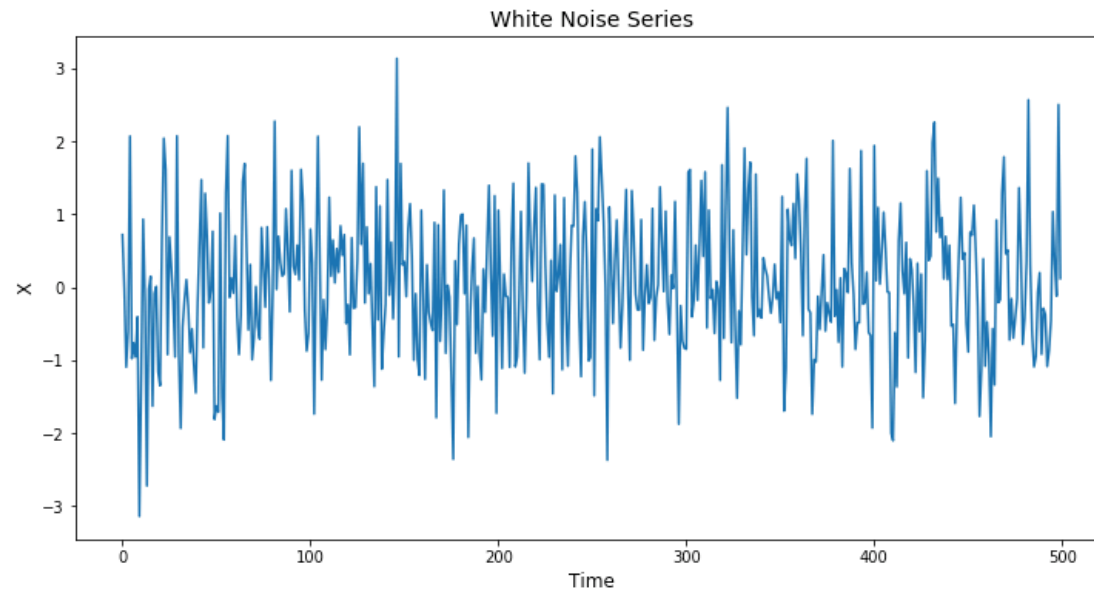
My elevation while running around Schenley Park seems to have a seasonal effect!

(Makes sense because running the same loop repeatedly).

# Stationarity

A time series is called **stationary** if one section of the data looks like any other section of the data, in terms of its distribution.

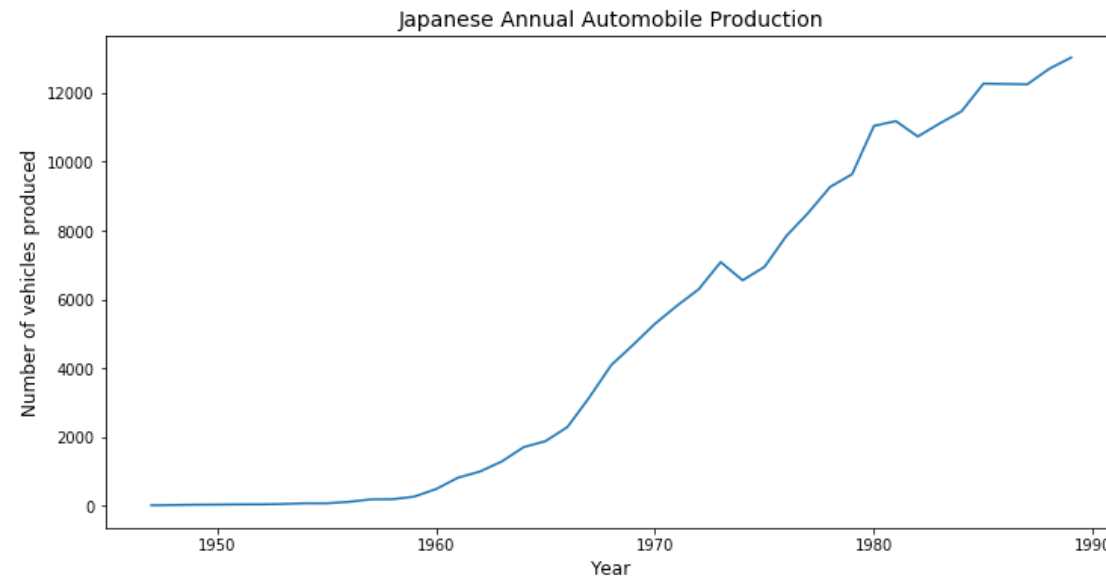
A white noise series  
(sequence of random  
numbers) is stationary.



More formally, a time series is stationary if  $X_{1:k}$  and  $X_{t+k-1}$  have the same distribution, for all  $k$  and  $t$ . (Every section of length  $k$  has the same distribution of values).

# Stationarity

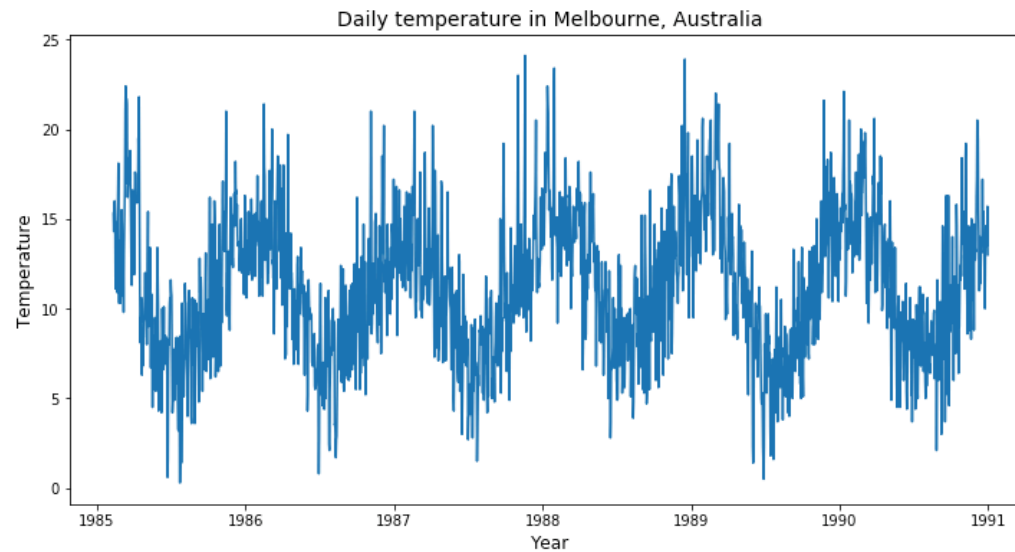
Is this time series stationary?



No, a series with a trend is non-stationary.

# Stationarity

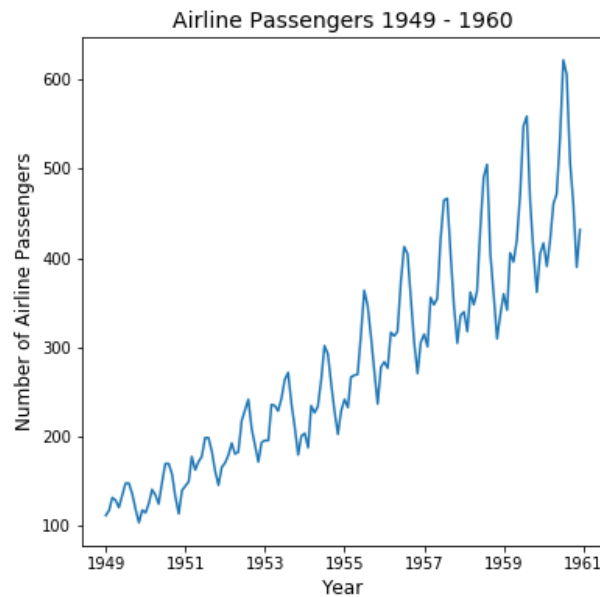
Is this time series stationary?



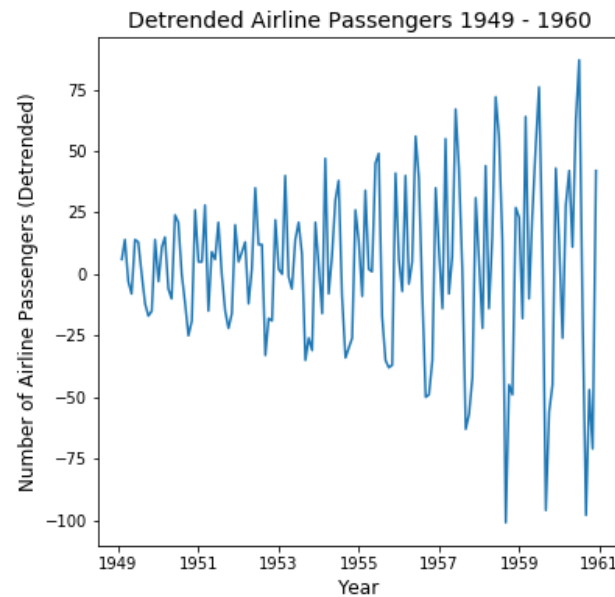
No, a series with seasonality is non-stationary.

# Stationarity

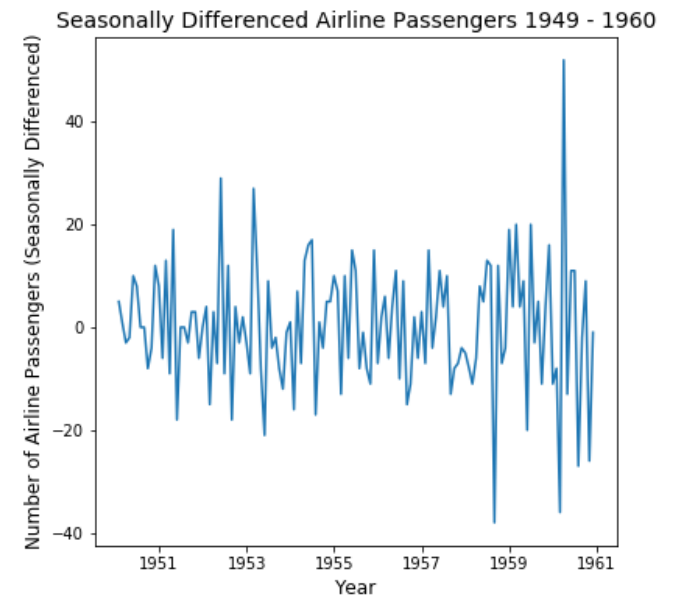
It's often useful to transform a non-stationary series into a stationary series for modeling.



Original series



Removing trend  
(First-order differencing)



Removing seasonality  
(Seasonal differencing)

This is stationary

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Applications and examples

Descriptive methods for understanding a time series

Forecasting

# Applications of Time Series

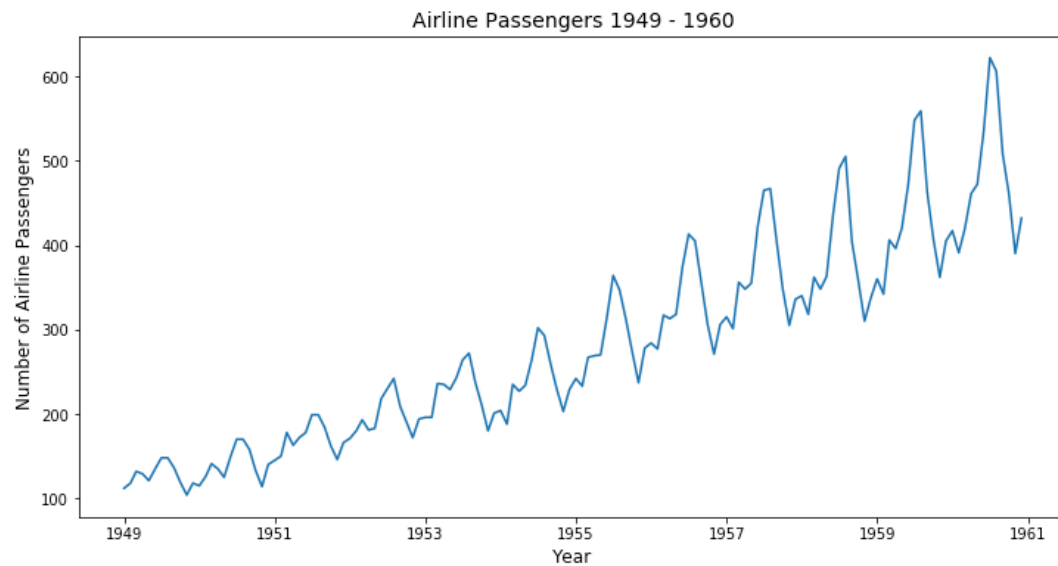
A few applications of time series data:

- Description
- Explanation
- Control
- Forecasting

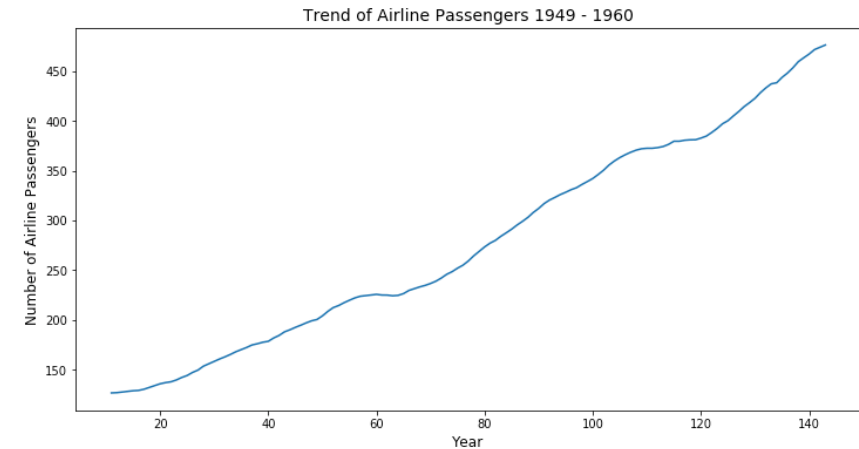


# Application: description

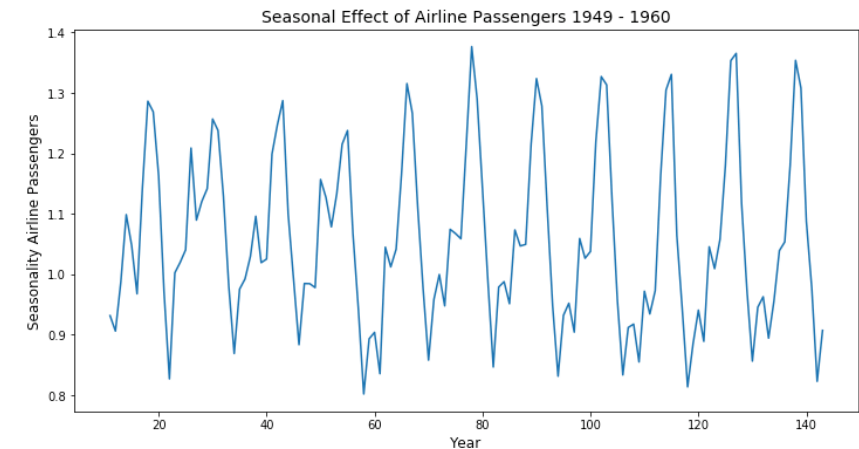
Can we identify and measure the trends, seasonal effects, and outliers in the series?



Original Series



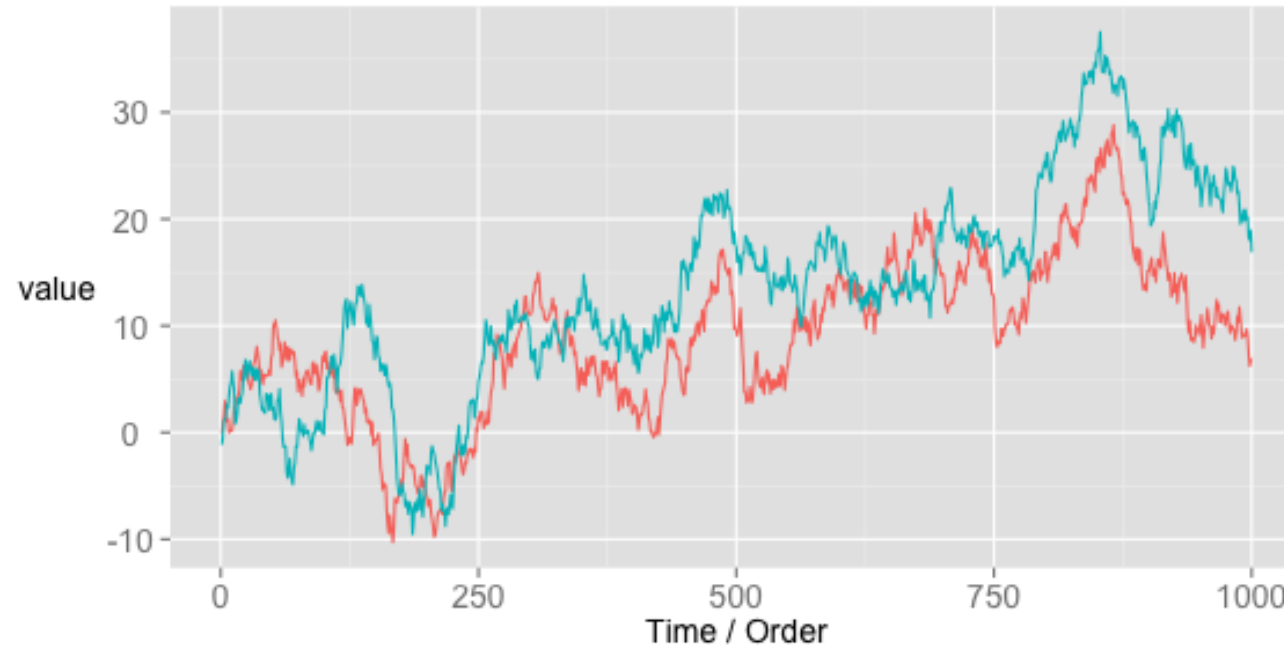
Trend component



Seasonal component

# Application: explanation

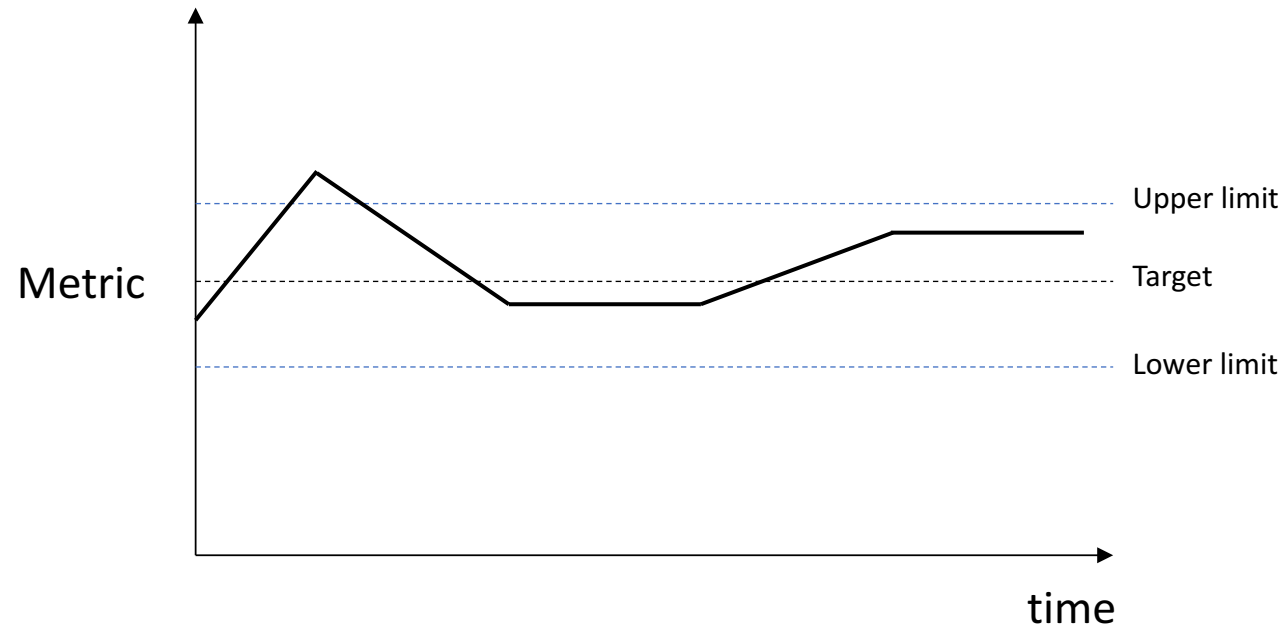
Can we use one time series to explain/predict values in another series?



Model using linear systems: convert one series to another using linear operations.

# Application: control

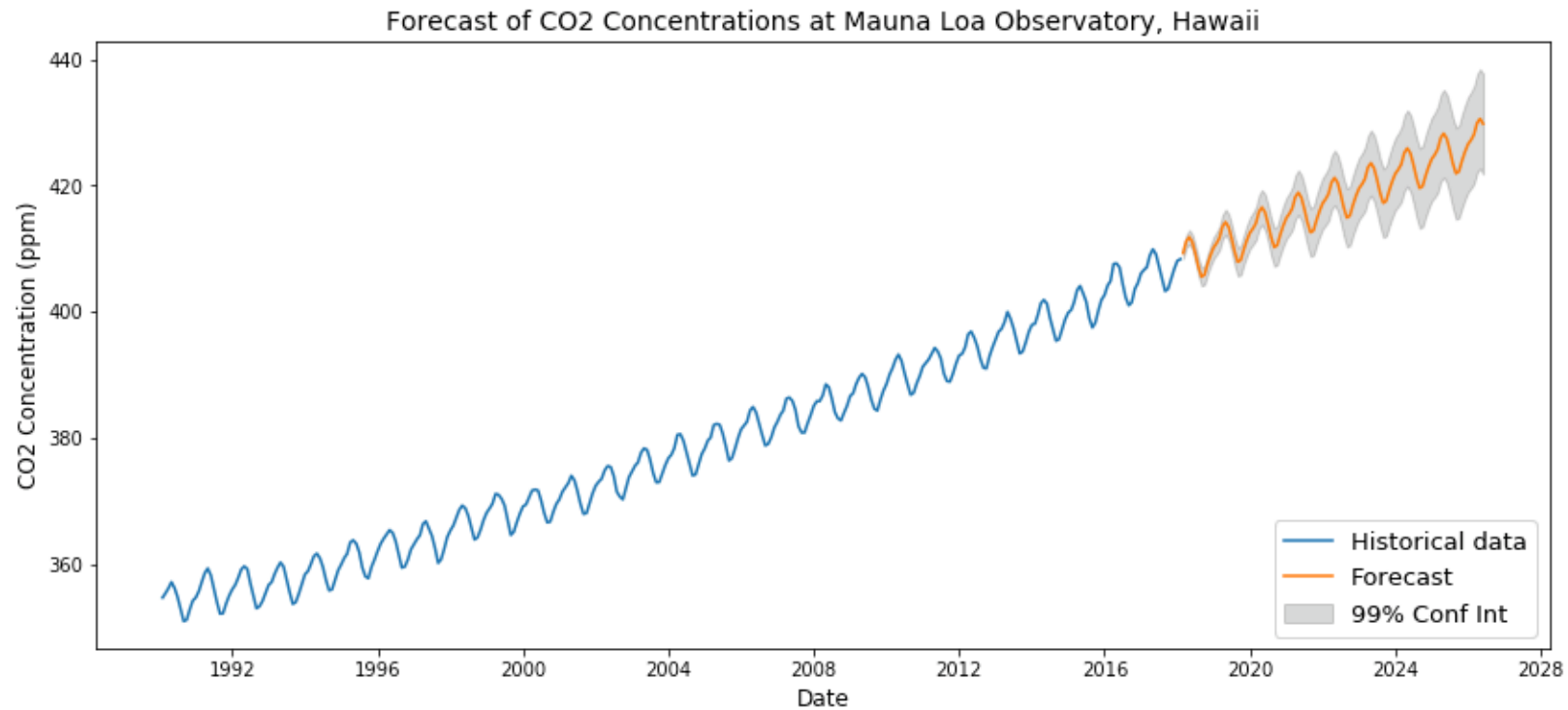
Can we identify when a time series is deviating away from a target?



Example: Manufacturing quality control

# Application: forecasting

Using observed values, can we predict future values of the series?



# Applications of Time Series

In this lecture:

- Description

Can we identify and measure the trends, seasonal effects, and outliers in the series?

- Explanation

- Control

- Forecasting

Using observed values, can we predict future values of the series?

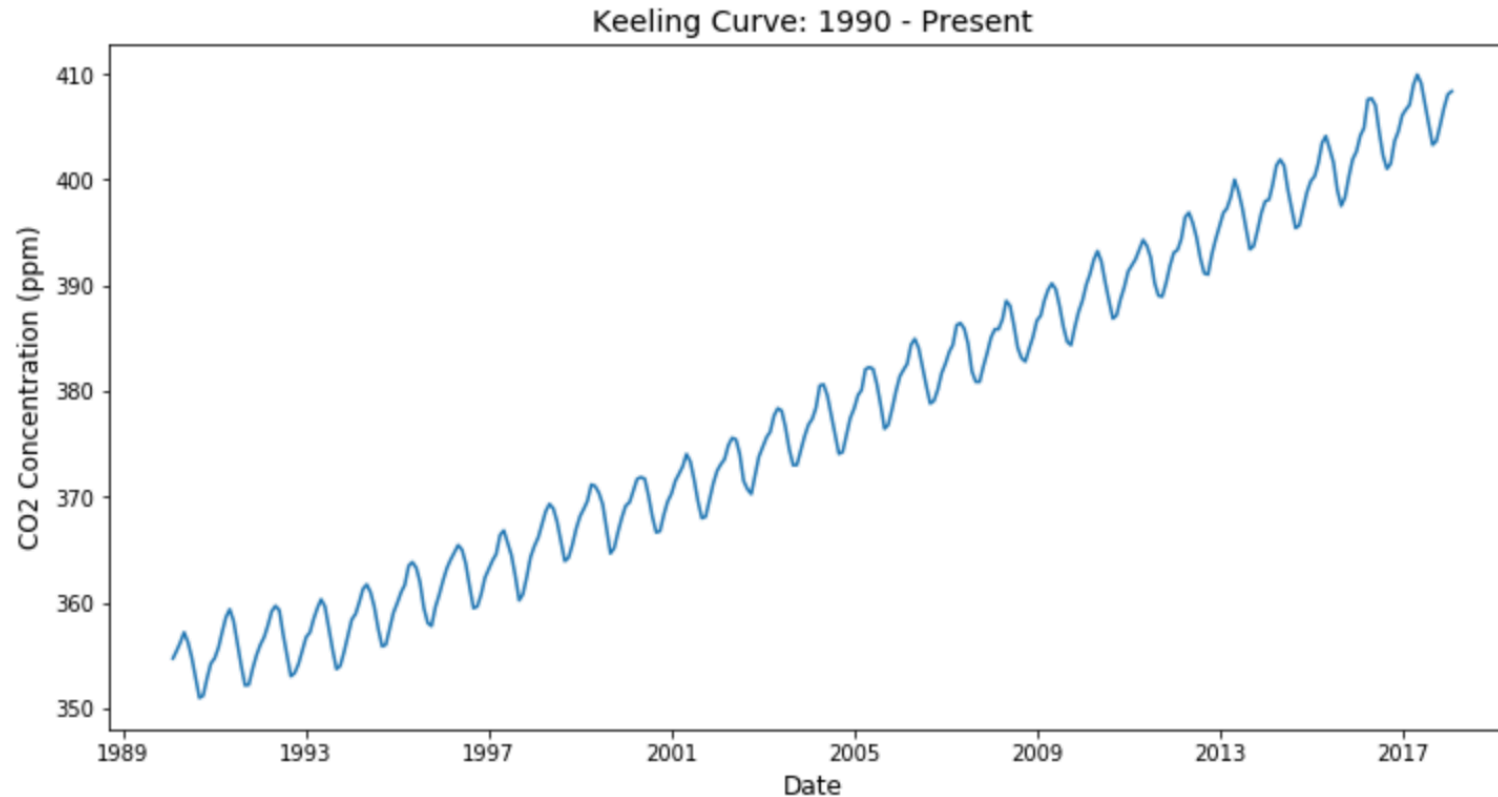
# Example: Keeling Curve

The Keeling Curve is the foundation of modern climate change research.



Daily observations of atmospheric CO<sub>2</sub> concentrations since 1958 at the Mauna Loa Observatory in Hawaii.

# Example: Keeling Curve



Why is there an annual season? Plants grow in spring, die in fall

Why is there a trend? Climate change

# Outline

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Applications and examples

Descriptive methods for understanding a time series

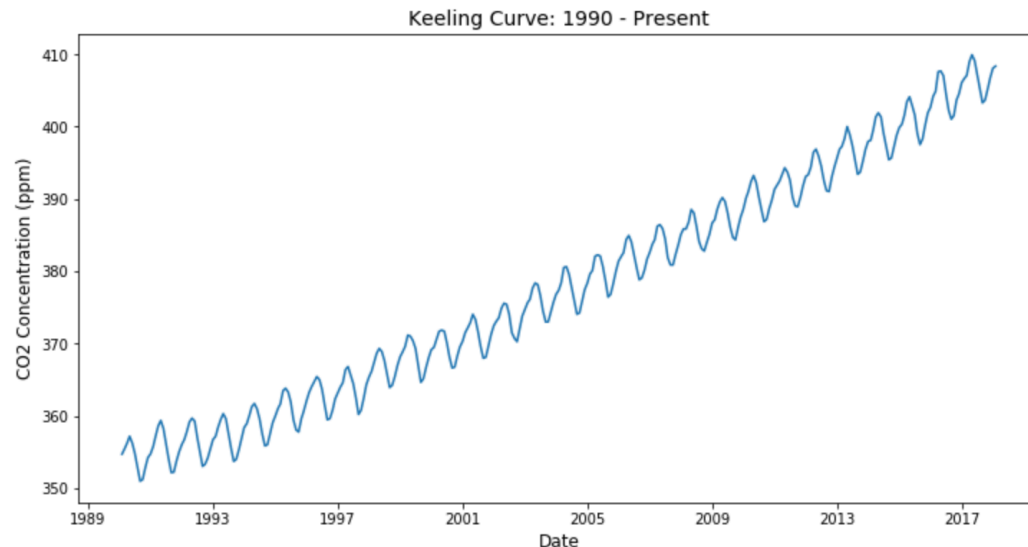
Forecasting



# Time plot

The first thing you should do in any time series analysis is plot the data.

```
plt.plot(df['date'], df['CO2'])  
plt.xlabel('Date', fontsize=12)  
plt.ylabel('CO2 Concentration (ppm)', fontsize=12)  
plt.title('Keeling Curve: 1990 - Present', fontsize=14)
```



Plotting helps us identify salient properties of the series:

- Trend
- Seasonality
- Outliers
- Missing data

# Measuring the trend

Next, we can take a more systematic approach in measuring the trend of the series.

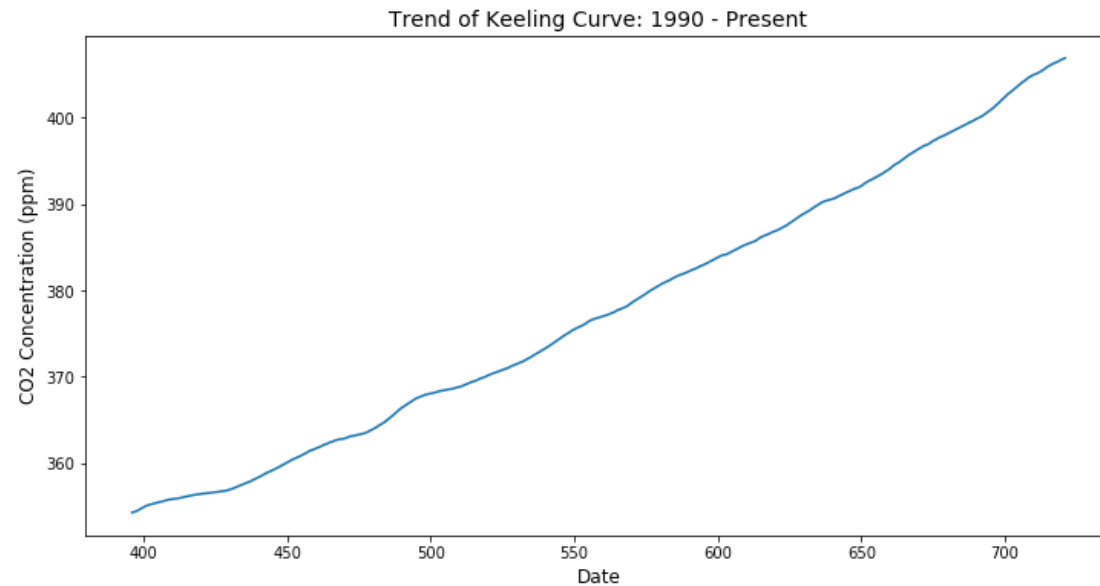
We can estimate a trend by using a moving average.

$$X_t = \frac{1}{2k} \sum_{i=-k}^k X_{t+i}$$

# Measuring the trend

Implementing the moving average is easy.

```
moving_avg = df['CO2'].rolling(12).mean()
fig = plt.figure(figsize=(12,6))
plt.plot(moving_avg.index, moving_avg)
plt.xlabel('Date', fontsize=12)
plt.ylabel('CO2 Concentration (ppm)', fontsize=12)
plt.title('Trend of Keeling Curve: 1990 - Present', fontsize=14)
```



# Removing the trend

We can also remove the trend by first-order differencing.

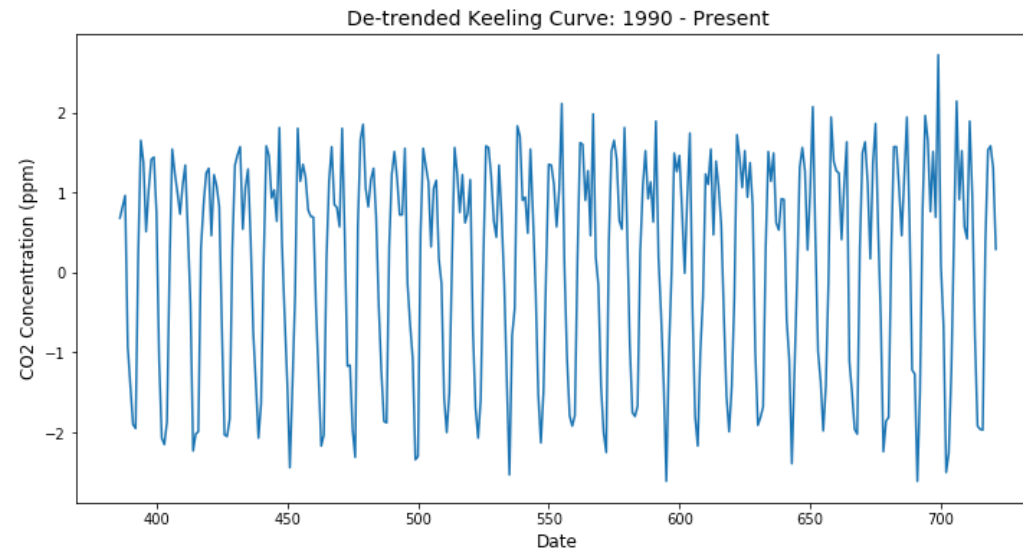
$$X'_t = X_t - X_{t-1}$$

$X'_t$  will be a de-trended series.

# Removing the trend

Implementing first-order differencing.

```
detrended = df['CO2'].diff()  
fig = plt.figure(figsize=(12,6))  
plt.plot(detrended.index, detrended)  
plt.xlabel('Date', fontsize=12)  
plt.ylabel('CO2 Concentration (ppm)', fontsize=12)  
plt.title('De-trended Keeling Curve: 1990 - Present', fontsize=14)
```



# Removing seasonality

We can also remove the seasonality through seasonal differencing.

$$X'_t = X_t - X_{t-m}$$

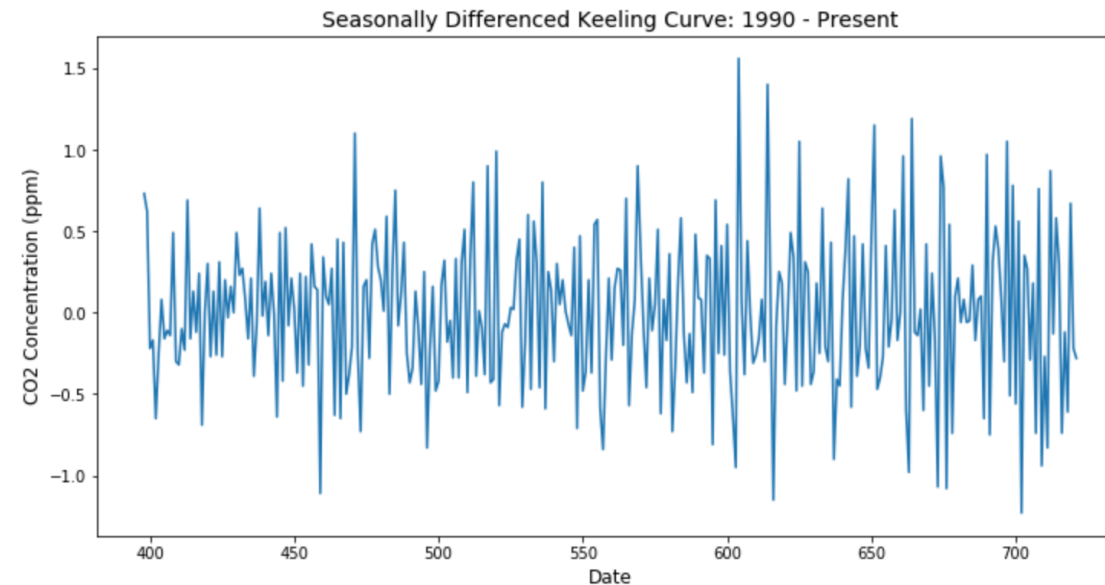
where  $m$  is the length of the season

$X'_t$  will be a de-seasonalized series

# Removing seasonality

Implementing seasonal differencing.

```
seasonal_diff = detrended.diff(12)
fig = plt.figure(figsize=(12,6))
plt.plot(seasonal_diff.index, seasonal_diff)
plt.xlabel('Date', fontsize=12)
plt.ylabel('CO2 Concentration (ppm)', fontsize=12)
plt.title('Seasonally Differenced Keeling Curve: 1990 - Present', fontsize=14)
```



# Outline

Properties of time series data

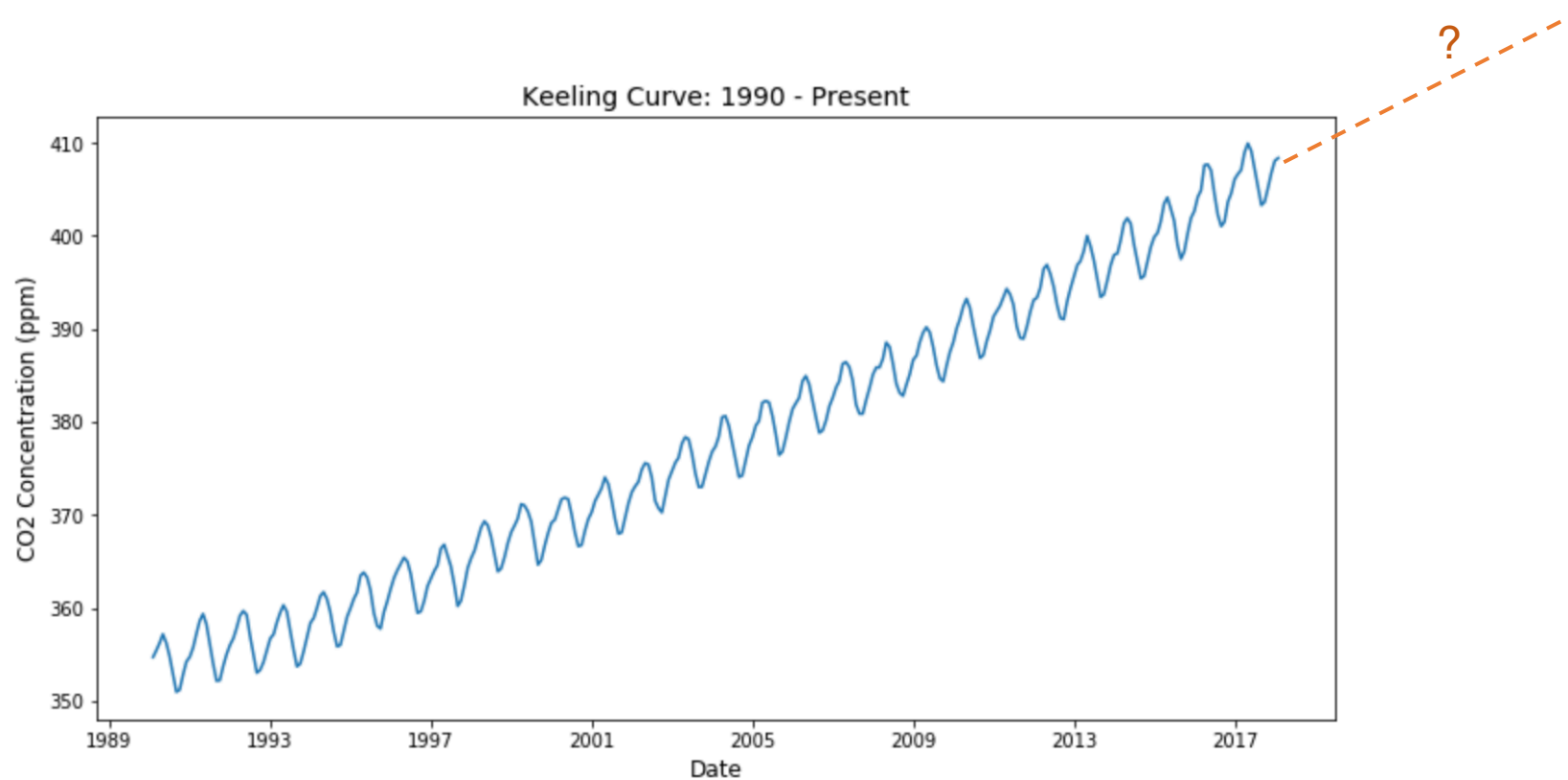
Applications and examples

Descriptive methods for understanding a time series

**Forecasting**



# Forecasting

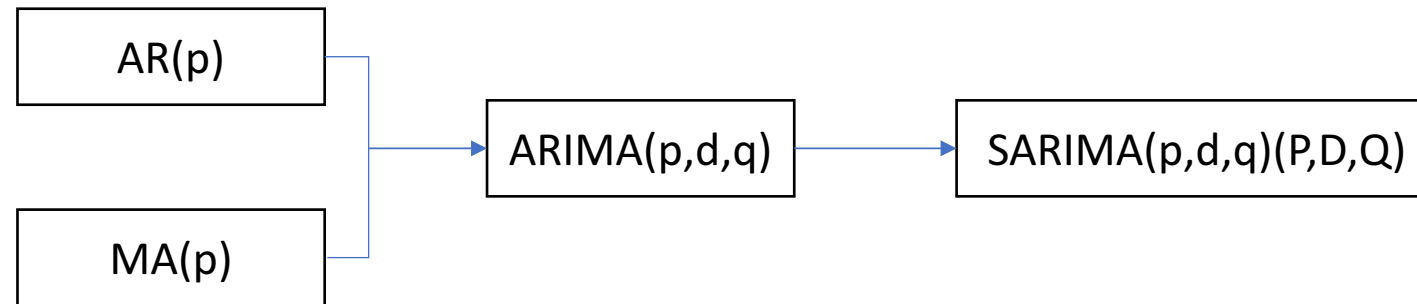


Can we predict future values of the Keeling curve using observed values?

# Forecasting

Now, we will introduce a class of linear models called the ARIMA models, which can be used for time series forecasting.

There are several variants of ARIMA models, and they build on each other.



ARIMA models work by modeling the autocorrelations (correlations between successive observations) in the data.


# Autoregressive Model: AR

An autoregressive model predicts the response  $X_t$  using a linear combination of past values of the variable. Parameterized by  $p$ , (the number of past values to include).

$$X_t = \theta_0 + \theta_1 X_{t-1} + \theta_2 X_{t-2} + \dots + \theta_p X_{t-p}$$

This is the same as doing linear regression with lagged features. For example, this is how you would set up your dataset to fit an autoregressive model with  $p = 2$ :

t	$X_t$
1	400
2	500
3	300
4	100
5	200



$X_{t-2}$	$X_{t-1}$	$X_t$
400	500	300
500	300	100
300	100	200

# Moving Average Model: MA

A moving average model predicts the response  $X_t$  using a linear combination of past forecast errors.

$$X_t = \beta_0 + \beta_1\epsilon_{t-1} + \beta_2\epsilon_{t-2} + \dots + \beta_q\epsilon_{t-q}$$

where  $\epsilon_i$  is normally distributed white noise (mean zero, variance one)

Parameterized by  $q$ , the number of past errors to include. The predictions  $X_t$  can be the weighted moving average of past forecast errors.

# AutoRegressive Integrated Moving Average Model: ARIMA

Combining a autoregressive (AR) and moving average (MA) model, we get the ARIMA model.

$$\begin{aligned} X'_t = & \theta_0 + \theta_1 X_{t-1} + \theta_2 X_{t-2} + \dots + \theta_p X_{t-p} \\ & + \beta_0 + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \dots + \beta_q \epsilon_{t-q} \end{aligned}$$

Note that now we are regressing on  $X'_t$ , which is the differenced series  $X_t$ . The order of difference is determined by the the parameter  $d$ . For example, if  $d = 1$ :

$$X'_t = X_t - X_{t-1} \text{ for } t = 2, 3, \dots, N$$

So the ARIMA model is parameterized by:  $p$  (order of the AR part),  $q$  (order of the MA part), and  $d$  (degree of differencing).

# Seasonal ARIMA: SARIMA

Extension of ARIMA to model seasonal data.

Includes a non-seasonal part (same as ARIMA) and a seasonal part. The seasonal part is similar to ARIMA, but involves backshifts of the seasonal period.

In total, 6 parameters:

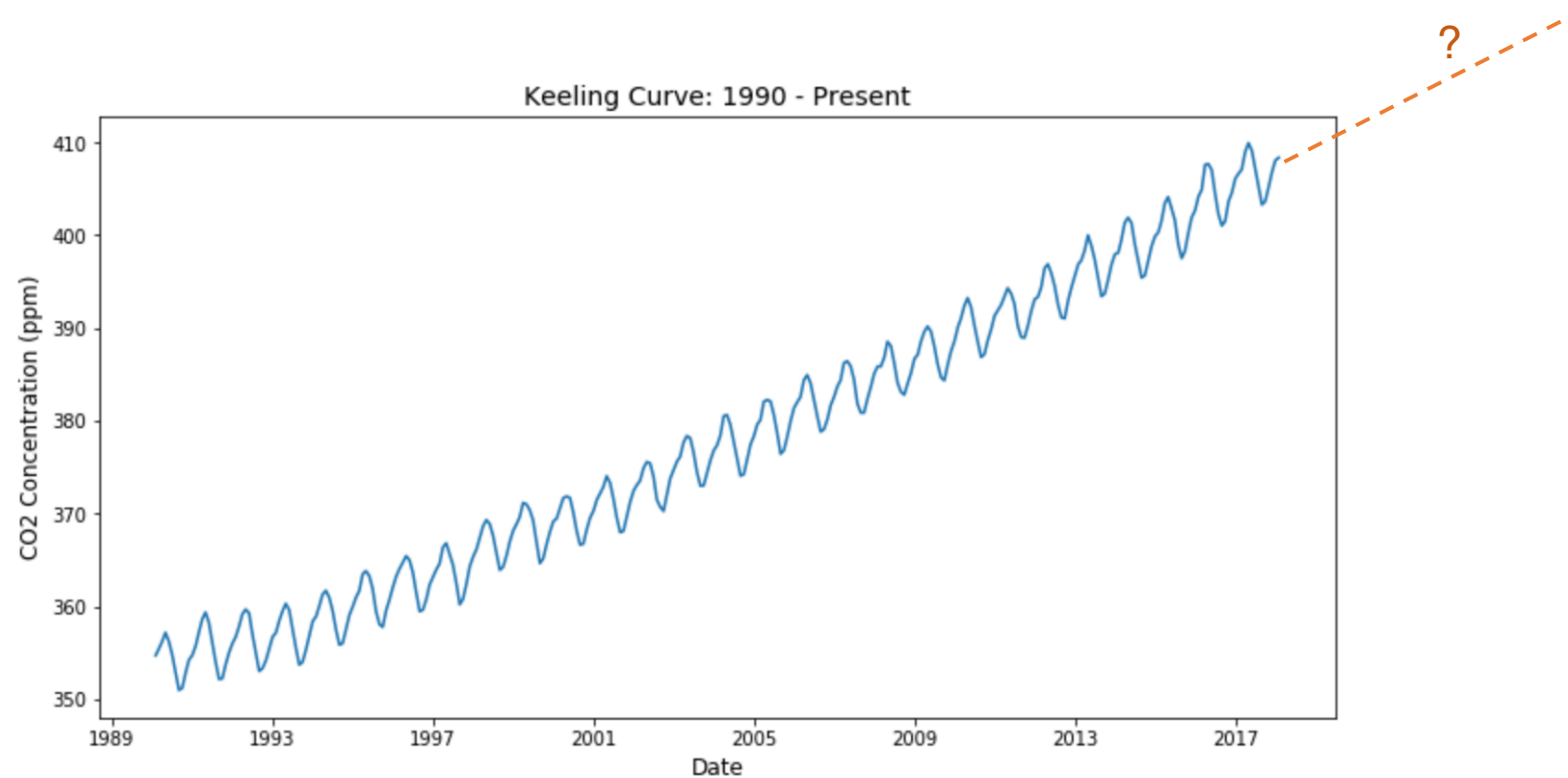
- $(p, d, q)$  for non-seasonal part
- $(P, D, Q)_s$  for seasonal part, where  $s$  is the length of season

# Implementing an ARIMA model

How to find the parameters  $(p, d, q)$  and  $(P, D, Q)_m$  that best fit the data?

- $m$  is known: just visualize the data to know season length
- $d$  and  $D$  are easy to determine:
  - Does your data need de-trending? If so,  $d = 1$  or  $2$ . If not,  $d = 0$ .
  - Does your data need seasonal differencing? If so,  $D = 1$  or  $2$ . If not,  $D = 0$ .
- $p$ ,  $P$ ,  $q$ , and  $Q$  can be estimated by looking at the autocorrelation and partial autocorrelation
- In practice, just do grid search over the  $(p, q)$  and  $(P, Q)$  values to find the parameters that optimize performance (usually minimize AIC).

# Implementing an ARIMA model



Lets fit an SARIMA model to the Keeling curve to forecast future values.



# Implementing an ARIMA model

Dataframe contains variable CO2, which we want to predict

```
df.head()
```

	CO2	seasonally adjusted [ppm]	fit [ppm]	seasonally adjusted fit [ppm]	CO2 filled [ppm]	seasonally adjusted filled [ppm]
date						
1990-02-01	354.70	354.02	354.37	353.69	354.70	354.02
1990-03-01	355.38	353.96	355.22	353.78	355.38	353.96
1990-04-01	356.20	353.62	356.48	353.88	356.20	353.62
1990-05-01	357.16	354.04	357.10	353.98	357.16	354.04
1990-06-01	356.23	353.87	356.43	354.10	356.23	353.87

# Implementing an ARIMA model

Fit SARIMA model using StatsModels library.

```
from statsmodels.tsa.statespace.sarimax import SARIMAX
```

```
model = SARIMAX(df['CO2'],  
                order=(1, 1, 1),  
                seasonal_order=(1, 1, 1, 12))
```

```
result = model.fit()  
print(result.summary().tables[1])
```

(p, d, q)

(P, D, Q, m)

	coef	std err	z	P> z	[ 0.025	0.975]
ar.L1	0.1748	0.114	1.534	0.125	-0.049	0.398
ma.L1	-0.5632	0.096	-5.879	0.000	-0.751	-0.375
ar.S.L12	-0.0157	0.073	-0.215	0.830	-0.159	0.128
ma.S.L12	-0.8441	0.050	-16.943	0.000	-0.942	-0.746
sigma2	0.1094	0.008	14.138	0.000	0.094	0.125

# Implementing an ARIMA model

Generating point forecasts and confidence intervals 100 time steps into the future.

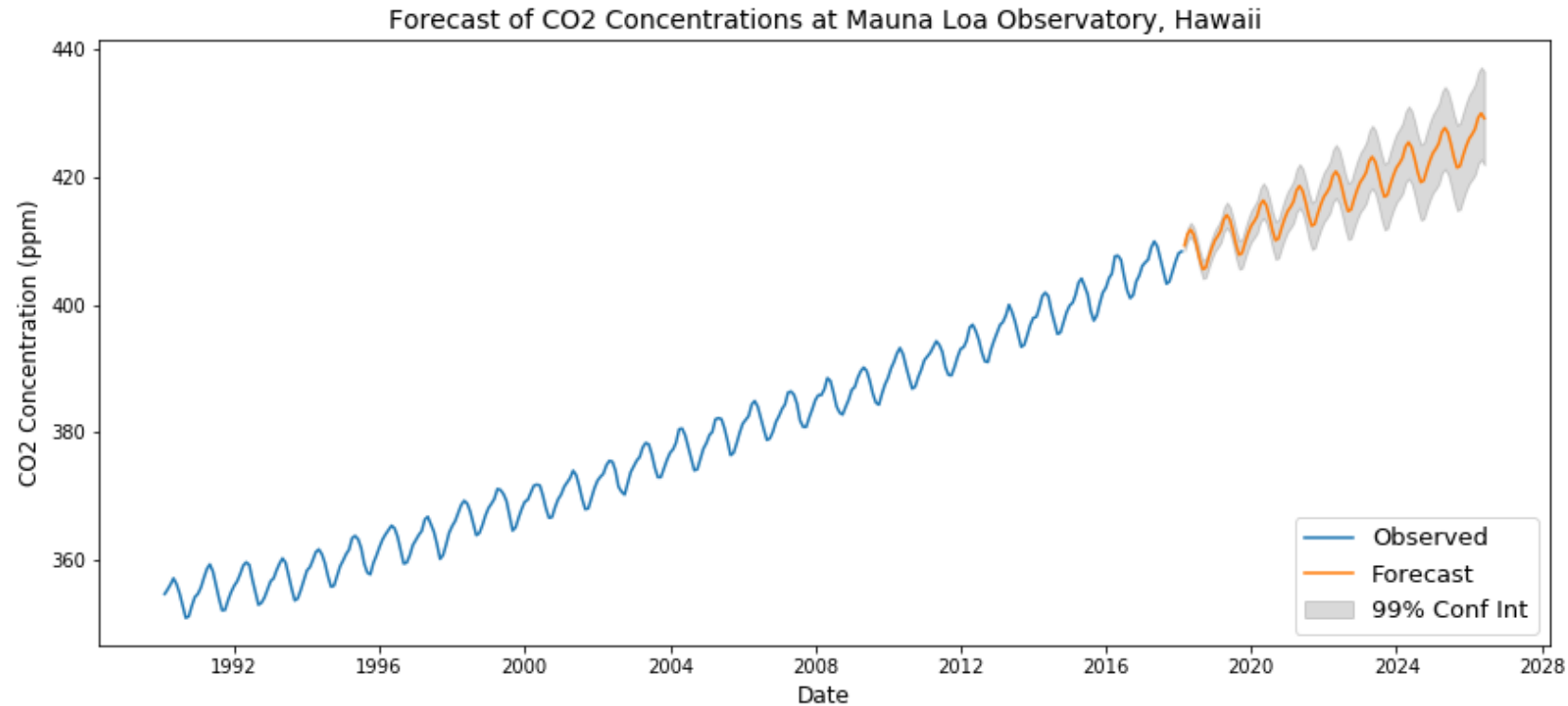
```
pred = result.get_forecast(steps=100)
pred_point = pred.predicted_mean
pred_ci = pred.conf_int(alpha=0.01)
```

Plot the forecast!

```
fig = plt.figure(figsize=(14,6))
plt.plot(df['CO2'], label='Observed')
plt.plot(pred_point, label='Forecast')
plt.fill_between(pred_ci.index, pred_ci.iloc[:, 0], pred_ci.iloc[:, 1],
                 color='k', alpha=.15, label='99% Conf Int')
plt.xlabel('Date', fontsize=12)
plt.ylabel('CO2 Concentration (ppm)', fontsize=12)
plt.title("Forecast of CO2 Concentrations at Mauna Loa Observatory, Hawaii",
          fontsize=14)
plt.legend(loc='lower right', fontsize=13)
```

# Implementing an ARIMA model

Result of the forecast



# Goals

After this lecture, you will be able to:

- Explain key properties of time series data
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- Understand the concept of **stationarity**
- Create and interpret **autocorrelation function** (acf) plots
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- Create your own time series forecast

Lots more to learn about time series

# References

Books (good for learning the theory)

- Forecasting: Principles and Practice by Hyndman, Athanasopoulos
- The Analysis of Time Series by Chris Chatfield
- Time Series Analysis and its Applications by Shumway, Stoffer

Articles (good for seeing examples in Python)

- [A Guide to Time Series Forecasting with ARIMA in Python](#)
- [Kaggle Time Series Notebook](#)