

# Ball Balancing on a Stewart Platform using Fuzzy Supervisory PID Visual Servo Control

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**Abstract**— In this paper, fuzzy supervisor of PID controller for ball balancing on a Stewart platform (ball and plate system) was implemented. Small-size Stewart platform was built and used for demonstrating the application of the controller. HD web camera was the visual sensor for sensing the ball position. After image processing, by using color threshold and object detection, the ball position was a feedback signal of the controller. In ball balancing control problem, the x-direction and y-direction can be decentralized and are separated into two PID control loops. Conventional PID control might give poor performance due to nonlinearity such as friction and error from the visual tracking. Model-based control design is complicated and could be suffered from modeling uncertainties. Fuzzy controller, which is a nonlinear control, is chosen to resolve the problem. By the way, the PID controller can be enhanced by using a fuzzy supervisory system. By using a few human intuitive rules, from ball speed and ball location, control performance can be improved. Each PID controller gains were adjusted by the fuzzy supervisor. The controller generated the pitch and roll references. The reference signals were transformed to six servo motor set-points by using the inverse kinematics of the platform. PID position controllers were used for the six servo motors. The ball position results showed enhancement by the proposed control system.

**Keywords**- *Stewart Platform; Fuzzy Supervisory Control; Ball and Plate; Robots; Machine vision*

## I. INTRODUCTION

Stewart platform mechanism, sometime called Hexapod, has been applied in the design of many robot manipulators and other applications. The platform consists of a platform and a base coupled together by six links and therefore is also known as parallel mechanism. Despite its relatively small workspace and low maneuverability, it is capable of 6-DOF, load distribution and high structural rigidity. The position control is significantly effective due to the non-serial accumulation of joint errors.

The motion of the platform can be produced by varying the lengths of the six parallel linear actuators, which are often referred to as legs of the platforms. Electrical drives like servo motors or fluid power drives such as hydraulic or pneumatic systems are the actuators. Length of the legs of the linear actuators can sometime be measured by LVDTs or optical encoders.

Stewart platform manipulator has been predominantly used since 1965, when Stewart introduced it for training the helicopter pilots. For many decades, Stewart platform manipulators, as a motion device, has been implemented as driving simulators or flight simulators [1].

Stewart platform has been introduced in a radio telescope such as AMIBA [2]; a radio telescope dedicated for cosmological studies. Servo motors with two independent encoders are installed on each leg. Six jackscrews have to work together while the antenna control unit (ACU) is responsible for coordinating the Hexapod. The control system consists of a programmable logic controller (PLC), an ACU, a pointing computer (PTC) and a stand-alone PC linked via RS232, RS485 and Ethernet communication protocol.

Stewart platform was presented in service robots such as the Stewart robot [3]. It has a mobile platform and two robotic arms to perform various tasks with mobility. It provides proactive services using its own manipulator. Intelligence systems like fuzzy systems can enhance human-friendliness in various tasks.

The drawback of the systems is that due to mechanical misalignments, the real process can significantly differ from its theoretically mathematical model. Because the precise model is difficult to obtain, the controller design should compensate for modeling uncertainties and preserve the closed-loop performance at various operating points.

Ball balancing on a plate is a typical illustration of a class of mechatronic systems. Various control algorithms have been proposed to deal with nonlinear dynamics of the mechatronic systems. [4] applied sliding mode control to balance a ball on a Stewart platform and compared with LQ control.

PID controller is the widely used owing to its simple structure and robustness. However, PID controller can be designed based on mathematical model or tuned by intelligence system. Ball balancing on a Stewart platform by using fractional PD controller [5] and tuned by genetic algorithm [6] can provide better performances than a conventional controller.

Fuzzy controller have been used widely in modern engineering problem such as in automotive industry [7] [8]. Fuzzy system is different to the other intelligence systems; such as artificial neural network (ANNs) which does not provide understandable knowledge but only accumulates data in numerical weight values between neurons.

Fuzzy if-then rules by a fuzzy system can be effective for tuning, because it is similar to human intuitive with fuzzy terms and association rules from uncertain data. This paper shows that the ball balancing problem can be resolved by using a few human intuitive rules.

Section II demonstrates inverse kinematics, dynamic model, and hardware design. Section III describes image processing for ball detection, and fuzzy supervisor of PID controller for ball balancing. Section IV shows the experimental results. Section V is conclusions section.

## II. STEWART PLATFORM

Balancing a ball on a Stewart platform is usually used for demonstrating the application of control theory.

### A. Inverse Kinematics

Inverse kinematics of parallel robots can be found in [9]. Closed-form solution for the inverse kinematics is presented. It is a transformation of the corresponding position and orientation of the platform, with respect to the base, to six servo motor angles.

Two right-hand coordinate frames  $\{A\}$  and  $\{B\}$  to the platform and base are assigned. The centroid A of the platform is the origin of the frame  $\{A\}$  and the  $x, y, z$  axes are shown in the Fig. 1.  $\theta_p$  denotes the angle between the joint  $A_1$  and  $A_2$ . Similarly, the centroid B of the platform is the origin of the frame  $\{B\}$  corresponding to the  $x_p, y_p, z_p$  axes.  $\theta_b$  denotes the angle between the universal joint  $B_1$  and  $B_2$ .  $\theta_p$  and  $\theta_b$  are  $20^\circ$ .  $r_p$  and  $r_b$  represent the radius of the platform and base, respectively.  $r_p$  and  $r_b$  are 105 mm.

$p_i$  is the vector from centroid A to the joint  $A_i$ ,  $b_i$  is the vector from centroid B to the universal joint  $B_i$ .

$$p_i = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{zi} \end{bmatrix} = \begin{bmatrix} r_p \cos a_i \\ r_p \sin a_i \\ 0 \end{bmatrix}$$

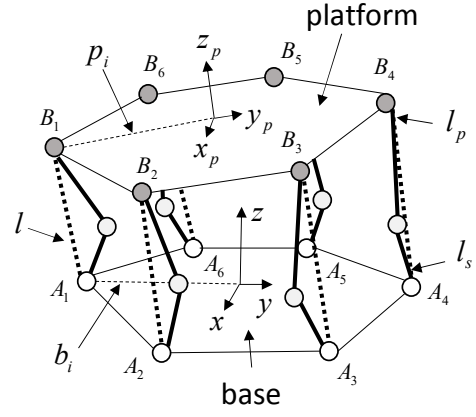


Figure 1. Stewart platform model

When,

$$a_i = \frac{i\pi}{3} - \frac{\theta_p}{2}, i = 1, 3, 5$$

$$a_i = a_{i+1} + \theta_p, i = 2, 4, 6$$

$$b_i = \begin{bmatrix} b_{xi} \\ b_{yi} \\ b_{zi} \end{bmatrix} = \begin{bmatrix} r_b \cos c_i \\ r_b \sin c_i \\ 0 \end{bmatrix}$$

When,

$$c_i = \frac{i\pi}{3} - \frac{\theta_b}{2}, i = 1, 3, 5$$

$$c_i = c_{i+1} + \theta_b, i = 2, 4, 6$$

The orientation of frame  $\{B\}$  with respect to frame  $\{A\}$  can be described by the orientation matrix  $R_T$  which requires  $r_{ij}$  when  $i, j = 1, 2, 3$ . Roll-Pitch-Yaw angles ( $\alpha$ ,  $\beta$ , and  $\gamma$ ) represent the orientation of Frame  $\{B\}$  about the  $x, y, z$ , respectively. The rotation matrix is

$$R_T = R_x(\alpha) R_y(\beta) R_z(\gamma) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In case of a linear actuator. The leg's length, from  $i = 1, 2, \dots, 6$ , can be computed from

$$l_{i,i} = \sqrt{(b_{xi} - S_{xi})^2 + (b_{yi} - S_{yi})^2 + (b_{zi} - S_{zi})^2} \quad (1)$$

$$\text{When, } S_i = \begin{bmatrix} S_{xi} \\ S_{yi} \\ S_{zi} \end{bmatrix} = \begin{bmatrix} X + p_{xi} r_{11} + P_{yi} r_{12} \\ Y + p_{xi} r_{21} + P_{yi} r_{22} \\ Z + p_{xi} r_{31} + P_{yi} r_{32} \end{bmatrix}$$

In this case, the legs consist of servo arm links and platform links which are engaged together with universal joints. The servo arm link's length is  $l_s = 40$  mm, the platform link's length is  $l_p = 80$  mm. The servo arm angles, from  $i = 1, 2, \dots, 6$ , can be written as

$$\theta_i = \sin^{-1} \left( \frac{k_i}{\sqrt{m_i^2 + n_i^2}} \right) - \tan^{-1} \left( \frac{n_i}{m_i} \right) \quad (2)$$

$$k_i = l_{i,i}^2 - (l_{p,i}^2 - l_{s,i}^2)$$

$$m_i = 2 \times l_s \times S_{xi}$$

$$n_i = 2 \times l_s \times (X_i \times b_{xi}) \times (S_{yi} - b_{yi})$$

### B. Dynamics Model

The dynamic model of ball and plate system can be described by using the Euler-Lagrange method:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} + \frac{D}{\partial q} = Q \quad (3)$$

The Lagrangian ( $L$ ) represents the difference between kinetic energy ( $T$ ) and potential energy ( $V$ ).  $D$  is the Rayleigh dissipation function.  $Q$  is the vector with external forces and  $q = [x, y]^T$  is the of generalized coordinates vector.

The kinetic energy is the sum of the translation ( $x, y$ ) of the ball and the rotation of the ball about its center. The energy of the rotation of the platform is neglected.  $m$  is the mass of the ball,  $r_b$  is the radius of the ball and  $J$  is the moment of inertia. The kinetic energy can be written as follows:

$$T = \frac{1}{2} \left( m + \frac{J}{r_b^2} \right) (\dot{x}^2 + \dot{y}^2) \quad (4)$$

$\theta_x$  and  $\theta_y$  are the rotations of the platform along the x-axis and y-axis. The potential energy can be written as follows:

$$V = mg (x \sin \theta_x + y \sin \theta_y) \quad (5)$$

From the Euler-Lagrange method, dynamic model is obtained:

$$\left( m + \frac{J}{r_b^2} \right) \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = -mg \sin \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} \quad (6)$$

### C. HARDWARE DESIGN AND SYSTEM SETUP

A small-size Stewart platform was built in our laboratory in order to perform ball balancing as shown in Fig. 2. Six digital servo motors, wCK-1108, are the actuators of the platform. They are small-size motors with the dimension of 51.6x27.1x36.4 mm and weight of 45 grams.

Gear ratio of the servo motors is 1:173. Operating Supply voltage is 11.2 VDC. Maximum speed at No Load is greater than 0.15 sec/60°. Maximum torque is over 8 kg.cm, maximum power exceeds 1.1 W. Inside the digital servo motor, PID controller is used for position control. The standard resolution is 1.055° and the error range is ±0.8°.

All servo motors are electrically connected together and received set-points command from a controller by Multi-drop-Full-Duplex-UART serial communication. Baud Rate is set to 115,200. Controller sends an 8-bit package command to each servo motor; consists of servo motor ID, position set-points, and required torque. Each servo feedbacks current position and torque back to the controller.

The base of the Stewart platform is made from stainless steel. The servo arm links are also made from stainless steel but they are painted to black. Platform links are made from aluminum. The servo arm link is engaged with a platform link by a universal joint. The camera is attached to an aluminum profile structure on the top of the platform. A ping pong ball is on the platform which has a smooth texture carpet on the top.

LabVIEW, as a graphical programming language, is applied for this research. The software has four parallel loops consist of online image processing, control loops, UART communication, and data logging.

### III. VISION AND CONTROL

In order to balance the ball on the Stewart platform, a feedback of the position of the ball is necessary. This feedback is obtained by using a visual sensor mounted on the top of the platform.

#### A. Image Processing

An inexpensive web camera is the visual sensor for sensing the ball position. Logitech c270 HD webcam captured an image by 432x240 pixels at 30 frame per second. The low-resolution image is sufficient to track the ball as well as minimizes computational effort. The USB camera is connected to a PC and the image is acquired by using NI Vision Acquisition Software (IMAQdx) in LabVIEW. Self-written vision algorithm determines the position of

the ball on the platform. The processing frame can be run continuously on a PC at 15 Hz.

The orientation of the image is adjusted by using image rotation function. Region of Interest (ROI) is chosen to crop the image and remove undesirable region; the processed image shows only the platform area. The ROI pixels is (90,50) to (270,230), the image resolution becomes 180x180 pixels. As a result, the center position of the platform is (180, 140).

The platform was setup in our laboratory; the bright was significantly controlled. RGB color threshold function can distinguish the ball on the platform. The function places the image into three 8-bit images. Red color range is set to 188 to 255. Green color range is set to 154 to 255. Blue color range is set to 0 to 244.

After color threshold, object detection function selects the bright color of the image and determines object size and location. The object is filled holes within object. The size of the object must be within 350 to 450 pixels. In case of no object is detected, the location of the object is the same as the previous one. The low pass filter, with cut-off frequency of 5 Hz, is used for smoothing the trajectory of the ball as well as rejecting noise and image detection errors. After image processing, the ball position is a feedback signal of the controller to balancing the ball on the platform.

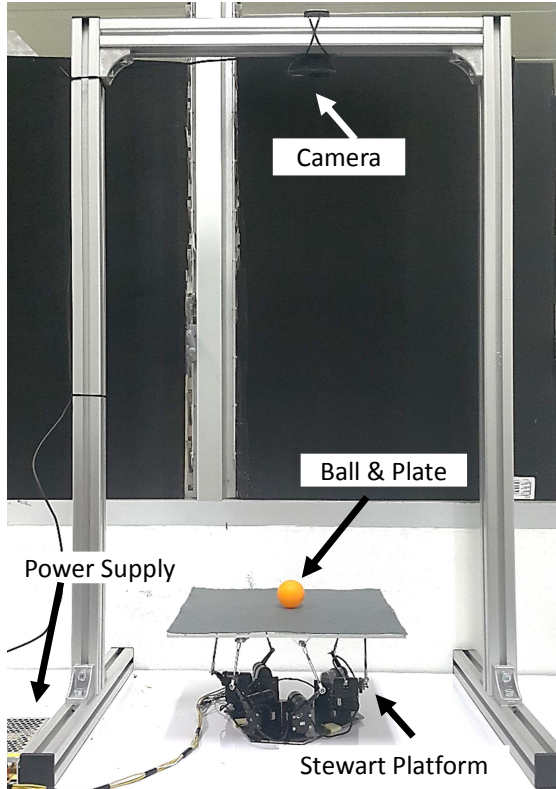


Figure 2. Stewart platform with a camera as vision feedback

### B. Fuzzy Supervisor of PID Controller

Feedback control system is discussed in this section. In ball balancing control problem, the x-direction and y-direction can be decentralized and is separated into two control loops.

The controller schematic is given in Fig. 3 and Fig. 4. The control system consists of two separated fuzzy supervisor of PID controller systems. X-direction position is controlled by adjusting the pitch of the platform. Y-direction position is controlled by adjusting the roll of the platform. In the upper control loop,  $x_{ref} = 0$  is set as the reference and  $x$  is the x-direction position of the ball. Similarly,  $y_{ref} = 0$  is set as the reference and  $y$  is the y-direction feedback for the lower control loop. That control loops has PID controllers which generate the reference pitch ( $\beta_{ref}$ ) and roll ( $\alpha_{ref}$ ) of the Stewart platform. The PID Gains of the both loops are the following.  $K_p = 0.5$ ,  $K_i = 0.015$ , and  $K_d = 0.4$ .

The inputs of the fuzzy supervisors are ball position and ball speed; or the derivative ( $S$ ) of the position. The input linguistic numeric values range from “-2” to “2”, the output linguistic numeric values range from “0”, “1”, and “2”. Input linguistic variables of both supervisors are “Position” and “Velocity”. Output linguistic variables of the supervisors are “Pitch Gain” and “Roll Gain”, respectively. The premise conjunction is “minimum”. The defuzzification method is “center of area”. In the x-direction, the input scaling gain  $g_{11} = 0.2$ ,  $g_{12} = 0.01$  and output scaling gain  $h_1 = 1$ . In the y-direction, input scaling gain  $g_{21} = 0.2$ ,  $g_{22} = 0.01$  and output scaling gain  $h_2 = 1$ .

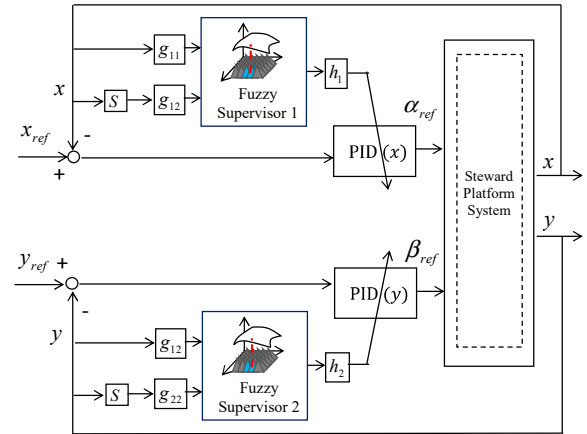


Figure 3. Fuzzy Supervisor of PID Controller

In Fig. 4, the Stewart platform system model contains the inverse kinematics model which transforms the reference pitch and roll to six servo motor angles ( $\theta_{i,ref}$ ). Each PID controller, inside digital servo motor, control the each motor position separately. As a result, the motion of the platform is

generated and the ping pong ball moves. The web camera vision system acts like a feedback sensor.

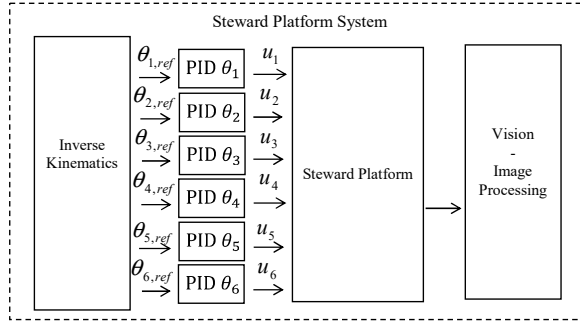


Figure 4. Stewart platform system

Fig. 5A contains the input membership functions of “Position” and “Velocity”. Fig. 5B contains the output membership functions of “Pitch Gain” and “Roll Gain”.

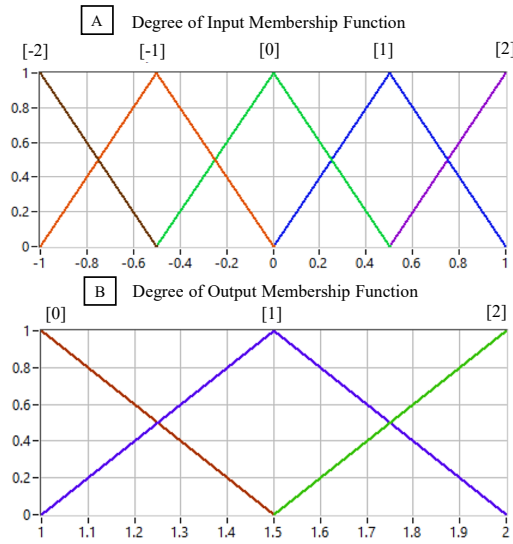


Figure 5. Membership Functions of fuzzy systems

Table 1 shows the fuzzy rule-based of the both fuzzy supervisors. Fig. 6 shows their corresponding surfaces; where the outputs of the fuzzy supervisors are plotted against their two inputs. The output of the supervisors adjust the PID scaling gains between 1 to 2 times.

TABLE I. FUZZY RULE-BASE

"Pitch/Roll Gains"		"Position"				
		-2	-1	0	1	2
"Velocity"	-2	2	2	1	1	0
	-1	2	2	1	0	1
	0	1	1	0	1	1
	1	1	0	1	2	2
	1	0	1	1	2	2

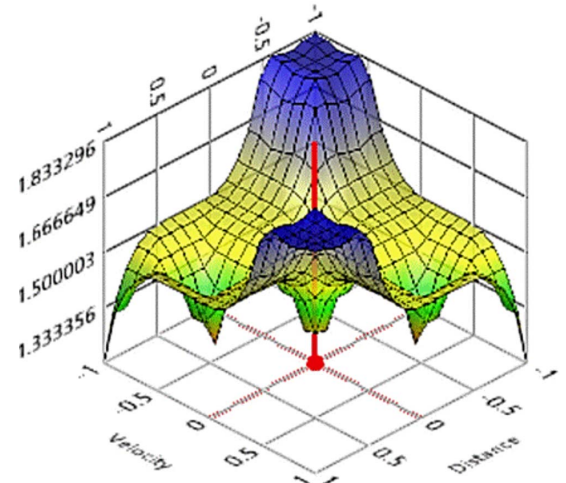


Figure 6. Control surface of the fuzzy supervisors

#### IV. EXPERIMENTAL RESULTS

##### A. Visual Tracking of a ball

As introduced in the section III-A, the ball position was detected by the web camera. Fig. 7A shows the original image from the camera. Fig. 7B shows the cropped image. Fig. 7C shows the image after the color threshold and object detection. The operation could run at 15 Hz with a typical PC, the position of the ball was found precisely.

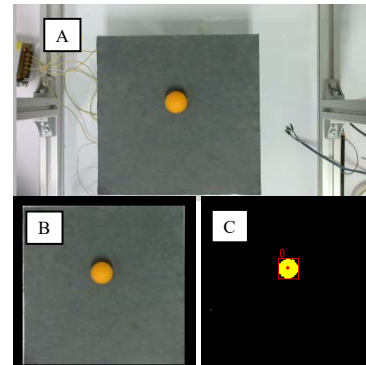


Figure 7. Ball detection using image processing

##### B. Ball Balancing and Control

The proposed controller was tested with ball balancing on the Stewart platform. The experimental results of the controller are compared with a well-tuned PID controller. During the tests, external force was applied to the ball at 1 second.

Fig. 8A and Fig. 9A show the ball positions in both x and y-directions. With the control, the orientation of the Stewart platform was changed to balance the ball. The position of the ball was controlled to the origin and was settled by 7 seconds by the PID control. With the proposed control, the ball was settled by 3.5 seconds. The blue solid lines show the x-positions, the red dashed lines show the y-positions.



In Fig. 8B and Fig. 9B, the pitch angles show as the blue solid lines, the roll angles show as the red dashed lines.

Fig. 8C and Fig. 9D show that the six servo motors changed their angular positions as the followings. #1 is the blue solid lines, #2 is the red dashed lines, #3 is the dark green dash-dot lines, #4 is the green dotted lines, #5 is the magenta dashed lines, and #6 is the black solid lines.

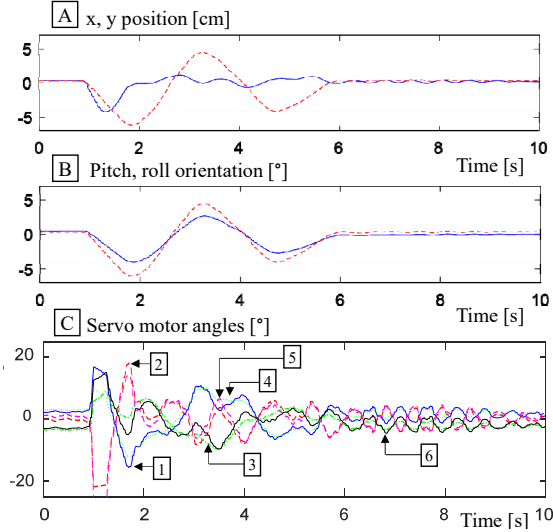


Figure 8. Experimental results of PID control

The fuzzy supervisors adjusted the PID controller gains as shown in Fig. 9C. The blue solid line is the pitch angle gain and the red dashed line is the roll angle gain.

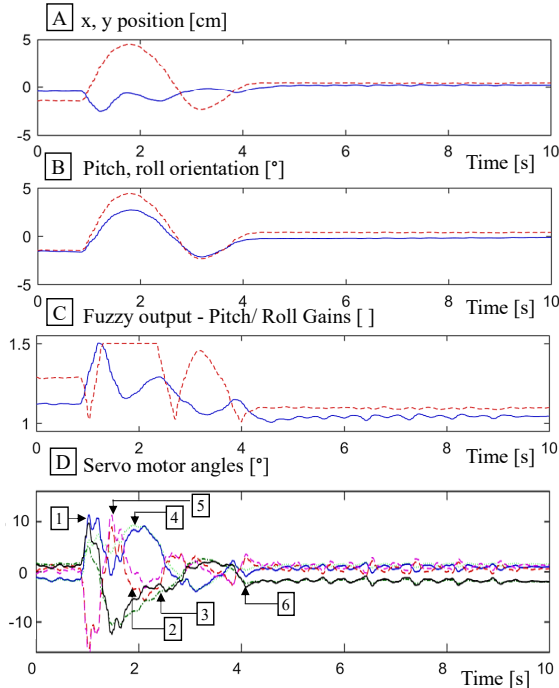


Figure 9. Experimental results of fuzzy supervisory PID visual servo control

## V. CONCLUSIONS

The fuzzy supervisory PID visual servo control system can balance a ball on a Stewart platform. From the experimental results, it shows that the setting time from the impulse response is less than 3.5 seconds. The results are excellent when compare with the results from online self-learning control algorithm called dual heuristic programming with global Laplacian Eigenmaps (GLEM-DHP) that requires 8 seconds to settle the ball [10].

The friction between the ball and the plate present the nonlinearity of the system, and steady-state errors were found. However, the ball was balanced and controlled at around the origin by using a few human intuitive rules.

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