A Two-Phase Evasive Strategy for a Pursuit-Evasion Problem Involving Two Non-Holonomic Agents with Incomplete Information

Suryadeep Nath[1] and Debasish Ghose[2]

Abstract— **In this paper, we consider the problem of pursuit-evasion between two non-holonomic agents. To make the problem more realistic, we assume that the pursuer and evader have incomplete information regarding each other's motion. We propose a novel, two-phase, evasive strategy against a higher speed pursuer, based on worst-case scenario planning and proximity-based maneuver. In the first phase, the evader assumes the pursuer to have zero turn radius and executes a best-response strategy, which is analytically shown to be movement along a tangent to the turn circle of the evader. In the second phase, when the pursuer gets close to the evader, the latter switches to movement along high-curvature paths to side-step the former. The effectiveness of the two phases is shown through simulations.**

Index Terms— **Pursuit, Evasion, Non-holonomic, Evasive Strategy, Differential game, Proportional Navigation**

I. INTRODUCTION

Scenarios of pursuit-evasion are very common in defense applications such as reconnaissance missions where a spy robot is made to venture into a hostile territory and has to circumvent attacks from enemy robots and return to safety. Often, such robots are non-holonomic systems like fixed-wing UAVs or differential drive ground vehicles. Therefore, in this work, we address a pursuit-evasion problem involving non-holonomic systems. We focus on the evader and design evasive strategies that will help it to escape the pursuers.

Our work is related to differential game theory [1]. The homicidal chauffeur game [2], [3], first proposed by Isaacs, is a classic pursuit-evasion game which motivates our work. Like the homicidal chauffeur game, the pursuer, in our problem, is of higher speed and has curvature constraints. We additionally consider the evader to have curvature constraints as well. Isaacs' two cutters and fugitive ship problem [4], [5] have given rise to many works related to pursuit-evasion games involving multiple pursuers against multiple evaders [6], [7], however, the pursuers and evaders are assumed to be holonomic agents, capable of instantaneously changing direction of heading. Geometric methods have been used to solve these problems [8], where, the concept of region of dominance is used to decide the strategies of the players. In most of these works, the pursuers have a speed advantage, similar to our problem.

Pursuit-evasion involving Dubins' vehicle [9], [10] or a game of two cars [1], resembles the problem addressed in this paper, where, similar to our problem, the cars are non-holonomic agents and have curvature constraints. The necessary and sufficient conditions for escape and capture was given in [11], [12]. The Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation is used to solve for the game of kind and game of degree with time of capture being the payoff of the game [13]–[15]. Recently, in [16]–[18], geometric approaches were used to obtain feedback strategies that can be implemented at negligible computation costs.

In most of the literature around pursuit-evasion games, differential games approach is adopted and the strategies are obtained by deciding on a payoff with the pursuer and the evader having conflicting objectives. Differential games where the control strategies are determined based on position of the players, with the control belonging to some compact sets are known as positional differential games [19]. Our approach has similarities with such games in the sense that the evasive strategy proposed in our work is determined based on relative position of the evader and the pursuer and the control input is chosen from a compact set. A major problem of formulating the pursuit-evasion problem as a differential game is its dependence on complete information. In a differential game setting, it is assumed that the evader's and the pursuers' kinematic properties are known to both parties. The optimal strategies are obtained based on such assumptions, which, may not always be practical. In our approach, we remove the dependence on complete information.

The key contribution of the paper is the two-phase evasive strategy against a higher speed pursuer. In the first phase, the evader assumes the worst-case that the pursuer has zero turn radius (as in [20]), and obtains its best response strategy. Dynamic programming is used to solve for the optimal strategy which is shown to be movement along the optimal tangent to one of the evader turn circles. The second phase is a proximity based maneuver, inspired from [21], where, once the pursuer gets in the proximity of the evader, the latter starts moving along high curvature paths. The evasive strategy proposed can be applied irrespective of the turning capabilities of the pursuer.

The rest of the paper is organised as follows. Section II describes the problem. Section III discusses the two-phase evasive strategy. Section IV shows the evasive strategy in action through simulations. Section V concludes the paper with a discussion on possible directions to extend the work.

II. PROBLEM DESCRIPTION

Fig. 1 presents the engagement geometry of a pursuer (P) chasing an evader (E). The evader (pursuer) is modelled as an

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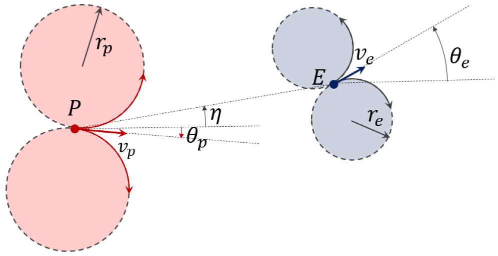


Fig. 1. Engagement geometry

unicycle non-holonomic agent having constant speed v_e (v_p) and turn radius constraint of r_e (r_p). The evader is considered to be of slower speed than the pursuer, $v_e < v_p$.

Definition 1. *A circle of minimum feasible turn radius r_e (r_p) is known as a minimum turn circle of the evader (pursuer).*

At every instant of time, the evader (pursuer) will have a pair of minimum turn circles. One for the anti-clockwise turn and another for clockwise turn.

The state of the evader and pursuer is given by

$$\mathbf{E}(t) = \begin{bmatrix} x_e(t) & y_e(t) & \theta_e(t) \end{bmatrix}^T$$
$$\mathbf{P}(t) = \begin{bmatrix} x_p(t) & y_p(t) & \theta_p(t) \end{bmatrix}^T$$

where $\mathbf{e}(t) = \begin{bmatrix} x_e(t) & y_e(t) \end{bmatrix}^T$ $\left(\mathbf{p}(t) = \begin{bmatrix} x_p(t) & y_p(t) \end{bmatrix}^T\right)$ is the position of the evader (pursuer) and θ_e (θ_p) is the heading of the evader (pursuer). The equations of motion for $i \in \{e, p\}$ are

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{y}_i(t) \\ \dot{\theta}_i(t) \end{bmatrix} = \begin{bmatrix} v_i \cos\theta_i(t) \\ v_i \sin\theta_i(t) \\ \omega_i(t) \end{bmatrix} \tag{1}$$

where ω_e (ω_p) is the turn rate and control input of the evader (pursuer). The control input of the evader (pursuer) is constrained to the compact set $|\omega_e(t)| \le \frac{v_e}{r_e}$ $\left(|\omega_p(t)| \le \frac{v_p}{r_p}\right)$. The maximum attainable evader turn rate is $\omega_m = \frac{v_e}{r_e}$.

Problem 1. *Find a feedback evasive control strategy ω_e that will help the evader escape, or delay capture, without explicit knowledge of pursuer's turn capabilities.*

The information structure is chosen to resemble a realistic problem as follows: (i) the positions $\mathbf{e}(t)$ and $\mathbf{p}(t)$ and speeds v_e and v_p are known to both evader and pursuer, and (ii) the evader does not know the minimum turn radii r_p of the pursuer and the pursuer is not aware of the minimum turn radius r_e of the evader.

A. Pursuer strategy

Under the given information structure, the pursuer is not aware of the evader's turning capabilities and therefore, is assumed to use Proportional Navigation (PN) guidance law against the evader as PN is shown to be an optimal pursuit strategy in the stochastic version of the game of two cars [22]. PN is optimal in the linearized framework for non-maneuvering targets and also effective against slower speed, weaving targets (like the evader in our case) [23]. Augmented PN [24] has high capturability for maneuvering targets but it requires knowing the evader lateral acceleration, which is not available according to the information structure we have chosen. Thus, we choose pure PN as a pursuit strategy.

The pursuer's control ω_p is given by

$$\omega_p = N\dot{\eta} \tag{2}$$

where η is the line-of-sight orientation and N is the navigation gain.

III. Two-phase evasive strategy

The evader executes a two-phased strategy based on relative position of the pursuer, with r_c being the critical phase-switching distance. In this section we will describe the two phases of the evasive strategy.

A. Worst-case scenario planning (WCSP) phase

WCSP strategy. *When $\|\mathbf{e}(t) - \mathbf{p}(t)\| \ge r_c$, the evader assumes that the pursuer has complete knowledge of evader's kinematic properties and has zero turn radius, thus capable of making instantaneous turns. Based on this worst-case assumption, the evader computes its best response strategy that will maximize the time of capture.*

The Hamiltonian analysis [25] is employed to find the best response strategy.

1) Hamiltonian analysis: Since, the evader perceives the pursuer to be holonomic, the pursuer's motion is given by

$$\dot{\mathbf{P}}(t) = \begin{bmatrix} v_p \cos\theta_p(t) & v_p \sin\theta_p(t) \end{bmatrix}^T \tag{3}$$

with θ_p being the perceived pursuer control. The Hamiltonian for the given state of the system is

$$H = 1 + \lambda_{xe} v_e \cos\theta_e + \lambda_{ye} v_e \sin\theta_e + \lambda_{\theta e}\omega_e + \\ \lambda_{xp} v_p \cos\theta_p + \lambda_{yp} v_p \sin\theta_p \tag{4}$$

where, $\begin{bmatrix} \lambda_{xe} & \lambda_{ye} & \lambda_{\theta_e} \end{bmatrix}^T$ denotes the adjoint vector corresponding to the evader and $\begin{bmatrix} \lambda_{xp} & \lambda_{yp} \end{bmatrix}^T$ denotes the adjoint vector corresponding to the pursuer. We rewrite Eq. 4 by introducing some new variables

$$\lambda_i = \sqrt{\lambda_{xi}^2 + \lambda_{yi}^2} \quad \tan\phi_i = \lambda_{yi}/\lambda_{xi} \tag{5}$$

where $i \in \{e, p\}$. After substitution we get,

$$H = 1 + \lambda_e v_e \cos(\theta_e - \phi_e) + \lambda_{\theta e}\omega_e + \lambda_p v_p \cos(\theta_p - \phi_p) \tag{6}$$

The HJBI equation must hold along the optimal path

$$\min_{\theta_p} \max_{\omega_e} H = 0 \tag{7}$$

The adjoint system is given by

$$\dot{\lambda}_{xe} = \dot{\lambda}_{ye} = \dot{\lambda}_{xp} = \dot{\lambda}_{yp} = 0 \tag{8}$$
$$\dot{\lambda}_{\theta e} = v_e \lambda_e \sin(\theta_e - \phi_e) \tag{9}$$

The final time constraint for point capture of the evader at time T is given by

$$\Phi(T) = \mu^T \begin{bmatrix} x_e(T) - x_p(T) \\ y_e(T) - y_p(T) \end{bmatrix} = 0 \tag{10}$$

where, $\mu = \begin{bmatrix}\mu_x & \mu_y\end{bmatrix}^T$ are the undetermined constants. By Hamiltonian analysis, we have

$$\begin{aligned}
&\lambda_{xe}(T) = \partial\Phi/\partial x_e = \mu_x \quad \lambda_{ye}(T) = \partial\Phi/\partial y_e = \mu_y \\
&\lambda_{\theta e}(T) = \partial\Phi/\partial\theta_e = 0 \quad \lambda_{xp}(T) = \partial\Phi/\partial x_p = -\mu_x \\
&\lambda_{yp}(T) = \partial\Phi/\partial y_p = -\mu_y
\end{aligned} \tag{11}$$

Thus, from Eq. 8 and 11 we can conclude that $\lambda_{xe}, \lambda_{ye}, \lambda_{xp}, \lambda_{yp}$ are constants and from Eq. 5

$$\lambda_e = \lambda_p = \lambda = \sqrt{\mu_x^2 + \mu_y^2} \tag{12}$$

$$\phi_e = \tan^{-1}\frac{\mu_x}{\mu_y} = \phi \quad \phi_p = \pi + \tan^{-1}\frac{\mu_x}{\mu_y} = \pi + \phi \tag{13}$$

which are also constants.

Lemma 1. *The pursuer moves in a straight line ($\bar{\theta}_p =$ constant) to capture the evader in minimum time.*

Proof: Substituting Eq. 13 in Eq. 6 we get,

$$H = 1 + \lambda_e v_e \cos(\theta_e - \phi) + \lambda_{\theta e}\omega_e - \lambda_p v_p \cos(\theta_p - \phi) \tag{14}$$

The pursuer seeks to capture the evader in minimum time. To minimize H, the optimal strategy of the pursuer must be $\bar{\theta}_p = \phi =$ constant. Therefore, the pursuer must move along a straight line oriented at an angle ϕ. □

The evader assumes the pursuer to have complete knowledge of the evader's kinematics and therefore, the perceived straight line motion of the pursuer is directed to a point on the evader's trajectory to capture it in minimum time.

Lemma 2. *The evader's best response strategy consists of movement along minimum turn circles and/or straight line motions.*

Proof: The evader seeks to maximize the time of capture. To maximize the Hamiltonian H, the optimal strategy of the evader will involve maximizing the term $S = \lambda_e v_e \cos(\theta_e - \phi) + \lambda_{\theta e}\omega_e$ from Eq. 14. When $|\lambda_{\theta e}| \neq 0$, then $\bar{\omega}_e = sign(\lambda_{\theta e})\omega_m$. When $|\lambda_{\theta e}| = 0$, the evader heading should be such that $\bar{\theta}_e = \phi =$ constant. And in this case $\bar{\omega}_e = 0$.

Therefore, the evader's optimal motion consists of motion along minimum turn circles ($\bar{\omega}_e = \pm\omega_m$) and/or along a straight line ($\bar{\omega}_e = 0$ and $\bar{\theta}_e = \phi =$ constant). □

Lemma 3. *Once $\lambda_{\theta e} = 0$, there is no change in its value and it corresponds to a straight line motion for the evader.*

Proof: When $\lambda_{\theta e} = 0$, then $\bar{\omega}_e = 0$ and the optimal strategy would demand that $\bar{\theta}_e = \phi$ to maximize the Hamiltonian in Eq. 14. In this case, all time-derivatives of $\dot{\lambda}_{\theta e}$ will be zero from Eq. 9. Therefore, once $\lambda_{\theta e}$ attains zero value, there is no change in its value and since $\bar{\theta}_e = \phi$, it implies that the evader moves in a straight line motion. □

Lemma 4. *The adjoint variable $\lambda_{\theta e}$ will attain zero value if $\|\mathbf{e}(t_0) - \mathbf{p}(t_0)\| \geq 2r_e + 2\pi r_e(v_p/v_e)$, where t_0 denotes the start of the WCSP phase.*

Proof: From Eq. 12, we know that $\lambda_e = \lambda_p$. Consider the Hamiltonian H in Eq. 14. When $\lambda_{\theta e} = 0$, the optimal strategies involve $\theta_e = \theta_p = \phi$. To satisfy the HJBI equation in Eq. 7,

$$\lambda_e = \lambda_p = \frac{1}{v_p - v_e} > 0 \tag{15}$$

Let Δ_e denote the angle $\theta_e - \phi$. Using Eq. 9 and Eq. 15, we can construct the following table

	$\Delta_e \in (0,\pi]$	$\Delta_e \in [-\pi,0)$
$\lambda_{\theta e} > 0, \omega_e = +\omega_m$	$\lvert\lambda_{\theta e}\rvert$ increases	$\lvert\lambda_{\theta e}\rvert$ decreases
$\lambda_{\theta e} < 0, \omega_e = -\omega_m$	$\lvert\lambda_{\theta e}\rvert$ decreases	$\lvert\lambda_{\theta e}\rvert$ increases

If the pursuit-evasion engagement starts with $\lambda_{\theta e} > 0$ and $\Delta_e \in (0,\pi]$, then $|\lambda_{\theta e}|$ increases. However, since $\omega_e = +\omega_m$, Δ_e will move into the region $\Delta_e \in [-\pi, 0)$, where $|\lambda_{\theta e}|$ decreases. When $\Delta_e = 0$, $\dot{\lambda}_{\theta e} = 0$ (Eq. 9) and from Eq. 14 and Eq. 15, we get $\lambda_{\theta e} = 0$ to satisfy the HJBI equation. Again, if $\lambda_{\theta e} < 0$ and $\Delta_e \in [-\pi, 0)$, then $|\lambda_{\theta e}|$ increases. But, as $\omega_e = -\omega_m$, Δ_e will move into the region $\Delta_e \in (0,\pi]$, where $|\lambda_{\theta e}|$ decreases. By similar reasoning it can be shown that $\lambda_{\theta e}$ will go to zero when $\Delta_e = 0$. From Lemma 3, we can say that $\lambda_{\theta e}$ will remain zero and that implies a straight line motion for the evader.

The state $\lambda_{\theta e} = 0$ is reached when Δ_e reaches zero. It happens if the pursuer's straight line trajectory is unable to reach the minimum turn circle of the evader before the evader heading reaches ϕ. This is guaranteed when the initial (at the start of the *WCSP* phase) distance between the evader and the pursuer is more than $2r_e + 2\pi r_e(v_p/v_e)$ [16]. Thus, $|\lambda_{\theta e}|$ attains zero when $\|\mathbf{e}(t_0) - \mathbf{p}(t_0)\| \geq 2r_e + 2\pi r_e(v_p/v_e)$. □

Theorem 1. *The evader's optimal strategy will consist of turning on a minimum turn circle followed by a straight line motion along a tangent to the circle.*

Proof: From Lemma 2 we established that the evader's motion will consist of turns along the minimum turn circles and/or motion along a straight line. Now from Lemma 3 and 4 we see that $\lambda_{\theta e}$ does not change sign. When $\lambda_{\theta e} > 0$ ($\lambda_{\theta e} < 0$), it implies $\omega_e = \omega_m$ ($\omega_e = -\omega_m$), which in turn implies a movement along the counterclockwise (clockwise) minimum turn circle. When $\lambda_{\theta e}$ reaches zero, it remains at zero value, implying a straight line motion as discussed in Lemma 3. Since, the motion is continuous, the straight line must be a tangent to one of the minimum turn circles depending on sign of $\lambda_{\theta e}$. Thus, the optimal evader strategy is to move along a minimum turn circle and then continue along the tangent to the circle. □

Theorem 2. *The pursuer's motion will be along a tangent to one of the minimum turn circles of the evader.*

Proof: We have established in Lemma 1 that the pursuer moves along a straight line at an angle $\theta_p = \phi$. Using Theorem 1, we established the evader will move along a minimum turn circle and then along the tangent to the circle at an angle $\theta_e = \phi$. Therefore, the pursuer moves parallel to the evader as the latter starts moving on a tangent. The straight line motion of the evader and the pursuer must be co-linear to ensure capture. Thus, the pursuer must follow the tangent to one of the evader's minimum turn circle. □

2) Selection of optimum valid tangent: From the previous section, we derived that the evader's optimal strategy would be to move along a minimum turn circle followed by motion along a tangent to the circle and the pursuer's optimal strategy would be to move straight along the same tangent. Therefore, the problem boils down to finding the optimal tangent. For that we introduce the concept of a valid tangent.

Definition 2. *A tangent, directed from the pursuer to the evader's minimum turn circles, such that the direction of motion along the circle is parallel the direction of the tangent, is a valid tangent.*

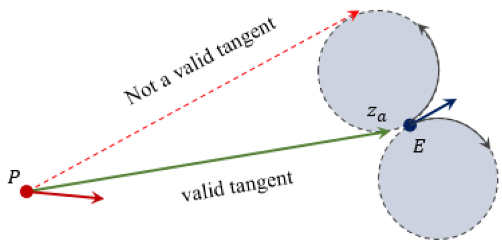


Fig. 2. Valid tangent selection

For each of the turn circles of the evader there is only one valid tangent out of the two possible tangents. Fig. 2 shows the valid tangent for the anti-clockwise turn circle. So there are a total of two valid tangents considering both the turn circles. Algorithm 1 gives the steps for selecting the optimal tangent. The evader's optimal strategy under the assumption that the pursuer is holonomic would be to move along the optimal tangent.

Algorithm 1 Selecting the optimal tangent

1) Find point of valid tangency z_a, from P to the anti-clockwise minimum turn circle of E. Let d_a be the distance between P and z_a.
2) Find time taken, t_a, by evader to reach from E to z_a by moving along the anti-clockwise minimum turn circle.
3) Find total capture time,
$$T_a = t_a + \frac{d_a - v_p t_a}{v_p - v_e}$$
.
4) In similar way, compute capture time along tangent to clockwise minimum turn circle, T_c by following steps similar to 1, 2 and 3.
5) The valid tangent corresponding to a higher time of capture $(\max\{T_a, T_c\})$ is the *optimal tangent*.

If the evader were to follow only the *WCSP* phase, against a pursuer using PN guidance, the trajectories would be as shown in Fig. 3. It consists of the evader and pursuer taking turn maneuvers and then moving co-linearly in a tail-chase engagement. Since, the evader is slower than the pursuer, capture will be inevitable. This situation arises because the evader makes a conservative assumption that the pursuer is holonomic.

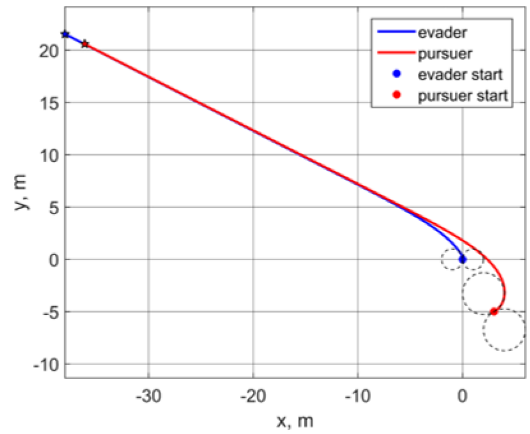


Fig. 3. Pursuit-evasion trajectory with evader following only *WCSP* and pursuer following PN guidance law with $N = 3$. The parameters are $v_e = 0.8$ m/s, $v_p = 1$ m/s, $r_e = 1$ m and $r_p = 2$ m. The initial conditions are $x_e = 0$, $y_e = 0$, $\theta_e = \pi/2$, $x_p = 2$, $y_e = -5$ and $\theta_p = \pi/6$.

B. Proximity-based maneuver (PBM) phase

In the *PBM* phase, the evader undergoes motion along high-curvature paths when it is within the critical phase-switching distance from the pursuer. A pre-condition for executing this phase is that the evader qualitatively knows the pursuer to have turn radius constraint as well. The inspiration behind this phase is that the pursuer will have difficulty in moving along the high-curvature paths taken by the evader and as a result, separation between the pursuer and the evader will be created and eventually, delay capture of the evader. The *PBM* phase is highly effective if the pursuer is less agile than the evader ($r_e < r_i$). This is conceptually similar to, although not the same as, the side-stepping maneuver by the evader in the homicidal chauffeur game, in case of a closed barrier [1].

A Dubins' vehicle is one that can move forward with constant speed and has turn curvature constraints [9]. The evader can, therefore, be assumed to be a Dubins' vehicle.

PBM strategy. *When $\|\mathbf{e}(t) - \mathbf{p}(t)\| < r_c$, the evader's feedback strategy is to reach the pursuer's instantaneous location in shortest time for a Dubins' vehicle.*

The paths taken to reach various points in shortest time by a Dubins' vehicle are described next.

a) *Points lying outside the minimum turn circles:* Move along one of the minimum turn circles and then along the tangent to that circle. Such paths are called curved-straight (*CS*) trajectories.
b) *Points lying inside one of the minimum turn circles:* Move away along a minimum turn circles and then turn along the opposite minimum turn circle. Such paths are called curved-curved (*CC*) trajectories.

The *CS* and *CC* trajectories, shown in Fig. 4, are the high-curvature paths taken by the evader. The evader's feedback strategy is described in Algorithm 2.

The *PBM* phase will not be very effective if the pursuer is more agile, that is, it can take sharper turns than the evader ($r_e \geq r_p$). However, the two-phased strategy proposed in

Algorithm 2 Feedback strategy to reach a point in shortest time by a Dubins' vehicle

1) If the point P lies outside or on the minimum turn circle.
   a) Draw a tangent from P to each of the two turn circles, directed from the turn circle to P.
   b) Select the tangents whose direction matches that of the turn direction. Let the points of tangency be p_a and p_c corresponding to anti-clockwise and clockwise turn circles.
   c) Compute the total time T_{Da} taken to reach P by moving along the anti-clockwise turn circle. Let t_{1a} be time taken to turn and reach p_a on the turn circle, and t_{2a} be time to follow along the tangent to reach from p_a to P.
   $$T_{Da} = t_{1a} + t_{2a}$$
   Similarly, compute the total time T_{Dc} taken to reach P by moving along the clockwise turn circle.
   d) Move along the turn circle that corresponds to the minimum between T_{Da} and T_{Dc}.
2) If the point P lies inside one of the minimum turn circles.
   a) Turn along the opposite minimum turn circle as long as the point P lies inside the minimum turn circle.
   b) When the point P moves to the boundary of the minimum turn circle in the relative frame of reference, turn along that minimum turn circle.

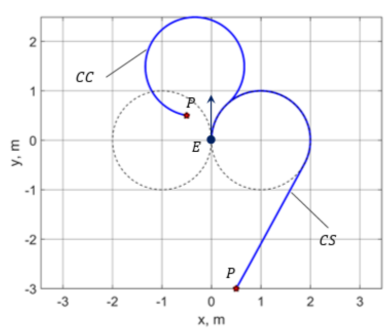


Fig. 4. Shortest time trajectories for the *PBM* phase

Algorithm 3, does not rely on knowing the pursuer's turning capabilities and therefore, can be applied irrespective of the pursuer's agility.

IV. Simulation results

In this section, we apply the two-phase ($WCSP + PBM$) evasive strategy and examine how they fare against a pursuer using PN guidance. Though the evasive strategy can be applied irrespective of the turning capabilities of the pursuer, we test our strategy in a scenario where the evader is slower than the pursuer ($v_e < v_p$) but is more agile ($r_e < r_p$). This creates a balanced setting where scenarios of both escape and capture are likely to occur.

The simulations are shown in Fig. 5. In Fig. 5(a), $r_c = 3.5$ m, the initial relative separation is more than the critical phase-switching distance for *PBM* phase. In this case, the evader executes the *WCSP* phase followed by the *PBM*. When applied in succession, the two phases lead to a periodic pattern in the trajectories. As discussed through Fig. 3, the *WCSP* phase leads to a tail chase scenario and then when the relative separation goes below r_c, *PBM* is executed to temporarily increase relative separation. Though the evader prevents capture, it cannot escape the pursuer indefinitely and is confined in movement. Fig. 5(b) differs from Fig. 5(a) in that the the pursuer is chosen to be more agile with $r_p = 1.5$ m. As a consequence, though similar patterns in the trajectories can be observed, the evader is confined to a smaller region with shorter straight line motions. In Fig. 5(c), $r_c = 7$ m, the initial relative separation is less than r_c and *PBM* phase is executed. Again, a periodic pattern is seen in the trajectories of both pursuer and evader. Interestingly, in this case, the *PBM* phase does not enable the evader to escape the threshold and it is stuck in the *PBM* phase. The pursuer is also stuck in a circular motion. The region of confinement in this scenario is much smaller compared to Fig. 5(a). In Fig. 6, $r_c = 3$ m, the initial relative separation is more r_c, which is comparatively lower. This is a critical case where the evader is caught by the pursuer. In this case, the evader executes the *WCSP* phase and then a tail chase by the pursuer which leads to the *PBM* phase. However, in the *PBM* phase, the pursuer gets within one of the evader's turn circles, resulting in a *CC* trajectory. The *CC* trajectory involves successive turns in opposite directions, as shown in Fig. 4, resulting in a net low curvature turn. This defeats the purpose of the *PBM* phase as the pursuer can move along the low-curvature path and eventually capture the evader.

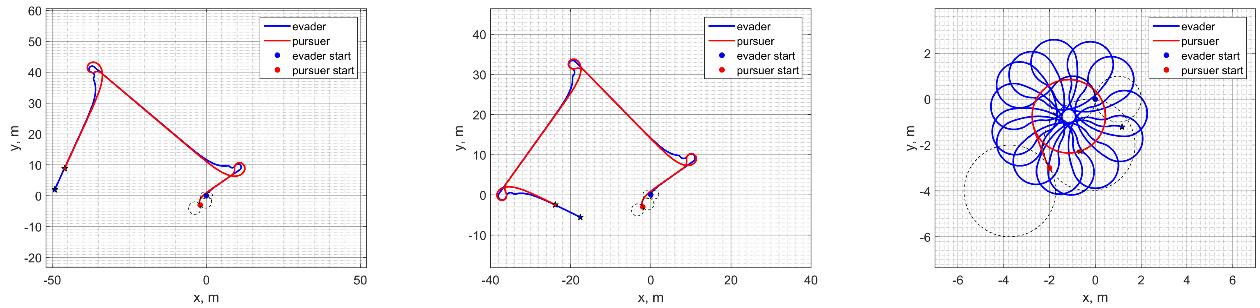
Algorithm 3 Two-phase feedback evasive strategy

1) When the relative separation between the evader and the pursuer is greater than r_c, execute *WCSP* phase and move along optimal tangent as described in Algorithm 1.
2) When the relative separation less than r_c, execute *PBM* phase by trying to reach the pursuer in shortest time by following Algorithm 2.

V. Conclusions

In this paper, we present a two-phase evasive strategy for a pursuit-evasion problem where the agents are non-holonomic and have incomplete information. The information structure is so chosen to make the problem realistic and the solution easy to implement. In the *WCSP* phase, the evader assumes that the pursuer is holonomic and executes a best-response strategy based on that assumption. This strategy leads to motion along the optimal tangent to a minimum turn circle. When the pursuer gets close to the evader (*PBM* phase), the evasive strategy switches to movement along a high-curvature trajectory. These phases, when executed in succession, provides an effective evasive strategy.

In future works, we will compare the evasive strategy against different pursuit strategies, study in detail about the



(a) $r_c = 3.5\ m$ (b) $r_c = 3.5\ m$ (c) $r_c = 7\ m$

Fig. 5. Two-phase evasive strategy against pursuer using PN with $N = 3$. In each of the simulations the initial conditions of the evader and pursuer are given as $e(0) = \begin{bmatrix} 0 & 0 & \pi/2 \end{bmatrix}^T$, $p(0) = \begin{bmatrix} -2 & -3 & 2\pi/3 \end{bmatrix}^T$. The speeds are $v_e = 0.8$ m/s and $v_p = 1.0$ m/s. The turn radii are $r_e = 1.0$ m and $r_p = 2.0$ m for (a) and (c) and $r_p = 1.5$ m for (b). The simulations were run for 150 s.

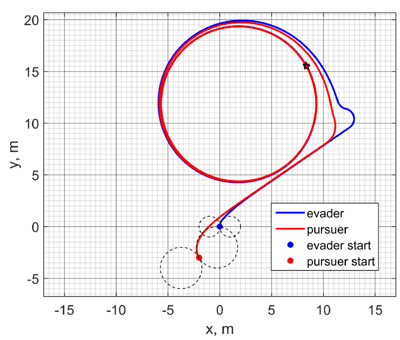


Fig. 6. A critical case when the evader gets captured. The initial conditions of the evader and pursuer are similar to Fig. 5 with $r_c = 3$ m and $r_p = 2$ m.

critical phase-switching distance r_c and analyse the different trajectory patterns for different kinematic properties of the pursuer and evader. We will also extend the analysis to the scenario of multiple pursuers.

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