**Lesson 2 - SPAN**

* vectors to represent coordinate system
* Any two vectors can be chosen to represent basis vectors, weighted sum of these two basis will results in any vector.
* **SPAN -** Set of all linear combinations of two vectors is span of v and w.
* Span of 2d vectors results in a plane
* Linear combination of 3 vectors is plane (if 3rd vector is in plane of first 2) else all 3d points in space
* If either of vectors line up with another (same or opposite direction) then they’re said to be linearly dependent
* Basis of vector space is set of linearly independent vectors that span the entire space

**Lesson 3 –Linear transformations and matrices**

* Transformation is same as function input a vector and output as vector
* Kind of how each point in space moves to which location once transformation is applied
* Linear transformations mean origin stays put and lines remain lines
* So just need to record where the basic vectors land and all other vectors can be computed as linear combination of these two basic vectors even after transformation
* Imagine as 2\*2 matrix where first column repsere3setns where I hat lands and second column represents where j hat lands
* [ac; b,d] any vector[x;y] gives [ax + by; cx+ dy] gives the transformed vector
* So if two columns which transform basis vectors are linearly dependent entire 2d space is transformed into a line

**Lesson 4 Matrix multiplication as composition**

* Composition of rotation and shear represented by two matrices and can be combined to one matrix
* Transformations applied from right to left same as functions composition were inner functions are evaluated first
* Order of multiplication matters not commutative but associative

**Lesson 5 Three-dimensional linear transformations**

* Same as 2d but extended by k hat vector in 3rd dimension
* 3d matrix multiplication to be done from right to left

**Lesson 6 Determinant**

* How much a transformation changes area
* Any area is scaled to same level as the area between the basis vectors
* Determinant is zero( area formed by transformation is zero is transformation vectors are linearly dependent – squishing them in to a line
* -ve values indicate flipping of basis vectors (in true cases I hat is to the right of j hat) incase its flipped j hat comes to the right of I hat
* The magnitude of the determinant gives the area scaling factor
* In case of 3d transformations it gives the area by which a unit volume is scaled.
* Usually orientation flip id determined by right hand rule; forefinger in I hat, middle finger in j hat then thumb has to point in k hat direction then it means orientation is not flipped. If this is possible with left hand then orientation is flipped

**Lesson 7 inverse matrices column space and null space**

* Linear system of equations
* So given a 2d transformation Ax = v, which means x is transformed to v vector, we have to find how to get to x given v;
* There can be two cases, if A squishes x to a line or just sheared to some 2d shape; can be checked with determinant
* If det(a) != 0; then we can be sure that only one vector lands on v rather than many (in case of det(a) == 0)
* So by finding an inverse, essentially we transform from v back to z you can see that A inverse A is identity (do nothing transformation)
* Easily extended to 3d
* Rank – if all points are squished into a line then rank1 if they’re transformed into a plane then rank is 2
* Rank is the no of dimensions in the output. so for 2\*2 2 is the max rank, for 3\*3 rank is 3
* Set of all possible Av is called column space - the span of all the columns in the transformation matrix
* For full rank, only origin gets transformed into origin, for less than full rank more vectors are transformed into origin; for 2 \* 2 transformation, if rank is 1, then a line full of vectors are transformed to origin; for 3\* 3 if rank is 1, then a plane full of vectors get into origin; these vectors which get transformed to origin other than origin is call null space or kernel of transformation

**Lesson 8 Non-square matrices as transformations between dimensions**

* Transformations can take input in 2d and output in 3d, but still be full rank; e.g. A 3\*2 matrix takes input 2d and gives 3d output meaning all possible vectors in plane are mapped to 3d space
* Similarly a 2\*3 matrix takes as input 3d point and squashes to 2d space; just look out for how the basis vectors are transformed. **The no of columns in transformation indicate whether transformed to 3d or 2d.**

**Lesson 9 dot products and Duality**

* **Traditionally dot product means sums up element wise multiplication of two lists of numbers**
* So dot product of v and w is product of length of (projected w on v) and length of v
* If w is pointing in roughly same directions as v then dot product is positive else fi perpendicular w is projected as a point no v hence dot product is 0; if in opposite direction then sign is negative
* Changing order of multiplication doesn’t change value as the vectors are just scaled
* Concept of duality – consider a 1 \* 2 matrix that transforms 2d points to a scalar value; multiplying this matrix with 2d vector (matrix vector multiplication) implies same as dot product
* Consider a 2d lien at some angle theta to x axis, assume unit vector u hat in that line, any 2d vector that gets projected on that line ensures linear transformation; so it is a special transformation that takes input 2d vectors and outputs a real number
* To find where I hat and j hat land; projecting u hat into I hat is same as projecting I hat into u hat line ( so projected length of u hat into I hat is just the x coordinate of u hat); similarly the projected length of u hat into j hat is y coordinate of u hat
* Duality natural correspondence of two mathematical things

**Lesson 10 Cross product part1**

* Cross product mag is the area of the parallelogram formed by two vectors
* If cross product gives anti clockwise direction then it's positive else negative
* Magnitude can be found by taking determinant of the matrix formed by stacking columns of the two vectors
* These columns  represent the transformation the basis vectors would undergo if these columns were taken as columns of  linear transformation of basis vectors
* In reality cross products depressant the vector perpendicular to the parallelogram formed by the two vectors and their direction is given by the right hand rule

**Lesson 11 Cross product part12**

* Think of a linear transformation where you have the second and third column to be the v and w vectors and the first column to be xyz
* So Duality of a vector means that you can transform from n-dimensional vector to a number line using that vector and it is same as taking a dot product that the columns represent where each of the basis vectors Woodland on the number line
* Think of a function which route map a vector x y z to a matrix and which is equal and to taking a dot product between a vector P1 P2 P3 and the vector x y z so essentially P1 P2 P3 become the constants described in a cross product multiplication
* So a cross product represents the volume of the parallelepiped formed where first the parallelogram is found by the two basis vectors and the area of the parallelogram is projected along the length of the vector such that the length of the vector matches the area of the parallelogram

**Lesson 12 change of basis vectors**

* Transformation a inverse m a where a is the change of basis vector from our coordinate to other coordinate and m is the transformation in our coordinate system
* This will carry out a transformation in the second coordinate system whereas we can visualize the change in the first coordinate system (represented by m)

**Lesson13 Eigen value**

* For any linear transformation basics access special vectors whose directions don't change after transformation; extend in their span or their direction and only stretch caused by scalar
* These special vectors for transformation are called as eigenvectors and the corresponding scalar value is called eigenvalue
* For example consider rotation. Any valid 3D rotation is a linear transformation where the eigenvector can be considered as the axis of rotation here; the Eigen value is just one cause rotations don't shrink or extend the eigenvectors
* Not all transformations need to have eigenvectors and values for example 2d rotation don't have any real number I can value since they rotate all vectors by certain degree there are no eigenvalues of vectors for rotations ( you get imaginary numbers to be Eigen values)
* Consider the case where the basis vectors happened to be eigenvectors; in that case the transformation matrix is a diagonal Matrix whose diagonal values are just Eigen values of two matrix; advantage of working with diagonal matrix is there power is essentially multiplying by an eigenvalue so if the any matrix is very it to be raised to a power can transform to the basis vector using the formula inverse and do the multiplication in the diagonal matrix anti transform to the original basis vectors
* Raises to concept of Eigen basis where basis vectors are Eigen vectors of transformation

**Lessons vector spaces**

* Functions are much like vectors; they do have operations of addition and scaling; for example taking a derivative is just like a linear transformation; in fact there are more vectors like things referred to as vector spaces;
* There are 8 axioms in total that apply to these vector spaces