Control system steps - notes

(1) A, B, C, D = constant means LTI system

(2) ~~Ae~~ Eigen values of $(SI - A)$ $\Rightarrow$ poles of system

(3) open-loop TF = $G(s)$

controller = $F(s)$

closed loop TF = $\dfrac{F(s)\,G(s)}{1 + F(s)\,G(s)}$

~~(4) $\dot{x} = Ax + Bu$~~

(4) Eigen values of A matrix = $-ve$ (stable)

if anyone is > 0 (unstable dynamics)

(5) state fb control design

modifying eigenvalues (via control design)

$$\boxed{u = -Kx(t) + K_r r(t)}$$

$$\dot{x} = Ax(t) + B(-K\,x(t) + K_r r(t))$$

$$\boxed{\dot{x}(t) = (A - BK)\,x(t) + BK_r r(t)}$$

closed loop eigen values, $\dot{x}$.

$$\boxed{K_r = \cancel{1/CA} = \frac{-1}{C(A - BK)^{-1}B}}$$

(6) ss-error = 0 if we have perfect knowledge of system (K_r is enough)

But in reality, we need an integral term

$$\boxed{z(t) = y(t) - r(t)}$$

$$\dot{x} = Ax + Bu$$
$$\dot{z} = y - r$$
$$= Cx - r$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r$$

~~u(t) = -K(t)~~

$$\boxed{u(t) = -Kx(t) + K_r r(t) - K_I z(t)}$$

(7) $|\lambda - (A - BK)|$ analyse this and compare with desired charac. eqn (polynomial)

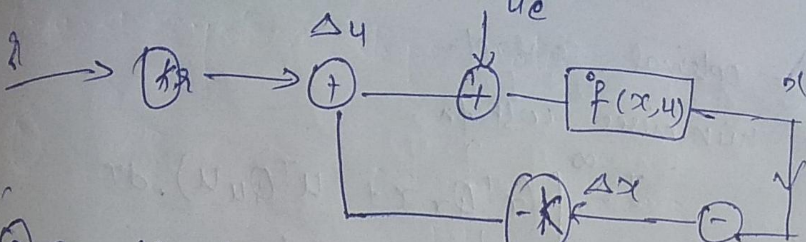
(8) $\dot{x} = f(x, u)$ — Linearization: Locally approximate around equilibrium points (x_e, u_e)

$y = h(x, u)$

$$\dot{x}_e = f(x_e, u_e) = 0$$

$$\dot{\Delta x} = A\Delta x + B\Delta u$$
$$\Delta y = C\Delta x + D\Delta u$$

$$\boxed{\begin{aligned} \Delta x &= x - x_e \\ \Delta y &= y - y_e \\ \Delta u &= u - u_e \end{aligned}}$$



(9) Equlibrium points
found by setting $\boxed{\text{o.d.e's to 0}}$

(10) Reachability $[B \; A^2B \; A^3B \ldots A^{n-1}B]$
has full rank only if $\det(A) \neq 0$

(11) Real values of Poles (as u go left, system response is faster)

(12) Zeros in LHP → more overshoot
Zeros in RHP → -ve undershoot (non-minimal phase)

(13) Ackerman's formula:
Design Desired char. poly for L.O. system

$$K = [(p_1 - a_1) \; (p_2 - a_2) \; \ldots \; (p_n - a_n)] \tilde{W}_r W_r^{-1}$$

$$\boxed{\tilde{W}_r = \begin{bmatrix} 1 & a_1 & a_2 & \ldots & a_{n-1} \\ \cdot & 1 & a_1 & \ldots & a_{n-2} \\ \cdot & \cdot & \cdot & \ldots 1 & a_1 \\ & & & & 1 \end{bmatrix}^{-1}}$$

$$W_r = [B \; AB \; A^2B \; A^{n-1}B]$$

(14) LQR - optimal control.

minimises cost fn

$$J = \int_0^\infty (\underbrace{x^T Q_x x}_{\text{state cost}} + \underbrace{u^T Q_u u}_{\text{control cost}}) \, dT$$

$u = -Kx$.

where $K = Q_u^{-1} B S$

S = soln. to Algebraic Ricatti eqn $\boxed{A^T S + SA - SBQ_u^{-1}B^T S + Q_x = 0}$

$Q_x \geq 0 \quad Q_u \geq 0$

(15) choosing Q_x, Q_u.

a) Simplest choice $Q_x = I$, $Q_u = \rho I$

$\|x\|^2$ vs $\rho \|u\|^2$ tradeoff

ρ = large u = small

b) $z = C_z x$ (be output u want to keep small)

$Q_x = C_z^T C_z \qquad Q_u = \rho I$.

(output weighting)

c) Diagonal weighting

$$Q_x = \begin{bmatrix} q_1 & & \\ & \ddots & \\ & & q_n \end{bmatrix} \qquad Q_u = \begin{bmatrix} \rho_1 & & & \\ & \rho_2 & & \\ & & \ddots & \\ & & & \rho_n \end{bmatrix}$$

Bryson's rule

$$\boxed{q_i = \frac{1}{x_{i,\max}^2}} \qquad \boxed{\rho_i = \frac{1}{u_{i,\max}^2}}$$

where $\boxed{\sum_{i=1}^{n} \alpha_i^2 = 1}$ $\boxed{\sum_{i=1}^{m} \beta_i^2 = 1}$

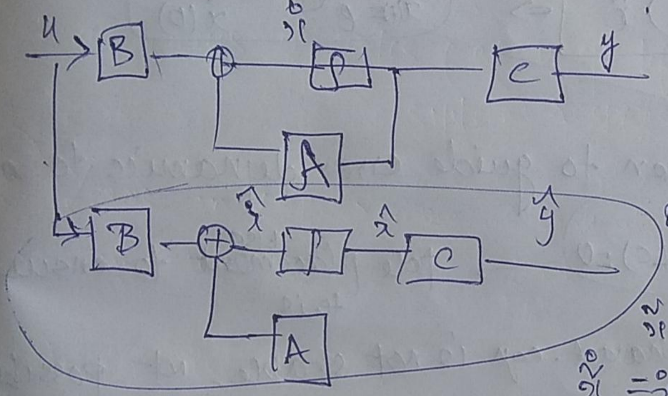
$\left.\begin{matrix} x_{i,max} \\ u_{i,max} \end{matrix}\right\} =$ Maximum response

α, β = individual weighting

d) Trial & error.

(16) <u>State estimation.</u>

$$\dot{\hat{x}} = A\hat{x} + Bu$$



open Loop Observer

$$\tilde{x} = x - \hat{x}$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

$$\dot{\tilde{x}} = Ax + Bu - (A\hat{x} + Bu)$$

$$= A(x - \hat{x}) = A\tilde{x}$$

$$\boxed{\dot{\tilde{x}} = A\tilde{x}}$$

$$\tilde{x}(t) = e^{At}\tilde{x}(0) = e^{At}(x(0) - \hat{x}(0))$$

if A matrix has -ve eigen values it'll drive $\tilde{x}(t) \to 0$

initial conditions of plant & observer

$\therefore$ We depend on A to be stable in observer $\Big\} \Rightarrow$ not always possible

(17) $\therefore \dot{\hat{x}} = A\hat{x} + Bu + L\tilde{y}$

where $\tilde{y} = (y - \hat{y})$

$\therefore \dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$

$= Ax + \cancel{Bu} - (A\hat{x} + \cancel{B\hat{u}} + L\tilde{y})$

$= A(x - \hat{x}) - L(y - \hat{y})$

$= A(x - \hat{x}) - L(Cx - C\hat{x})$

$\dot{\tilde{x}} = (A - LC)(x - \hat{x})$

$\boxed{\dot{\tilde{x}} = (A - LC)\tilde{x}} \Rightarrow \boxed{\tilde{x}(t) = e^{(A - LC)t}\tilde{x}(0)}$

L can be chosen to guide error dynamics to 0.

$\det(sI - A + LC) = 0 \longrightarrow$ use pole placement to ensure this

But if charact. eqn is not stable, or possible

$\boxed{\text{observable}} \rightarrow$ determines if system char. eqn is

$\boxed{✓ \text{ or } ✗}$

(18) Control using est. state

$\dot{x} = Ax + Bu$

$y = Cx$

controller $\Rightarrow u = -K\hat{x} + K_r r$.

~~complete~~ state estimator $\dot{\hat{x}} = A\hat{x} + Bu + L\tilde{y}$

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} BK_r \\ 0 \end{bmatrix} r$$

$\lambda(s) = \det(sI - A + BK) \cdot \det(sI - A + LC)$

can be assigned arbitrary roots if system is reachable , Observable

estimator poles = (4-5) times state feedback

19) All discussed was in cont. domain

Discrete form

$\hat{x}(t_{k+1}) = \hat{x}(t_k) + \underbrace{(t_{k+1} - t_k)}_{\text{Sampling time}}(A\hat{x}(t_k) + Bu(t_k) + L(y(t_k) - \hat{x}(t_k)))$

19) Kalman filter (continuous & discrete time system)

state estimator (Optimal best LMMSE estimator for linear gaussian models)

LQR + kalman = LQG (Kalman Bucy filter)

$A \leftrightarrow A^T$ $B \leftrightarrow C^T$ Solving Algebraic Ricatti equation

$S \leftrightarrow P$ $K \leftrightarrow L^T$

$Q_x \leftrightarrow R_v$ $Q_u \leftrightarrow R_w$