

FE 1

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} y \\ v_y \\ \psi \\ \psi \\ \delta \end{pmatrix}$$

$$\dot{x}_1 = \dot{y} = v_x \sin \psi + v_y \cos \psi$$

$$x_1 = v_x \sin(x_3) + x_2 \cos(x_3)$$

$$\ddot{x}_2 = \ddot{y} = \left(\frac{-c_1 + c_2}{m v_x} \right) (v_y)$$

$$+ \left(\frac{b c_2 - a c_1}{m v_x} - v_x \right) \dot{\psi} + \left(\frac{c_1}{m} \right) (\delta)$$

$$\ddot{x}_2 = - \left(\frac{c_1 + c_2}{m v_x} \right) (x_2) + \left(\frac{b c_2 - a c_1}{m v_x} - v_x \right) (x_4) + \left(\frac{c_1}{m} \right) x_5$$

$$\ddot{x}_3 = \dot{\psi} = x_4$$

$$\ddot{x}_4 = \ddot{\psi} = - \left(\frac{a c_1 - b c_2}{J v_x} \right) (v_y) - \left(\frac{a^2 c_1 + b^2 c_2}{J v_x} \right) (\dot{\psi}) + \left(\frac{a c_1}{J} \right) \delta$$

$$= - \left(\frac{a c_1 - b c_2}{J v_x} \right) (x_2) - \left(\frac{a^2 c_1 + b^2 c_2}{J v_x} \right) (x_4) + \left(\frac{a c_1}{J} \right) x_5$$

$$\ddot{\delta} = \ddot{x}_5 = \left(\frac{-1}{\tau} \right) \delta + \frac{k_1}{\tau} u + \frac{k_2}{\tau} w$$

$$\ddot{x}_5 = \ddot{\delta} = - \frac{1}{\tau} x_5 + \frac{k_1}{\tau} u + \frac{k_2}{\tau} w$$

Linearise around $\bar{x} = (0, 0, 0, 0, 0)$ and $u_0 = 0$
 straight fwd driving.

$$\Delta \dot{x} = A \Delta x + B \Delta u$$

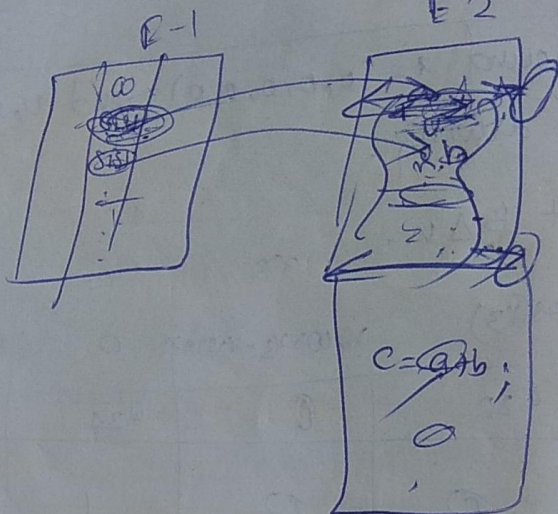
$$A = \begin{bmatrix} 0 & \cos(x_3) & v_x \cos(x_3) - x_5 \sin(x_3) & 0 & 0 \\ 0 & a_{22} & 0 & a_{24} & (c_1/m) \\ 0 & 0 & 0 & 1 & 0 \\ 0 & a_{42} & 0 & a_{44} & (a_{c1}/J) \\ 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix}$$

$$a_{22} = -\frac{(c_1 + c_2)}{m v_x} \quad a_{24} = \left(\frac{b c_2 - a c_1}{m v_x} - v_x \right)$$

$$a_{42} = -\left(\frac{a c_1 - b c_2}{J v_x} \right) \quad a_{44} = -\left(\frac{a^2 c_1 + b^2 c_2}{J v_x} \right)$$

$$B = \left[0; 0; 0; 0; \frac{K_1}{\tau} + 3 \frac{K_2 u^2}{\tau} \right]$$

System is reachable and controllable



DE
DE

10 - (3)

Pole placement.

$$\Delta u = -k \Delta x + K_r \Delta r$$

$$P(s) = (s + p_1)^2 (s + p_2)^3$$

$p_1 \Rightarrow$ satisfies settling time of 2s. (lane change maneuver).

$p_2 \Rightarrow 3 \text{ times } (p_1)$

$\phi_p = y$ position

$$C = [1 \ 0 \ 0 \ 0 \ 0]$$

$$\begin{array}{c|ccccc} s & -1 & -10 & 0 & 0 & 0 \\ \hline 1 & s+10.588 & 0 & 9.411 & -47.05 & 0 \\ 0 & 0 & s & -1 & 0 & 0 \\ p & -0.5 & 0 & s+17.45 & 60.00 & 0 \\ 0 & 0 & 0 & 0 & s+10.00 & 0 \end{array}$$

ΣS

Steering

Lateral offset

Steering angle

$$= \frac{K_p \omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} (K_p)$$

K

$$= \frac{K_p}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{s^2 + 2\zeta\omega_n s + (K_p + \omega_n^2)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$= \frac{K_p}{s^2 + 2\zeta\omega_n s + (K_p + \omega_n^2)}$$

$$K_p = 0.002166$$

L

K_p

$$s^2 + 2\zeta\omega_n s + (K_p + \omega_n^2)$$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{0.002166}{s^2 + 2\zeta}$$

$$\omega_n = 0.046541$$

$$\zeta = 0.13066$$

$$\zeta = 0.1306564$$

$$\omega_n = 0.046454$$

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{0.0170735}{s^2 + 0.01216s + 0.001707}$$

(97) $\frac{K_p}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{K_p + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$K_p = 0.01707$$

$$2\zeta\omega_n = 0.01216$$

$$\omega_n^2 + K_p = 0.01707$$

$$\omega_n^2 =$$

$$\frac{(s^2 + 2\zeta\omega_n s + \omega_n^2) + K_p}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{K_p + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$K_p = 0.001707$$

$$K_p$$

$$\frac{K_p + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$0.001707$$

$$s^2 + s$$

$$\frac{K_p + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$1 + 2\zeta + 0.0 = 1.26$$