Experiment No.: 04

1 Aim

1. Implementation of discrete Fourier transform (DFT) and inverse DFT (IDFT) algorithm.
2. Implementation of autocorrelation and cross correlation algorithm.

2 Software Used

1. MATLAB

3 Theory

Discrete Fourier transform (DFT): The sequence of N complex numbers $x_0, x_1, x_2, \cdots, x_{N-1}$ is transformed into an N-periodic sequence of complex numbers:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi\, k\, n/N}; \qquad k \in \mathbb{Z} \tag{1}$$

Inverse discrete Fourier transform (IDFT) is given as

$$X[n] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{j2\pi\, k\, n/N}; \qquad n \in \mathbb{Z} \tag{2}$$

In signals and systems analysis, the relationships between signals often indicate whether the physical phenomena which produced the signals are related or whether one signal is a modified version of the other. The relationship between two signals indicates whether one depends on the other, both depend on some common phenomenon, or they are independent. Correlation function indicates how correlated two signals are as a function of how much one of them is shifted in time.

The autocorrelation function (ACF) of a random signal describes the general dependence of the values of the samples at one time on the values of the samples at another time. Consider a random process $x(t)$ (i.e. continuous-time), its autocorrelation function is computed as

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)dt; \tag{3}$$

where T is the period of observation. $R_{xx}(\tau)$ is always real-valued and an even function. For sampled signal, the ACF is defined as

$$R_{xx}[m] = \frac{1}{N} \sum_{n=1}^{N-m+1} x[n]x[n+m-1]; \qquad m = 1, 2, ..., N+1 \tag{4}$$

where N is the number of samples. The cross-correlation function (CCF) measures the dependence of the values of one signal on another signal. For two wide sense stationary (WSS) processes $x(t)$ and $y(t)$, the CCF is defined as

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t+\tau)dt; \tag{5}$$

where T is the period of observation.
For sampled signal, the CCF is defined as

$$R_{xx}[m] = \frac{1}{N}\sum_{n=1}^{N-m+1} x[n]y[n+m-1]; \qquad m = 1, 2, ..., N+1 \tag{6}$$

where N is the number of samples. ACF is simply a special case of the CCF.

4 Procedure

1. Write a simple MATLAB program to perform the DFT. Use your program to determine and plot the DFT of the following signal:

   $$x(t) = \cos(2000\pi t) + \cos(800\pi t)$$

   Let $F_s = 8000Hz$ and $N = 128$ and plot the magnitude and phase of $X(k)$.

   Write a simple MATLAB routine to perform the inverse DFT.

2. Use $x_1[n] = \{1, 1, 1, 1\}$, and $N = 4$, determine the DFT. Plot the Magnitude and phase spectra of the $x_1[n]$. Use the IDFT to transfer the DFT results (i.e. $X[k]$ sequence) to its original sequence.

3. Take a speech signal and plot the magnitude and phase spectra.

4. Write a simple MATLAB program to perform the CCF. Use your routine to plot the crosscorrelation of the following signal:

   $$\begin{aligned} x(t) &= \sin(2\,\pi\,f\,t), \\ y(t) &= x(t) + w(t), \end{aligned} \tag{7}$$

   where $w(t)$ is a zero-mean, unit variance of the Gaussian random process. Sampled version is written as

   $$\begin{aligned} x[n] &= \sin\left(\frac{2\,\pi\,f\,n}{F_s}\right), \\ y[n] &= x[n] + w[n], \end{aligned} \tag{8}$$

   Let f = 1Hz, F_s = 200Hz, N = 1024 and $w[n]$ is generated by $randn(1, N)$ MATLAB function.

5 Observation

Write/ Plot Your Own With Observation Table (If Required).

6 Analysis of Results

Write Your own.

7 Conclusions

Write Your Own.

Precautions

Observation should be taken properly.