Experiment No: 5

1 Aim

1. To Simulate Continuous-time Sinusoidal Signals in Discrete-time.
2. To illustrate DSB-SC modulation and demodulation.
3. To illustrate FM modulation and demodulation.

2 Software Used

1. MATLAB.

3 Theory

3.1 Time Domain Simulation

Suppose you want to simulate the analog signal $c(t) = cos(\Omega_c t + \theta_c) = cos(2\pi F_c t + \theta_c)$ for over the time interval $0 \leq t \leq 2$ in Matlab where $F_c = 10Hz$ and $\theta_c = \pi/3$ radians. According to the sampling theorem, $c(t)$ can be represented by discrete-time samples provided the sample rate Fs is more than twice the largest frequency in $c(t)$. The highest frequency (and only frequency) in $c(t)$ is $F_c = 10Hz$. Therefore the sample frequency must be greater than $20Hz$ to avoid aliasing. If visualization or demonstration is the purpose of the simulation, it is a good idea to choose the sample rate 10 (or more) times faster than is required by the sampling theorem. On the other hand, if representation by discrete-time samples is all that is required, then sampling at $21Hz$ will do.

Let's suppose that we choose $F_s = 5000Hz$ as the sample rate. In Matlab, we will work with the discrete-time sequence $c_n = c(nT)$ instead of $c(t)$ where $T = 1/F_s$. Before generating the sequence c_n, we must generate the "time" vector $t_n = nT$ to represent time over the interval $[0; 3]$ seconds. The following Matlab commands will accomplish the construction of c_n.

$fs = 5000;$ % Sample frequency
$T = 1/fs;$ % Sample period
$t_n = [0 : T : 3];$ % Time vector with samples spaced T seconds appart
$fc = 10;$ % Frequency of the sinusoid
$theta = pi/3;$ % Phase of the sinusoid
$c_n = cos(2 * pi * fc * t_n + theta);$ % Construct the sinusoid
$plot(t_n, c_n);$ % Plot the sinusoid

The final command plots the sinusoid. Here is what the plot looks like.

You can zoom in and use the time axis to measure the period of the sinusoid. Invert the period measurement and verify that the frequency really is $10Hz$. You can also measure the frequency of the sinusoid by looking for a peak of the spectrum of the signal.

3.2 Viewing the Spectrum of the Sampled Signal

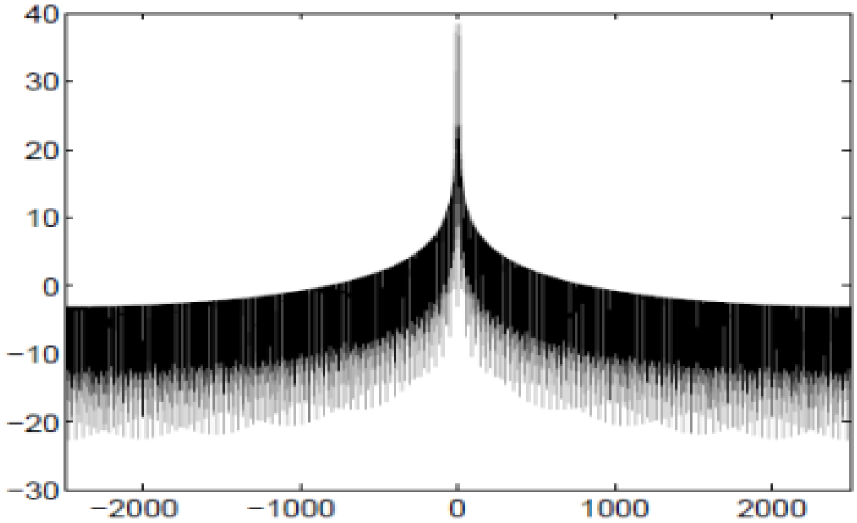
Matlab's fft function may be used to compute the discrete Fourier transform (DFT) of the sampled signal. Recall that discrete frequencies occupy the interval $[\frac{-1}{2}, \frac{1}{2}]$. The correspondence between discrete-time frequencies f and continuous-time frequencies F is given by the formula,

$$F = f * F_s$$

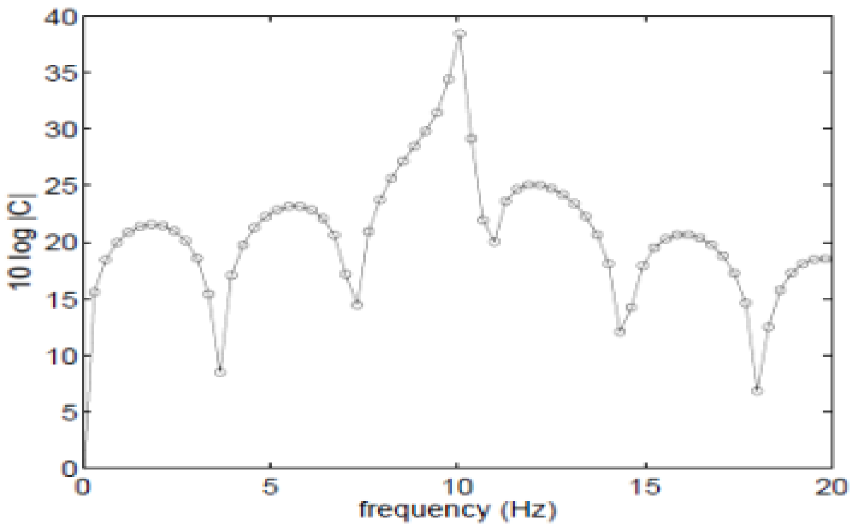
If we want frequency components in the sampled signal to appear in a plot of the spectrum at their true continuous time frequencies (assuming no aliasing), we have to generate a frequency vector that covers the range $[\frac{-1}{2}F_s, \frac{1}{2}F_s]$ with steps of F_s/N which is the sample frequency times the FFT bin width. Plotting the spectrum can be accomplished in Matlab using the following commands.

$N = 2^{14}$;   % FFT size
$f = ([0 : N-1]/N - 0.5) * fs$;   % The frequency vector for plotting
C = fftshift(fft(c_n, N));   % Compute the FFT and rearrange the output
$Plot(f, 10 * log10(abs(C)))$;   % Plot the magnitude of the spectrum on a log scale

Here is what the spectrum looks like. Note that the x-axis has the correct frequency labelling and scaling.



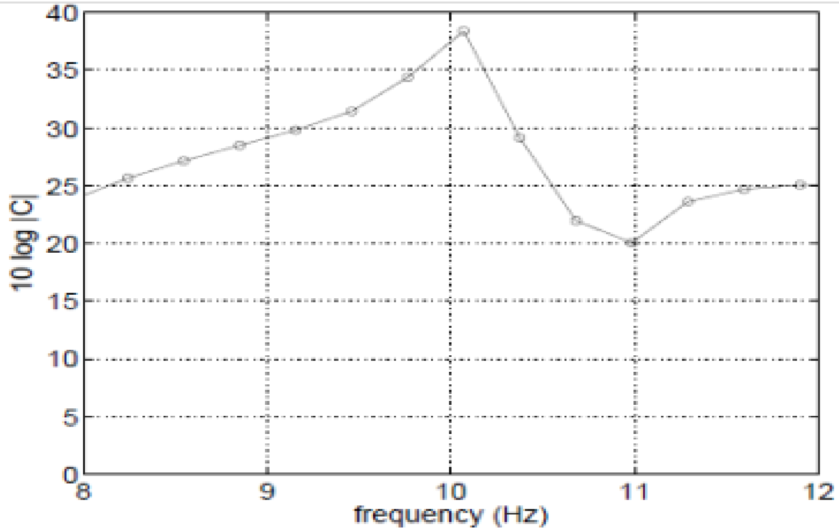
It is difficult to see the details of the spectrum around $10Hz$. The plot below shows the details around $10Hz$. Circles have been added around the data points which are connected by straight lines by the plot functions.

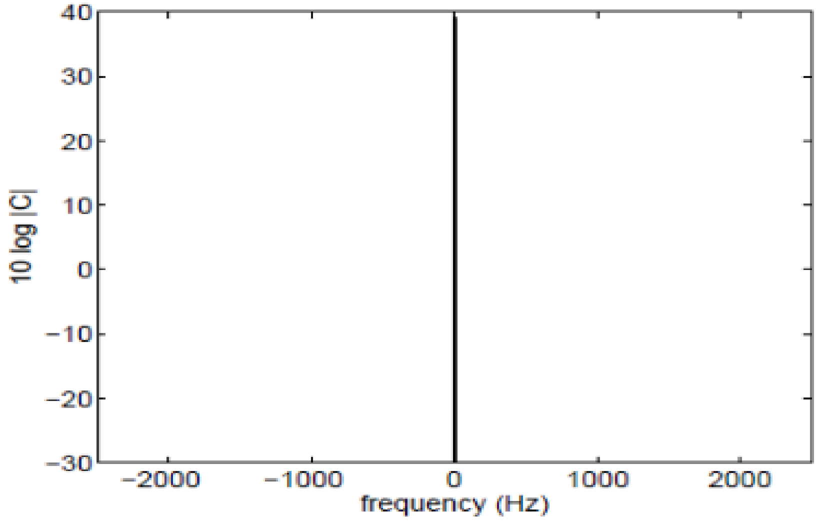


It appears that there is a peak in the spectrum at $10Hz$. However, zooming in to the interval $[8, 12]$ Hz as shown in the figure below reveals that the peak is not actually at $10Hz$. This is due to the "spectral leakage" phenomenon.

If the frequency of the continuous-time signal is changed to 9.765625 Hertz (which is a FFT bin center frequency mapped back to a continuous time frequency), then the plot looks like this.

Note that except for the two peaks (which are irresolvable because they are at low frequency, the spectrum is effectively zero (less than $-100dB$). Here is what the picture looks like zoomed to the frequency range 8 to $12Hz$.





3.3 Constructing Bin Center Discrete-time Sinusoids

Spectral leakage due to processing finite length data records smears the spectral lines of pure tones across several bins around the tonal frequency. Sinusoids with frequencies which are FFT bin centers are not smeared however. The FFT bin center frequencies are $0, \frac{1}{N}, \frac{2}{N}, \cdots, \frac{(N-1)}{N}$ cycles per sample. The "fundamental range" can be shifted by 1/2 to the following set of frequencies, $-\frac{1}{2}, -\frac{1}{2}+\frac{1}{N}, \cdots, -\frac{1}{2}+\frac{(N-1)}{N}$. Using the correspondence between continuous and discrete frequencies, the corresponding continuous time frequencies that are not smeared are given by

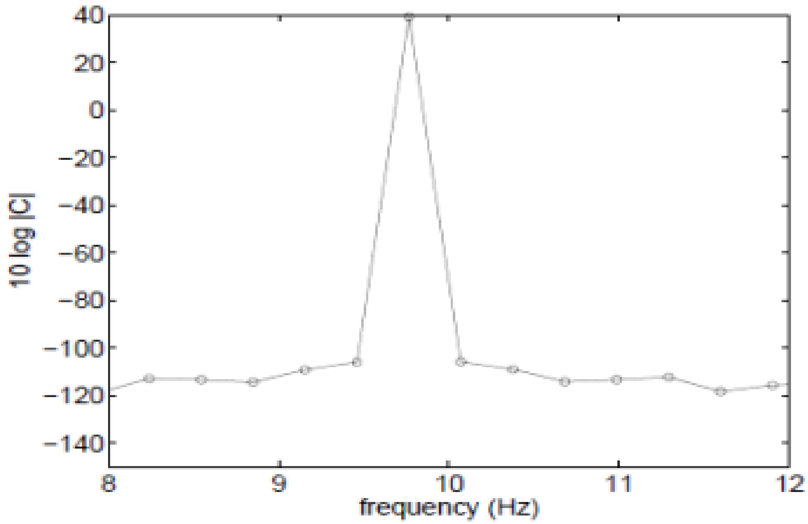
$$F_{bin\ center} = -\frac{1}{2}F_s + \frac{k}{2}F_s; \qquad k = 0, 1, \cdots, N-1.$$

4 Procedure

Exercise 1

1. What is the frequency (in Hertz) of the sinusoidal signal $x(t) = sin(19\pi^2 t)$? If this signal is sampled at a rate of 3 samples per second, what is the sampled signal $x(n/3)$ and what is the perceived frequency?

2. What is the fundamental period (in samples) of $(\frac{n}{3})$.



3. Simulate the continuous-time signal $c(t) = cos(2\pi F_c t + \theta_c)$? in Matlab using a sample rate $Fs = 15Hz$. Let $\theta_c = 2\pi/3$ radians. Let $N = 2^{12}$ be the size of the FFT that will be used for analyzing the sampled signal.
   (a) Choose Fc so that the sampled sinusoid $c_n = c(nT)$ has a discrete time frequency that is an FFT bin center frequency ($F_c = 0$ is not allowed). Remember that this is a demonstration. So, we want F_s to be much greater than F_c. What value of F_c did you choose? What FFT bin center frequency did you choose?
   (b) How many seconds of data should you generate in Matlab so that you will have exactly N samples of the sinusoid?
   (c) Generate the sinusoid in Matlab and plot it. The x-axis should be properly scaled so that the units are in seconds.
   (d) Using the fft function, compute the spectrum and plot it. The x-axis in this plot should be properly scaled so that the units are in Hertz. Does your plot appear as predicted?

Exercise 2

DSB-SC modulation and demodulation

The message signal $m(t)$ in the interval $[-0.04, 0.04]$ is given as

$$m(t) = \triangle\left(\frac{t+0.01}{0.01}\right) - \triangle\left(\frac{t-0.01}{0.01}\right); \tag{1}$$

where $\triangle\left(\frac{t}{\tau}\right)$ is a triangular function of width τ , the carrier signal $c(t)$ is given as

$$c(t) = cos(2\pi f_c\ t); \tag{2}$$

where f_c is the carrier frequnecy and the DSB-SC signal is defined as

$$\phi(t) = m(t)\ cos(2\pi f_c\ t) \tag{3}$$

The demodulated signal $e(t) = \phi(t) \times 2cos(2\pi f_c\ t + \theta)$, where $2cos(2\pi f_c\ t + \theta)$ is a local carrier generated at the demodulator. $\hat{m}(t)$ denotes the recovered message signal after low-pass filtering. Coherent demodulation is also implemented with a finite impulse response (FIR) low-pass filter of order 40. The low-pass filter at the demodulator has bandwidth of $150Hz$.

Write a simple MATLAB routine to illustrate DSB-SC modulation and demodulation schemes for $f_c = 300Hz$ and an arbitrary θ. Plot $m(t)$, $\phi(t)$, $e(t)$ and $\hat{m}(t)$ in time and frequency domains. In the ideal coherent demodulation we assume that the phase of the local carrier is equal to the phase of the carrier (i.e. $\theta = 0$). If that is not case- i.e. if there exists a phase shift θ between the local carrier and the carrier- how would the demodulation process change?

Exercise 3

FM modulation and demodulation

Once again, we use the same message signal $m(t)$ and carrier signal $c(t)$. The FM signal is defined as

$$\phi_{FM}(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right); \tag{4}$$

where k_f is the FM coefficient. We first integrate the message signal and then use (4) to find $\phi_{FM}(t)$. To demodulate the FM signal, a differentiator is first applied to convert the frequency-modulated signal an amplitude- and frequency modulated signal (i.e. $d(t) = \frac{d\phi_{FM}(t)}{dt}$. By applying envelope detection (non-coherent AM demodulation), we recover the message signal $\hat{m}(t)$.

Write a simple MATLAB routine to illustrate FM modulation and demodulation schemes for $f_c = 300Hz$ and $k_f = 80$. Plot $m(t)$, $\phi_{FM}(t)$, $d(t)$ and $\hat{m}(t)$ in time and frequency domains.

5 Observation

Write/ Plot Your Own With Observation Table (If Required).

6 Analysis of Results

Write Your own.

7 Conclusions

Write Your Own.

Precautions

Observation should be taken properly.