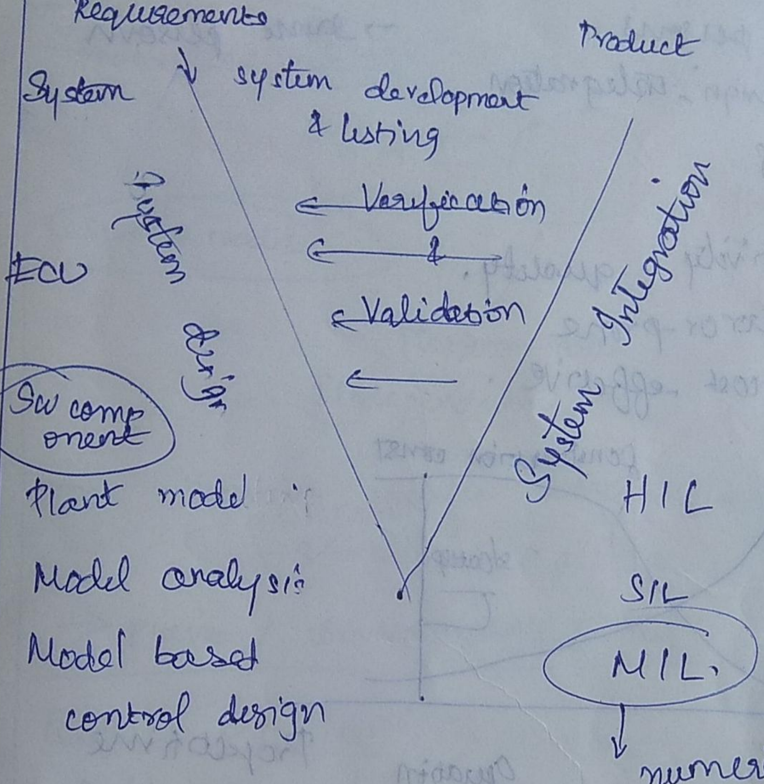
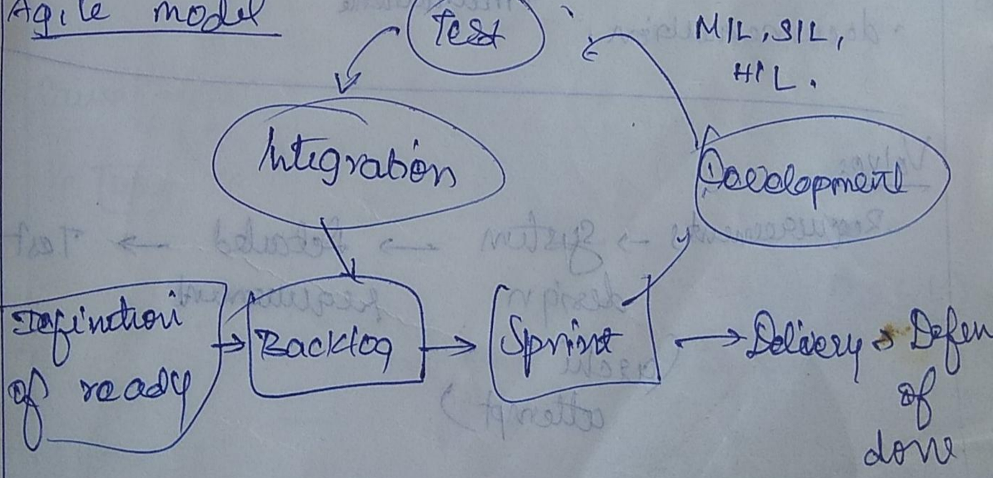
Model based Automotive design:



Plant model
Model analysis
Model based
control design

Agile model

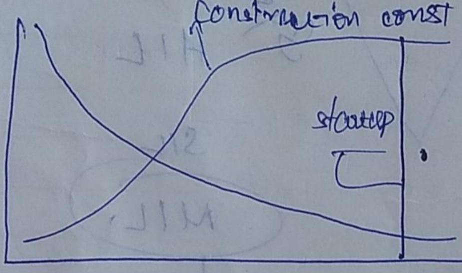


loop time, resources.

- V-model
→ Larger loop time
→ Diff. persons
for design, integration
testing

Agile model
→ smaller loop time
→ same person

→ Productivity & quality.
→ Less error-prone
↓ cost-effective.



· documentation.

<u>Volvo:</u>

Requirements → System design (archi attempt) → Detailed requirement → Test

Executable model as core object.

Red Continuous Integration.

~~Architecture~~
~~moderating~~ rule

Power supply
Network

s. Plant & Environment model

1. System models

Platform model
(independent)

DSL - Domain specific languages
- simulink / stateflow

→ Plant / Environment model

Feedback principles:

Control

Dynamic systems

feedback

Cause → effect

I/P $\boxed{\text{system}}$ O/P

memor

$y(t) = f(u(t))$

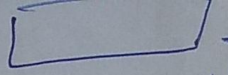
static

$u \rightarrow \boxed{f(\cdot)} \rightarrow y$

$y(t) = f(u(s), s \leq t)$

dynamic

?

$u \longrightarrow$  $\overset{y}{\longrightarrow}$

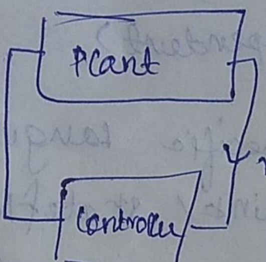
acc. pedal velocity

$$\frac{dv}{dt} = \left(-\frac{a}{m}\right)v + \left(\frac{b}{m}\right)u. \quad \underline{\text{Dynamics}}$$

$y = v$

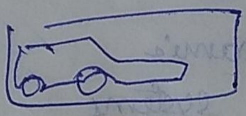
u = acc. pedal

<u>feedback:</u>



<u>eg</u>:- Cruise control.

disturbance ↓ slope

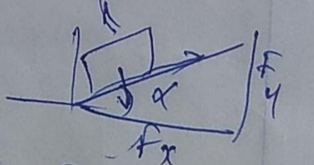
control input →  $\xrightarrow{\text{o/p}}$

throttle angle velocity

$$\frac{dv}{dt} = \left(-\frac{a}{m}\right)v + \left(\frac{b}{m}\right)u - g\sin\alpha$$

$y = v$

$$\Sigma = Ma = m\left(\frac{dv}{dt}\right).$$



F_x $\quad \sin\alpha = \frac{F_y}{W}$

$F_y = ma\sin\alpha$

$$(bu) - av = m.\frac{dv}{dt}$$

(bu) ↓ i/p force

av ↓ Friction

$$-mg\sin\alpha = m\frac{dv}{dt}.$$

$$\therefore \frac{dv}{dt} = \left(\frac{b}{m}\right)u - \left(\frac{a}{m}\right)v - g\sin\alpha$$

$\dot{v} = r$

$r = 30\ m/s$.

Destabilisation: $\rightarrow$ [Measurement noise] added in CL control.

~~Specifications~~

exercise: assume.

Open-loop: $\alpha = 0$

At steady state

$$a = 0; \quad 0 = \left(-\frac{a}{m}\right)v + \left(\frac{b}{m}\right)u$$

$$\boxed{u = \left(\frac{a}{b}\right)r} \Rightarrow \text{open loop}$$

Specs: Qlty — min energy

Quantity ~~spec~~ response time

control.

reference I/P

State feedback ctrl

$u = -Kx + Lr$

• state - space
→ optimisation problem
→ require mathematical model.

What

$$\dot{x} = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t))$$

Linear

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

1) Systematic design process
2) analysis
3) state - feedback design
4) Observer/ estimator design

Linear systems;

ODEs / TF / SS.

ODEs soln is homogenous eqn + Part. soln

$u = 0$

$$y(t) = \sum_{k=1}^{n} c_k e^{s_k t}$$

$a(s) = s^n + a_1 s^{n-1} +$

s_k = -ve decaying exp. fn
$s_k = 0$ constant
s_k = +ve → ∞

Particular soln i/p signal.

$s = \sigma + j\omega$

$y(t) = G(s) e^{st}$.

$\frac{d^k u}{dt^k} = s^k e^{st}$.

$\frac{d^k y}{dt^k} = s^k G(s) e^{st}$.

$(s^n + a_1 s^{n-1} + \dots + a_n) G(s) e^{st}$

$= (b_1 s^{n-1} + b_2 + \dots b_n) e^{st}$

$G(s) = TF$.

$y(t) = \sum_{k=1}^{n} C_k e^{s_k t} + G(s) e^{st}$.

initial condition response.

i/p signal

TF.

$\dot{x} = ax + bu \qquad x(0) = x_0$

$\cancel{X(s)[s-a] = bU(s)}$

$\cancel{\frac{X(s)}{U(s)} = \left(\frac{b}{s-a}\right) = be^{at}}$.

$\cancel{x(t) = u(t) \cdot be^{at}}$.

$\cancel{x_0 =}$

I homogenous eqn:

$\dot{x} - ax = 0$

$(s-a) = 0$.

$\boxed{x = Ce^{at}}$

$x_0 = Ce^{a(0)} = C$.

$C = x_0$

$\therefore x_h(t) = x_0 e^{at}$

Particular solution:

$x_p(t) = \int_0^t g(t) \cdot u(\tau) \cdot d\tau$

$\mathcal{L}^{-1}\left\{\frac{b}{s-a}\right\} = e^{at} b$

$$\boxed{x_h(t) = e^{at}x_0}$$

$$\boxed{x_p(t) = \int_0^t e^{a(t-\tau)}\, b\, u(\tau)\cdot \tau}$$

soln is Homog + Part. I

$$x(t) = x_h(t) + x_p(t) = e^{at}x_0 + \int_0^t e^{a(t-\tau)}\cdot b\, u(\tau)\cdot d\tau$$

$$\boxed{\text{PDE}} \xrightarrow{L} \boxed{\text{TF}}$$

$$\ddot{x} + 2\dot{x} + x = u$$

$$X(s)\,[s^2 + 2s + 1] = B\,U(s)$$

$$X(s) = \frac{B\,U(s)}{(s+1)^2}$$

$$s^2 + 2s + 2$$

$$s = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$= \frac{-2 \pm 2j}{2}$$

$$= -1 \pm j$$

$$y = Cx + Bu.$$

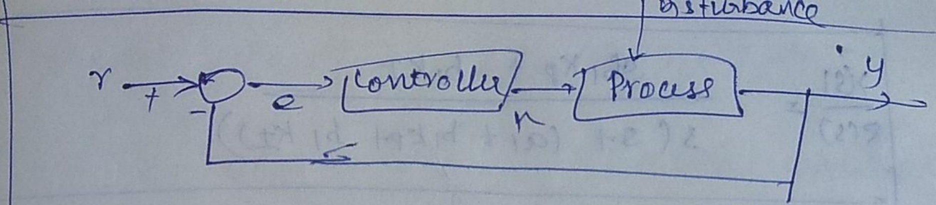
(2)

$x = 3\times 1$

$C =$

$2\times 1 = C$

$3\times$



$$Y(s)\,[s+1] = b_1\,U(s)$$

$\cancel{U(s)=}$

$\cancel{e = r - y}$

$\cancel{U = K_p(e) + \frac{K_I}{s}(}$

$$Y(s)\,[s+a_1] = b_1\,U(s)$$

$$U(s) = K_p\,[R(s) - Y(s)] + \frac{k_i}{s}\,[R(s) - Y(s)]$$

$$Y(s)\,[s+a_1] = b_1 K_p\,[R(s) - Y(s)] + b_1\left(\frac{K_I}{s}\right)[R(s) - Y(s)]$$

$$Y(s)\,[s+a_1] = b_1 K_p R(s) + \frac{b_1 K_I R(s) - b_1 K_I Y(s)}{s} + b_1 K_p Y(s)$$

$$Y(s)\,(s)\,[s + a_1 + b_1 K_p] = s \cdot b_1 \cdot K_p R(s) + b_1 K_I R(s) + b_1 K_I Y(s)$$

(s)

$$Y(s)\,[s + a_1 + b_1 K_p + b_1 K_I] = R(s)\,[s\,b_1 K_p + b_1 K_I]$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{sb_1 K_p + b_1 K_I}{s(s + (a_1 + b_1 K_p + b_1 K_I))}}$$

SS-models:

- A = system / dynamics matrix
  C = O/P (sensor) matrix
  D = feedthrough / direct

A, B, C, D ⇒ const (LTI).

$x_1 = y \quad x_2 = \frac{dy}{dt}$

$\dot{x}_1 = \dot{y} = x_2$

$\dot{x}_2 = \ddot{y} = -a_1 \frac{dy}{dt} - a_2 y + bu$

$$\dot{x}_2 = -a_1 x_2 - a_2 x_1 + bu$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

$y = Cx + Bu$

$$\begin{aligned} \dot{x} &= Ax + Bu \longrightarrow G(s). \\ y &= Cx + Du \end{aligned}$$

$\mathcal{L}\{\dot{x} = Ax + Bu\} \Rightarrow sX(s) - x_0 = AX(s) + BU(s)$

$\mathcal{L}\{y = Cx + Du\} \Rightarrow Y(s) = CX(s) + DU(s).$

$(sI - A)X(s) = BU(s) + x_0.$

$$\boxed{X(s) = (sI - A)^{-1} BU(s) + (sI - A)^{-1} x_0}$$

$$Y(s) = \underbrace{\left[C(SI-A)^{-1}B + D\right]}_{G(s)} U(s) + \underbrace{C(SI-A)^{-1}x_0}_{\text{initial condition response.}}$$

Matrix soln:

$\dot{x} = Ax + Bu \qquad x(0) = x_0$

$x_h(t) = e^{At} x_0.$

PI: $x_p(t) = \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau$

Soln: $x(t) = e^{At} x_0 + \int_0^t e^{A(t-\tau)} \cdot Bu(\tau) \cdot d\tau$

$y(t) = Ce^{At} x_0 + \int Ce^{A(t-\tau)} \cdot Bu(\tau) \cdot d\tau + Du(t)$

e^{At} = transition matrix / matrix exponential.

$$G(s) = C(SI-A)^{-1}B + D.$$

$$= \frac{\det\left(\begin{bmatrix} SI-A & -B \\ C & D \end{bmatrix}\right) \Rightarrow \text{zeros}}{\det(SI-A). \longrightarrow \text{poles of system}}$$

poles are roots of $|SI-A|$ of characteristic eqn

A eigen values are poles

Cruise control example

$$\boxed{\dot{V} = \frac{-aV}{m} + \left(\frac{b}{m}\right)u}$$

$T_s = 4s$.

$ss = 5\%$

no overshoot

$$V(s)\left[s + \frac{a}{m}\right] = \left(\frac{b}{m}\right)U(s)$$

$$\frac{V(s)}{U(s)} = \frac{b/m}{(s + a/m)}$$

$$G(s) = \frac{b/m}{(s + a/m)} \quad F(s) = K_p + K_I/s \Rightarrow PI$$ controller.

$$\frac{F(s)\,G(s)}{1 + G(s)F(s)} = \frac{\frac{b/m}{(s+a/m)}\left(K_p + \frac{K_I}{s}\right)}{1 + \frac{b/m}{(s+a/m)}\left(\frac{sK_p + K_I}{s}\right)}$$

$$= \frac{\frac{b/m}{(s+a/m)}\left(K_p + \frac{K_I}{s}\right)}{\frac{s(s+a/m) + (b/m)(sK_p + K_I)}{s(s+a/m)}}$$

$$= \frac{\cancel{s(b/m)}}{\cancel{s\left\{(s + a/m) + \left(\frac{b}{m}\right)(sK_p + K_I)\right\}}}$$

$$= \frac{\frac{(sK_p + K_I)(b/m)}{\cancel{s(s+a/m)}}}{s^2 + s\left(\frac{a}{m}\right) + \left(\frac{b}{m}\right)K_p s + K_I\left(\frac{b}{m}\right)}$$

$$\text{CL, TF} = \frac{(sK_p + K_I)(b/m)}{\left[s^2 + s\left(\frac{a}{m} + K_p\frac{b}{m}\right) + K_I\left(\frac{b}{m}\right)\right]}$$

I) $(s+0.2)(s+0.8)$

$s^2 + 1s + 0.16$

$a = 200;$

$b = 10{,}000;$

$m = 1000;$

$g = 9.82;$

$K_I\left(\frac{b}{m}\right) = 0.16 \Rightarrow K_I(10) = 0.16$

$$\boxed{K_I = 0.016}$$

$\left(\frac{a + K_p b}{m}\right) = 1.$

$\Rightarrow 200 + (K_p)(10{,}000) = 1000$

$0.2 + 10K_p = 1.$

$K_p = \frac{0.8}{10} = 0.08.$

II $s = -0.5 \pm j\sqrt{2}$

$(s + 0.5 - j\sqrt{2})(s + 0.5 + j\sqrt{2}) = 0.$

$s^2 + 0.5s + j\sqrt{2}s + 0.5s + 0.25 + j\sqrt{2}(0.5)$

$- j\sqrt{2}(s) - j\sqrt{2}(0.5) + 2 = 0$

$s^2 +$

$$s^2 + (0.5 + j\sqrt{2} + 0.5 + j\sqrt{2})s + (0.5 + j\sqrt{2})^2$$

$$s^2 + 1s + (0.5)^2 + (j\sqrt{2})^2 + 2j\sqrt{2} = 0$$

$$s^2 + s + (0.25) - 2 + j\sqrt{2} = 0$$

$$s^2 + s - (1.75 + j\sqrt{2})$$

$$s^2 + (1s) + [(0.5)^2 - (j\sqrt{2})^2]$$

$$s^2 + s + (0.25 - 2(-1))$$

$$\boxed{s^2 + s + 2.25}$$

$$s = \frac{-1 \pm \sqrt{1 - 4(1)(2.25)}}{2(1)}$$

$$s = \frac{-0.5 \pm 2\sqrt{2}j}{2} = -0.5 + \sqrt{2}j$$

$$\frac{(a + k_p b)}{m} = 1 \qquad k_I(b/m) = 2.25$$

$$\frac{k_I(10)}{10}$$

$$0.2 + 10k_p = 1.$$

$$k_p = 0.8$$

$$\dot{v} = \left(-\frac{a}{m}\right)v + \left(\frac{b}{m}\right)u$$

$$\dot{x} = ax + bu.$$

$$a = \frac{-a}{m} \qquad b = \left(\frac{b}{m}\right)$$

$$y = Cx + Du.$$

ACC example:

State vector: $d, v \rightarrow$ set velocity point.

$d \downarrow$ distance to front vehicle.

$$\begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -a/m \end{bmatrix} \begin{bmatrix} d \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ b/m \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w$$

$$y = [1 \quad 0] \begin{bmatrix} d \\ v \end{bmatrix} + [0]u + [0]w$$

d = relative dist b/w 2 vehicles

v = ego vehicle's current velocity

w = velocity of vehicle-in-front.

$$\boxed{\dot{d} = -v + w.}$$

$$\boxed{\dot{v} = \left(-\frac{a}{m}\right) v + \left(\frac{b}{m}\right) u.}$$

$1 \times 2 \quad 2 \times 1$

$$y = [1 \quad 0] \begin{bmatrix} d \\ v \end{bmatrix}$$

d is measured.

m = 1000, a = 200, b = 10,000 N/rad
kg N/m

$$A = \begin{bmatrix} 0 & -1 \\ 0 & -a/m \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -0.2 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

D = 0

$$\frac{Y(s)}{U(s)} = \frac{\text{Distance to front vehicle}}{\text{throttle angle}}$$

C

$$= [1 \;\; 0]_{1\times 2} \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -a/m \end{bmatrix} \right]^{-1} \begin{bmatrix} 0 \\ +b/m \end{bmatrix}$$

$$= [1 \;\; 0] \begin{bmatrix} s-0 & +1 \\ 0 & s+a/m \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ b/m \end{bmatrix}$$

$$= [1 \;\; 0] \underbrace{\begin{bmatrix} s+a/m & 1 \\ 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ b/m \end{bmatrix} \cdot \frac{1}{|A|}}$$

$$= \frac{1}{\|A\|} [1 \;\; 0]_{1\times 2} \begin{bmatrix} s+a/m & 1 \\ 0 & s \end{bmatrix}_{2\times 2} \begin{bmatrix} 0 \\ b/m \end{bmatrix}_{2\times 1}$$

$$= \frac{1}{\|A\|} [s + a/m \;\; 1]_{1\times 2} \begin{bmatrix} 0 \\ b/m \end{bmatrix}_{2\times 1}$$

$$= \frac{b/m}{\|A\|} = \frac{b/m}{s(s+a/m)}$$

$$\dot{d} = -v + \omega \quad \text{— (1)}$$

$$\dot{v} = \left(\frac{-a}{m}\right) v + \left(\frac{b}{m}\right) u$$

$$s \cdot V(s) = \left(\frac{-a}{m}\right) V(s) + \left(\frac{b}{m}\right) U(s).$$

$$V(s)\left[s + \frac{a}{m}\right] = \left(\frac{b}{m}\right) U(s).$$

$$\boxed{V(s) = \left(\frac{b}{m}\right)\left[\frac{m}{ms+a}\right] U(s)} \quad \text{— (2)}$$

② in (1) $\quad s.D(s) = -V(s) + \cancel{W(s)}\ \left(\frac{c}{s}\right).$

$$\frac{a}{m} D(s) = \frac{-V(s)}{(s)} + \left(\frac{c}{s^2}\right).$$

$$= \left(\frac{-b}{m}\right)\left(\frac{m}{ms+a}\right)\left(\frac{1}{s}\right) L f(s) + \frac{c}{(s^2)}$$

$$\boxed{\frac{D(s)}{V(s)} = \frac{-b}{(ms+a)}\left(\frac{1}{s}\right).}$$

$$s.D(s) = -V(s) + W(s).$$

$$(sI - A)^{-1} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -a/m \end{bmatrix}$$

$$= \begin{bmatrix} s & 1 \\ 0 & s+a/m \end{bmatrix}^{-1}$$

$$\boxed{|A| = s(s+a/m) = \frac{s(ms+a)}{m}.}$$

$$= \frac{adj(sI-A)}{|A|}$$

$$= \frac{\begin{bmatrix} s+\frac{a}{m} & -1 \\ 0 & s \end{bmatrix}(m).}{s(ms+a)}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$C(sI-A)^{-1}B$

$1\times2 \quad 2\times2 \quad 2\times1.$

$$= [1 \quad 0] \begin{bmatrix} s+\frac{a}{m} & -1 \\ 0 & s \end{bmatrix}\left[\frac{m}{s(ms+a)}\right]\begin{bmatrix} 0 \\ b/m \end{bmatrix}$$

=

$$[1 \quad 0]\begin{bmatrix} s+\frac{a}{m} & -1 \\ 0 & s \end{bmatrix} \cancel{\left(\frac{m}{ms+a}\right)\left(\frac{1}{s}\right)} \frac{1}{s\cdot\left(s+\frac{a}{m}\right)} \begin{bmatrix} 0 \\ b/m \end{bmatrix}$$

$$= \left[s+\frac{a}{m} \quad -1\right]\begin{bmatrix} 0 \\ b/m \end{bmatrix}\left(\frac{1}{s\cdot\left(s+\frac{a}{m}\right)}\right)$$

$$= \left(\frac{-b}{m}\right)\left[\frac{1}{s\left(s+\frac{a}{m}\right)}\right]$$

$$= \left(-\frac{b}{m}\right)\left(\frac{1}{s}\right)\left(\frac{m}{ms+a}\right)$$

$$\boxed{= \left(\frac{-b}{ms+a}\right)\left(\frac{1}{s}\right)}$$

~~disturbance~~

TF (rel. distance / disturbance) $\left(\frac{d}{w}\right)$

disturbance to relative distance

$$G_{wy}(s) = C(sI-A)^{-1}B + D$$

$$= [1 \quad 0]\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -a/m \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

is the B matrix for input 'w'

Poles $a = 200$ Ns/m ; $m = 1000$ kg $b = 10{,}000$ N/rad.

$$\boxed{G_{uy}(s) = \frac{-b}{s(ms+a)}} \qquad \boxed{G_{ud} = \frac{1}{s}}$$

poles @ $s = 0$

$s = -a/m$

$= \frac{-200}{1000} = -0.2$

$|s| = 0, 0.2$

PID control

$F(s) = K_p + \frac{K_I}{s} + K_d(s)$

$$\frac{F(s)G_{uy}(s)}{1 + F(s)G_{uy}(s)} = \frac{K_p + \frac{K_I}{s} + K_d(s) \cdot \left[\frac{-b}{s(ms+a)}\right]}{1 + \left[K_p + \frac{K_I}{s} + K_d(s)\right]\left[\frac{-b}{s(ms+a)}\right]}$$

$$= \frac{\left(\frac{s^2K_d + K_ps + K_I}{\cancel{s}}\right)\left(\frac{-b}{s(ms+a)}\right)}{\frac{s^2(ms+a) + (K_ps + K_I + K_ds^2)(-b)}{\cancel{s^2(ms+a)}}}$$

$$= \frac{-b(s^2K_d + K_ps + K_I)}{s^3m + as^2 - bK_ps - bK_I - bK_ds^2}$$

polynomial $= s^3(m) + s^2(a - bK_d) - s(K_p b) - bK_I$

pole placement

$a_{des} = (s+p_1)(s+p_2)(s+p_3)$

$p_1 = p_2 = p_3$

$= (s+p)^3 = s^3 + 3ps^2 + 3p^2 s + p^3$

$s^2(ms+a) - b(s^2 kd + skp + ki)$

$s^3(m) + s^2(a - bkd) - (bkp)s - bki$

comparing

$$s^3 + s^2\frac{(a - kd.b)}{m} - \left(\frac{bkp}{m}\right)s - \left(\frac{bki}{m}\right)$$

p^2 s constant

$3p = \left(\frac{a - bkd}{m}\right)$ $3p^2 = \frac{-bkp}{m}$ $\frac{-bki}{(m)} = p^3$

~~$kd = -3(pm)$~~ $kp = \left(\frac{-3p^2 m}{b}\right)$ $ki = \left(\frac{-p^3 m}{b}\right)$

~~$a - bkd =$~~

$bkd = a - 3pm$

$$kd = \left(\frac{a - 3pm}{b}\right)$$

$$s^3 + s^2\left(\frac{a - bkd}{m}\right) - \left(\frac{b.kp}{m}\right)s - \left(\frac{bki}{m}\right).$$

stability

$\left(\frac{a - bkd}{m}\right) > 0$ $\left(\frac{-bkp}{m}\right) > 0$ $\frac{-bki}{m} > 0$

$ki, kp > 0$