

2D Simulation :(Heat/Pressure Distribution)

Problem statement:

Consider the diffusion equation applied to a metal plate initially at temperature T_{cold} apart from a disc of a specified size which is at temperature T_{hot} . We suppose that the edges of the plate are held fixed at T_{cool} . The following code applies the above formula to follow the evolution of the temperature of the plate. It can be shown that the maximum time step, Δt that we can allow without the process becoming unstable is $\Delta t = 1/2D * (\Delta x \Delta y)^2 / (\Delta x)^2 + (\Delta y)^2$

Approach 1: Loops.(SLOW)

Approach 2: Vectorization (FAST)

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In [1]: import numpy as np
import matplotlib.pyplot as plt
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In [2]: plt.figure(figsize=(10,10))
plt.style.use('default')

<Figure size 1000x1000 with 0 Axes>
```

```
In [3]: #plate size, mm
w=h=10

#intervals in x-, y- directions, mm

dx=dy=0.1
#thermal diffusivity of steel, mm2.s-1
D=4
```

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In [11]: Tcool,Thot=300,700

nx,ny=int(w/dx),int(h/dy)
dx2,dy2=dx*dx, dy*dy
dt=dx2*dy2/(2*D*(dx2+dy2))

u0=Tcool*np.ones((nx,ny))
u=u0.copy()

#initial conditions -ring of inner radius r, width dr centred at(cx,cy) (mm)
r,cx,cy=2,5,5

r2=r*2

for i in range(nx):
    for j in range(ny):
        p2=(i*dx-cx)**2+(j*dy-cy)**2
        if p2<r2:
            u0[i,j]=Thot
```

```

def do_timestep(u0,u):
    # propagate with forward-difference in time, central-difference in space
    u[1:-1, 1:-1]=u0[1:-1,1:-1]+D*dt*(
        (u0[2:,1:-1]-2*u0[1:-1,1:-1]+u0[:-2,1:-1])/dx2
        +(u0[1:-1,2:]-2*u0[1:-1,1:-1]+u0[1:-1, :-2])/dy2)

    u0=u.copy()

    return u0, u

# Number of timesteps

nsteps=101

#output 4 figures at these timesteps

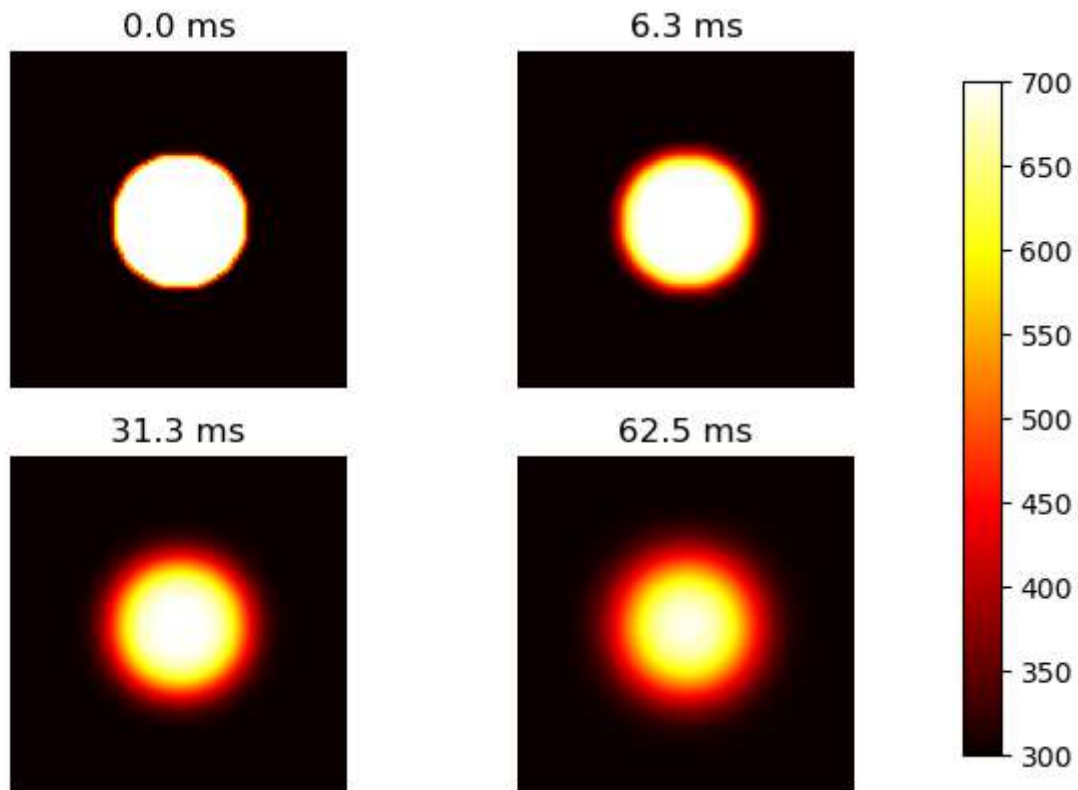
mfig=[0,10,50,100]
fignum=0
fig=plt.figure()
for m in range(nsteps):
    u0,u=do_timestep(u0,u)
    if m in mfig:
        fignum+=1
        print(m,fignum)
        ax=fig.add_subplot(220+fignum)
        im=ax.imshow(u.copy(), cmap=plt.get_cmap('hot'), vmin=Tcool,vmax=Thot)
        ax.set_axis_off()
        ax.set_title('{:.1f} ms'.format(m*dt*1000))
fig.subplots_adjust(right=0.85)
cbar_ax=fig.add_axes([0.9,0.15,0.03,0.7])
cbar_ax.set_xlabel('$T$/K',labelpad=20)
fig.colorbar(im,cax=cbar_ax)
plt.show()

```

```

0 1
10 2
50 3
100 4

```



In []: