INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Electronics & Eletrical Communication Engineering

Vision and Intelligent Systems

EC69211 – Image and Video Processing Laboratory

Experiment – 4.2

Frequency domain filtering

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CONTENTS

Objective:

1. Applying frequency domain filtering on input images with LPF and HPF ideal, Gaussian and Butterworth filters
2. Creating hybrid images from two input images so that perception of the image changes with viewing angle, time spent viewing, or image size
3. Removing noise from images contaminated with spatially periodic noise

Theory:

Ideal Low Pass Filter (LPF)

An ideal low pass filter is the one which transmits all the signal of frequencies less than a certain frequency ωc radians per second without any distortion and blocks all the signals of frequencies above ωc radians per second. Where, the frequency ωc radians per second is called the cut-off frequency.

$$|H(\omega)| = \{1 \text{ for } |\omega| < \omega_c \; 2 \text{ for } |\omega| > \omega_c$$

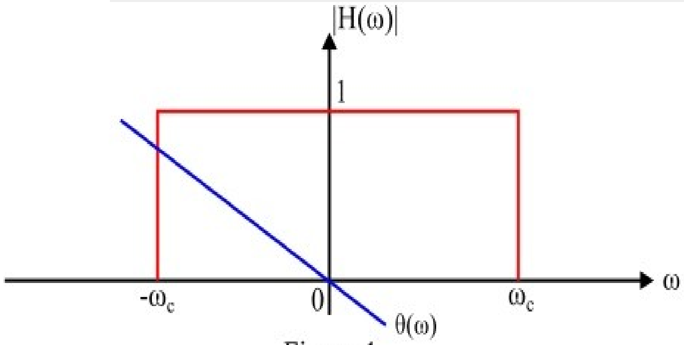


Figure-1

Ideal High Pass Filter (HPF)

An ideal high pass filter transmits all the signals of frequencies above a certain frequency ωc radians per second without any distortion and blocks completely all the signals of frequencies below the frequency ωc radians per second. Here, the frequency ωc radians per second is called the cut-off frequency.

$$|H(\omega)| = \{0 \text{ for } |\omega| < \omega_c \; 1 \text{ for } |\omega| > \omega_c$$

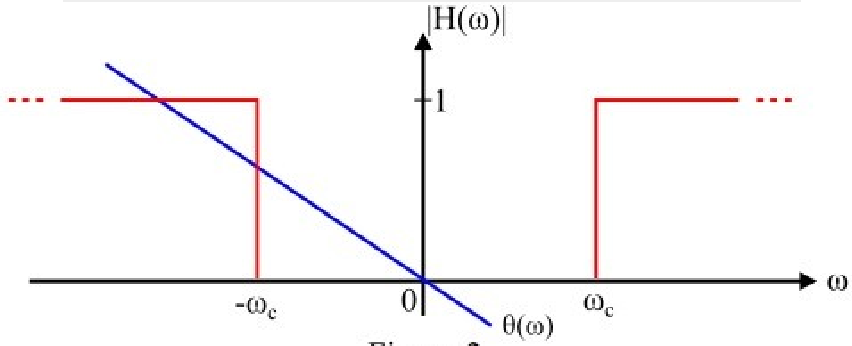
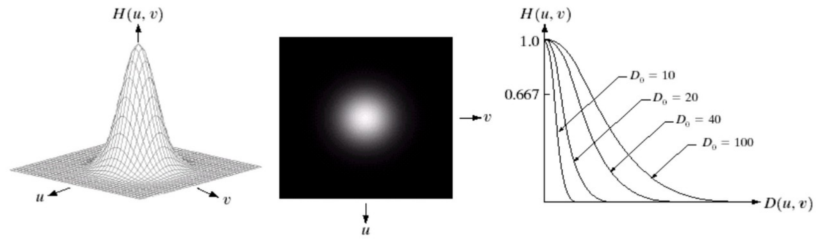


Figure-2

Gaussian Low Pass Filter

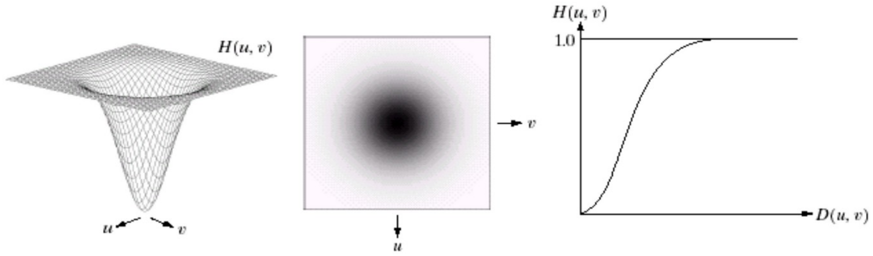
The transfer function of a Gaussian lowpass filter is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0^{\,2}}$$



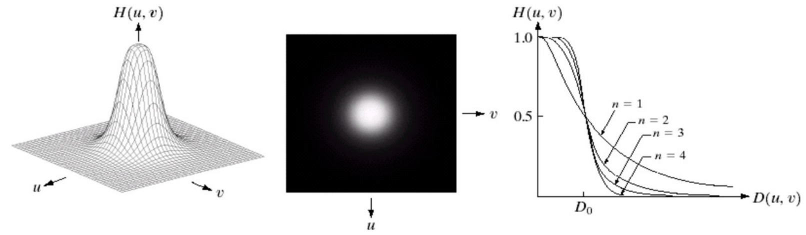
<u>Gaussian High Pass Filter</u>
The Gaussian high pass filter is given as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^{\,2}}$$



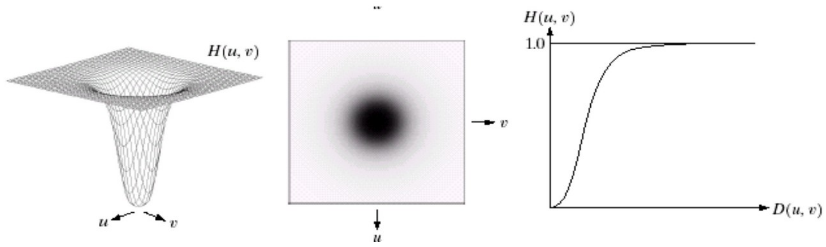
<u>Butterworth Low Pass Filter</u>
The transfer function of a Butterworth lowpass filter of order n with cutoff frequency at distance D0 from the origin is defined as:

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



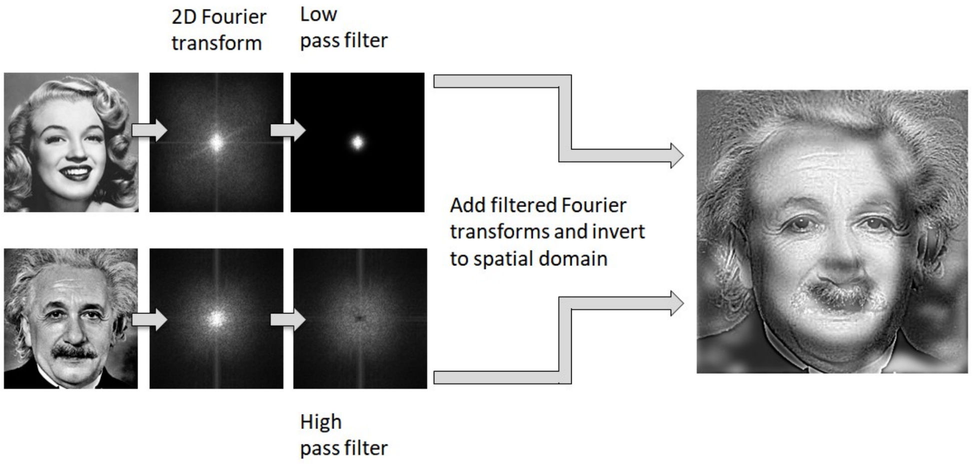
<u>Butterworth High Pass Filter</u>
The transfer function of a Butterworth highpass filter of order n with cutoff frequency at distance D0 from the origin is defined as:

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$



Hybrid Images

A hybrid image is a picture that combines the low-spatial frequencies of one picture with the high spatial frequencies of another picture producing an image with an interpretation that changes with viewing distance.



Algorithm:

Following algorithm is used to implement different filters with different cut off frequencies –

- Take as input the file path of the image, filter name, type and cut off frequency.
- Read the given image and generate the required filter kernel using the given specifications.
- Compute the FFT of the input image and perform FFT shift to center the frequency spectrum
- Perform elementwise product of the image and the filter kernel.
- Generate and store the magnitude spectrum plots of the input image, filter kernel and resultant image.

Following algorithm is used to generate hybrid images –

- Read both the images and store it in img1 and img2.
- Apply the high pass gaussian filter with cut off frequency as 50 with the img1
- Apply the low pass gaussian filter with cut off frequency as 10 with the img2
- Add both the images in frequency domain to generate the hybrid image.
- Compute the inverse FFT and store the resultant image.

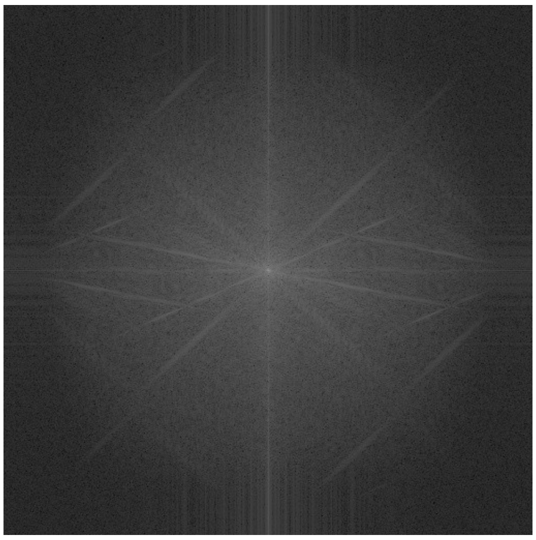
Following algorithm is used to eliminate noise from the images

- Take as input the file path the image.
- Read the image and choose the kernel size (r) and threshold (th) ratio between original spectrum and median filtered spectrum.
- Compute the FFT of the image and apply median filter on the frequency spectrum.
- For each element in the spectrum, if its distance from center of image is less than kernel's size or ratio of absolute value of original spectrum to absolute value of median filtered spectrum is less than threshold, then do not modify that element. Else, modify that element to the value obtained in median filtered spectrum.
- Compute the inverse FFT to generate the denoised image.
- Store the resultant plots and generated images.

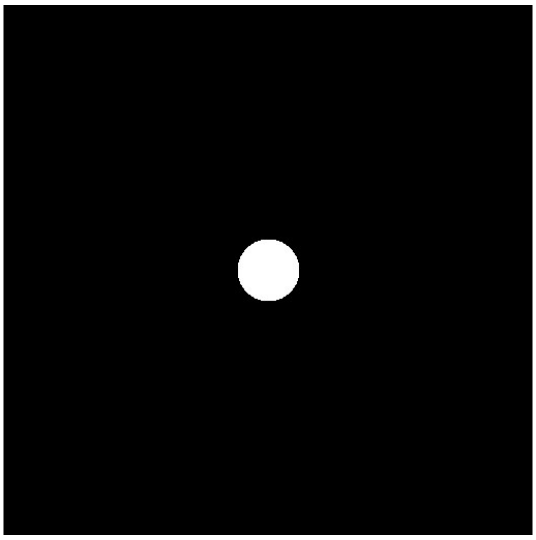
Results:

The given images were passed through ideal, Gaussian and Butterworth LPF and HPF with cutoff frequency of 30. Ideal filter had sharp passband to stopband transition due to which the filtered spectrum suddenly became zero after certain point. Gaussian filter's frequency response was found to have highest spread. When the image was passed through ideal filters, the output images were found to have ringing effects around the edges while this was not the case for other filters.

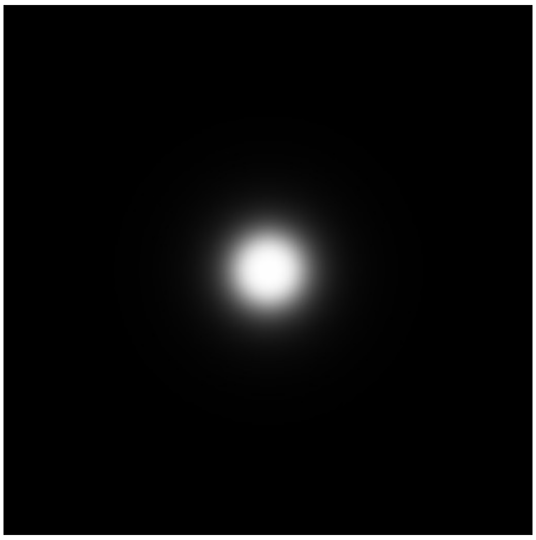
cameraman.jpg



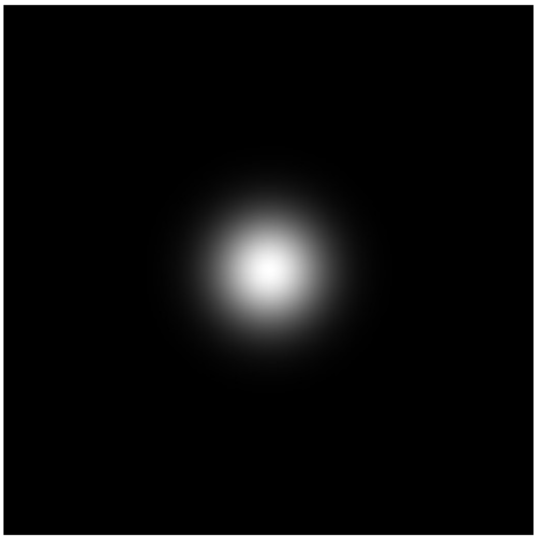
log magnitude spectrum of cameraman.jpg



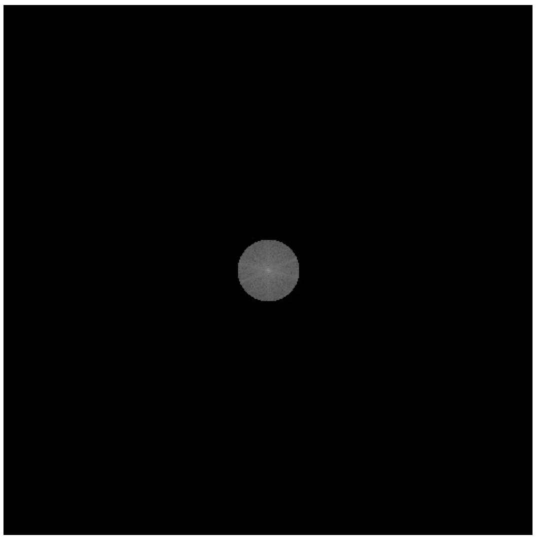
Ideal LPF frequency response



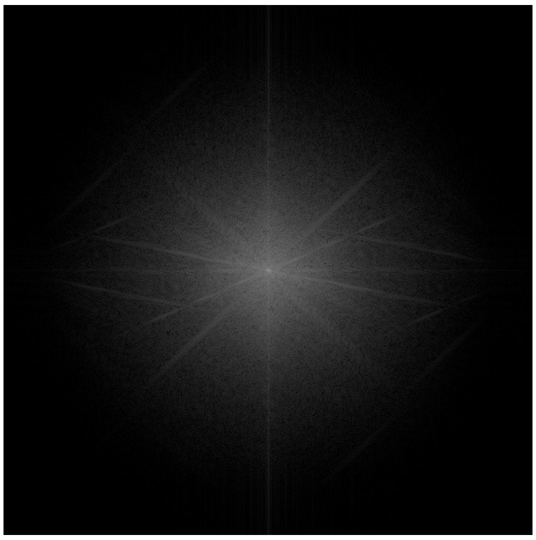
Butterworth LPF frequency response



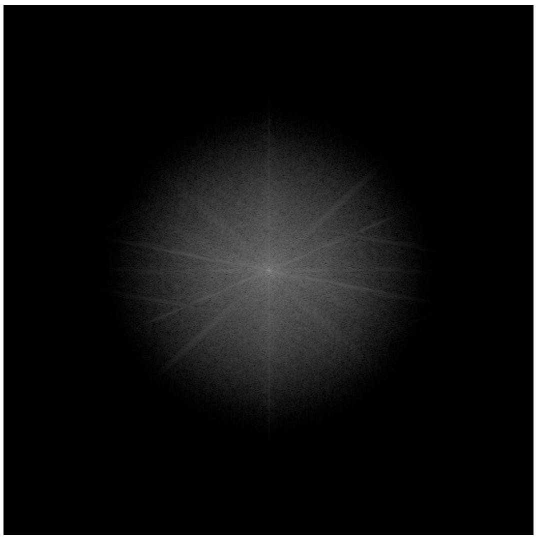
Gaussian LPF frequency response



Ideal LPF filtered spectrum



Butterworth LPF filtered spectrum

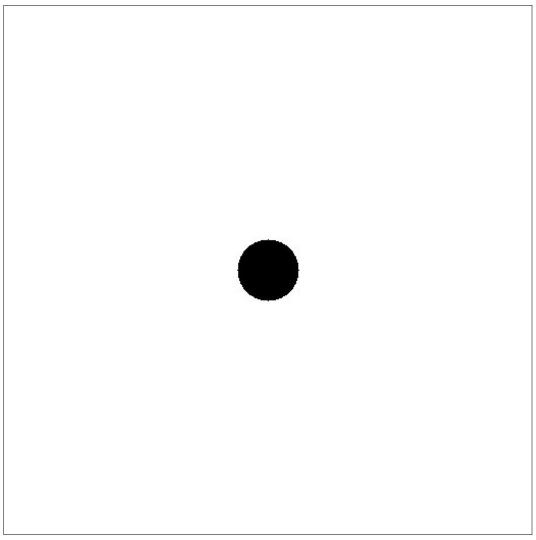


Gaussian LPF filtered spectrum

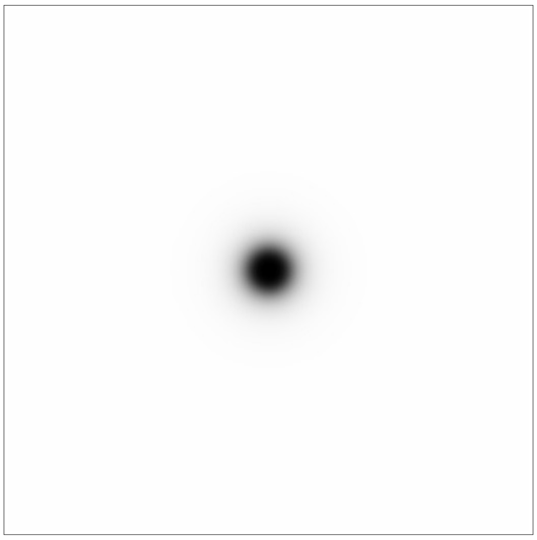
Ideal LPF filtered output

Butterworth LPF filtered output

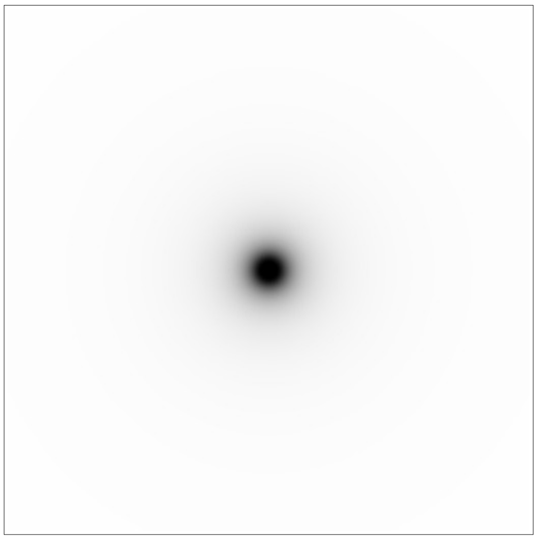
Gaussian LPF filtered output



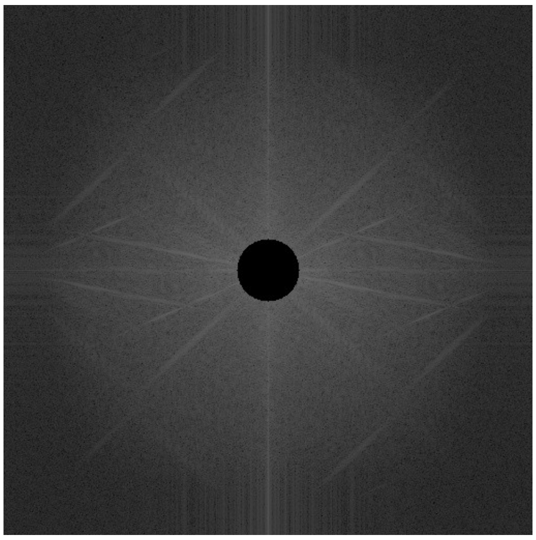
Ideal HPF frequency response



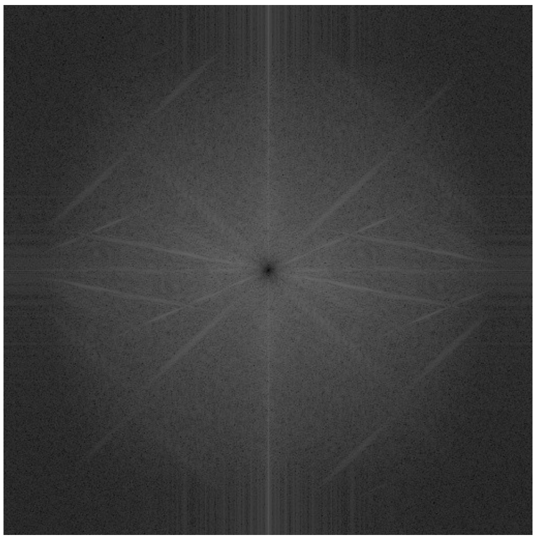
Butterworth HPF frequency response



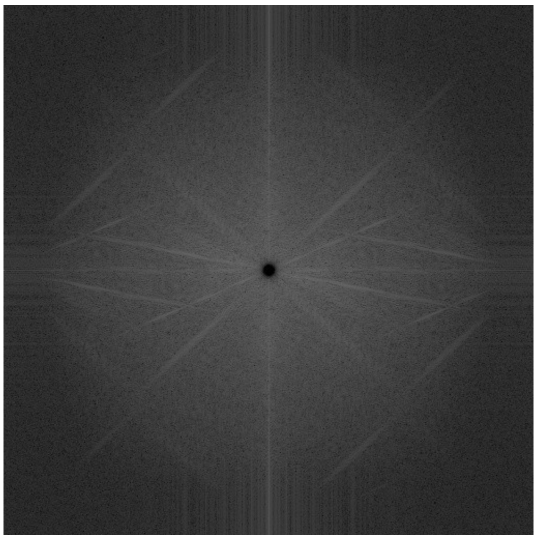
Gaussian HPF frequency response



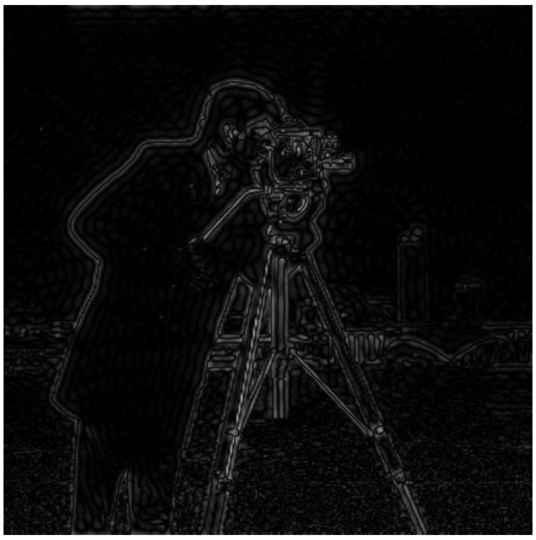
Ideal HPF filtered spectrum



Butterworth HPF filtered spectrum



Gaussian HPF filtered spectrum



Ideal HPF filtered output

Butterworth HPF filtered output

Gaussian HPF filtered output

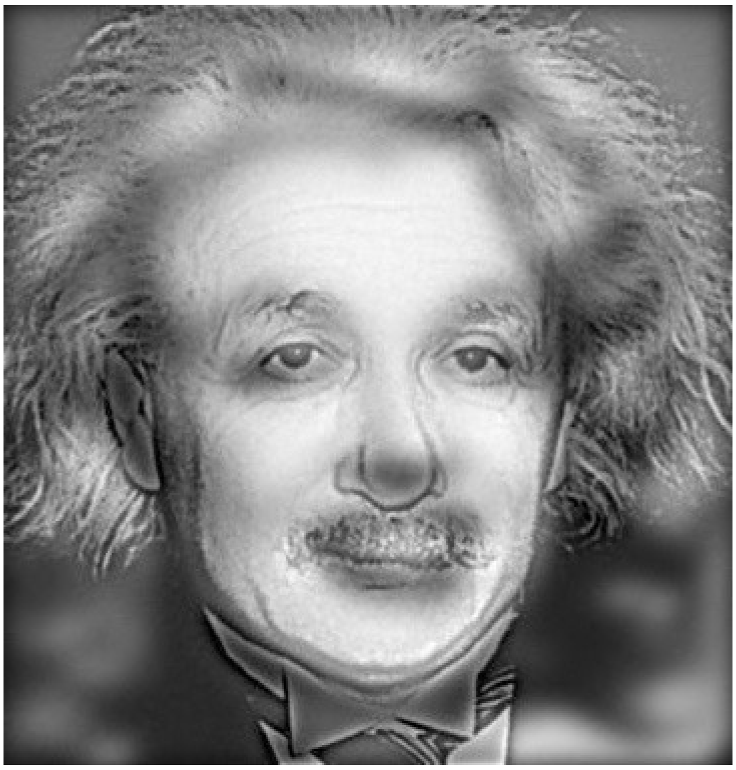
Butterworth HPF filtered output at cutoff 100

Butterworth HPF filtered output at cutoff 10

Butterworth LPF filtered output of order 2

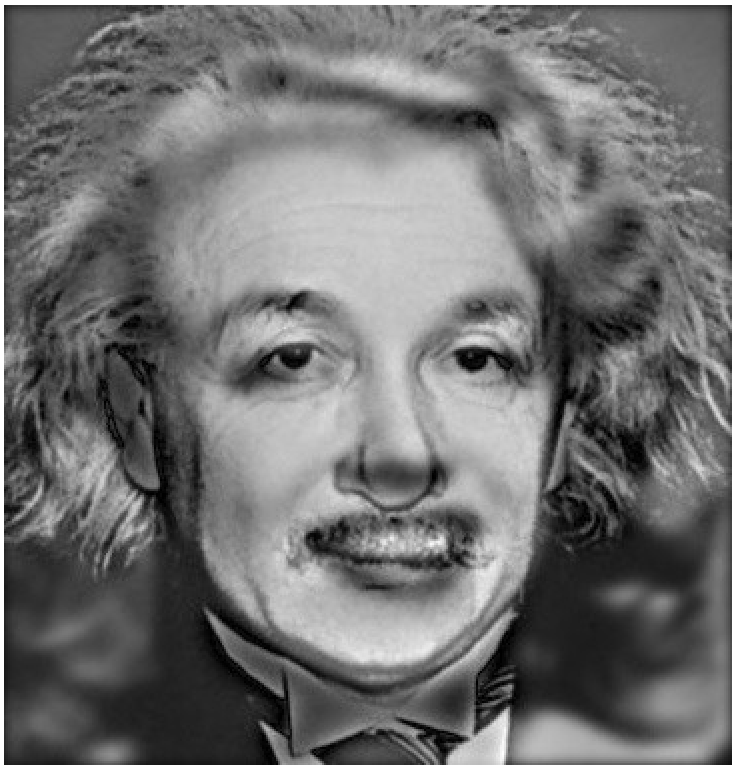
Butterworth LPF filtered output of order 12

The images *einstein.png* and *marilyn.png* were passed through HPF and LPF respectively and then elementwise added to get the hybrid image with the illusion. The image contains blurred marilyn and sharper features of einstein. marilyn can be perceived properly when viewed from far distance as that will lead to reduction of blurriness while einstein can be perceived properly when viewed from closer distance as in that case, moustache and wrinkles on face of einstein will be more prominent. Changing the cutoff frequency of the filters were also found to have impact on how much einstein or marilyn was perceived at various resolutions. For example, from the image obtained using HPF cutoff as 25 and LPF cutoff as 20, some wrinkles can be observed even when resolution was much decreased which was not the case with HPF cutoff as 50 and LPF cutoff as 10.



Hybrid image using Gaussian filter at higher resolution with HPF cutoff 50 and LPF cutoff 10

Hybrid image using Gaussian filter at lower resolution with HPF cutoff 50 and LPF cutoff 10

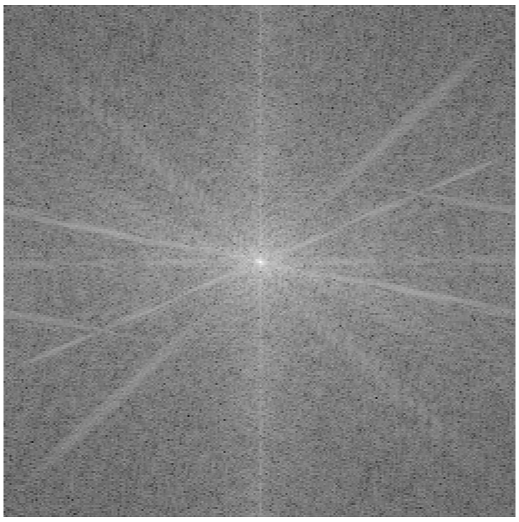


Hybrid image using Gaussian filter at higher resolution with HPF cutoff 25 and LPF cutoff 20

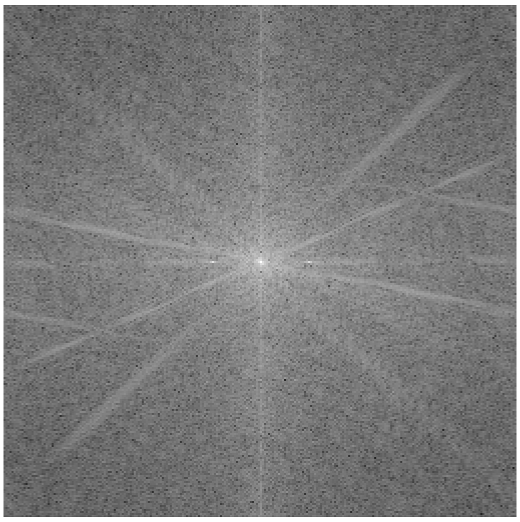
Hybrid image using Gaussian filter at lower resolution with HPF cutoff 25 and LPF cutoff 20

By observing the images in spatial domain, we find that some spatially periodic noise has been introduced in the image. From the spectrums of the noisy images it can be seen that for *cameraman_noisy1.jpg*, there are two bright points along the horizontal axis passing through the center which correspond to a sinusoid of particular frequency being introduced in the image. For *cameraman_noisy2.jpg*, additional bright points as well as some horizontal and vertical lines have been introduced in the spectrum. A source of such noise can be electrical or electromagnetic interference during the image capturing process which introduces such sinusoid at particular frequency.

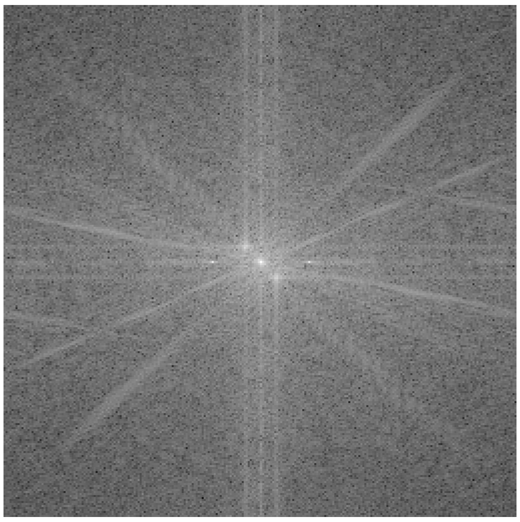
In order to remove such noise, first the spectrum of the noisy image is passed through a median filter so that the isolated peaks are removed. Then only those parts of the original spectrum are updated whose ratio with median is modified significantly, i.e., more than some threshold. The detailed algorithm can be found in the *Algorithm* section. When the threshold was increased, the noisy strips were more prominent while when it was decreased, the strips were almost removed but the overall image became somewhat darker and smudged. A threshold of 2.5 was found to give good result.



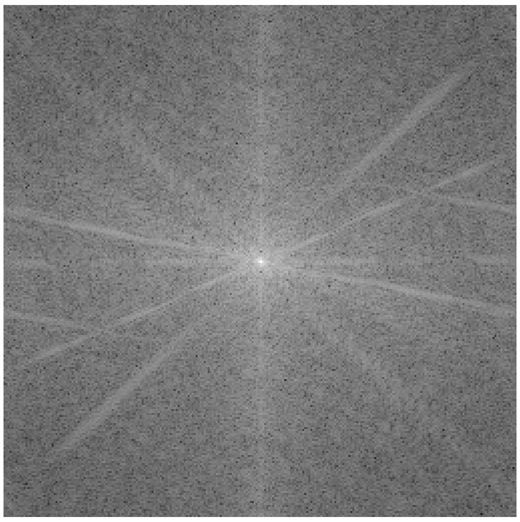
cameraman_256.jpg spectrum



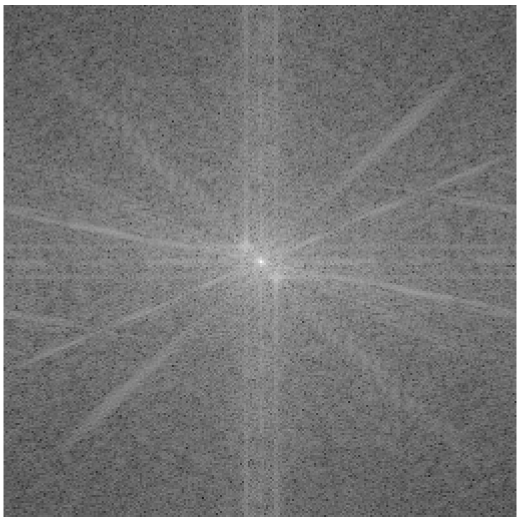
cameraman_noisy1.jpg spectrum



cameraman_noisy2.jpg spectrum



cameraman_noisy1.jpg filtered spectrum



cameraman_noisy2.jpg filtered spectrum

cameraman_noisy1.jpg

denoised cameraman_noisy1.jpg with threshold ratio 2.5

cameraman_noisy2.jpg

denoised cameraman_noisy2.jpg with threshold ratio 2.5

Discussion:

- When images were passed through ideal filters, the output images were found to have ringing effects around the edges. This happened due to the presence of ripples in sinc function of the impulse response of ideal filter which on convolution with input image produced such patterns. The same phenomenon was observed when butterworth filter of high order (say 12) was used and the reason being that at higher orders, butterworth filter has steep transition and hence becomes close to ideal.
- For low pass filters, reducing the cutoff frequency made the image more blurry. For high pass filters, increasing the cutoff frequency made stronger edges prominent while eliminating finer details of the image.
- The Gaussian filter has smoother transition from passband to stopband as compared to other filters. However, the passband transfer function is not constant and has significant variation. Due to this we incur passband attenuation while using Gaussian low pass filter and stopband leakage while using Gaussian high pass filter.
- To create the hybrid image, the two original images were passed through HPF and LPF respectively and then added. We could have also added weights to individual images and get the final output as an weighted sum of those images. That would cause one image to be more prominent than the other.
- In order to generate the hybrid image, the cutoff values for HPF and LPF had to be chosen in a way so that for higher resolution the image can be easily perceived as einstein and for lower resolution it is easily perceived as marilyn. If the cutoff for LPF was highly reduced, the marilyn image will become highly blurred which can make einstein very prominent at higher resolution but marilyn might not be perceived properly even at low enough resolution. On the other hand, if we take higher cutoff for LPF, we will start seeing marilyn even at higher resolution. Similar effects will take place when the cutoff for HPF is changed. Hence, we must choose an appropriate balance between the two cutoff frequencies to get desired output.
- For the algorithm designed to remove the periodic noise, the choice of neighborhood dimension and threshold were chosen based on experimentation with different values. It was found that for high values of threshold, the noisy strips were not being removed properly but the image retained its details while for lower value of threshold, the strips were removed but the image became somewhat smudgy. Hence we must choose the threshold appropriately based on the applicaiton so as to balance between the trade-off.
- The designed algorithm was found to remove some noise from central region of *cameraman_noisy2.jpg* but the edges still contained the noise. Hence, it was not very robust and did not perform well on the noise of second image.