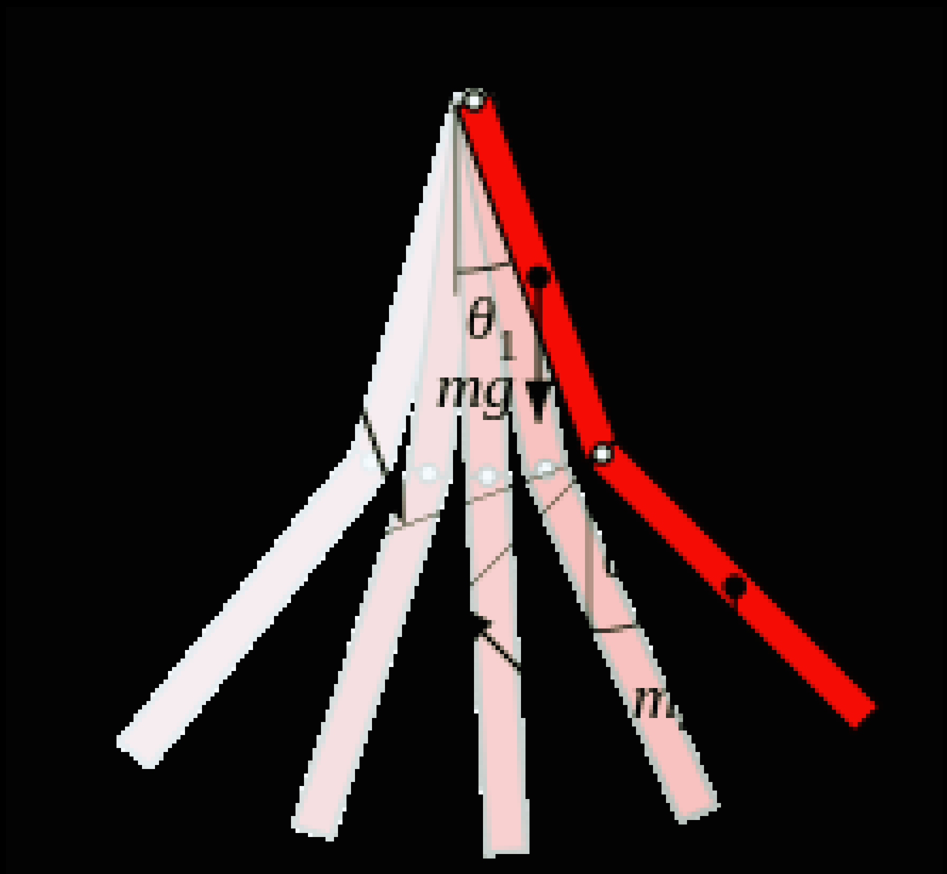
DEPARTMENT OF PHYSICS

IIT KHARAGPUR

MINI PROJECT FOR CDSR

TOPIC-"DOUBLE PENDULUM"



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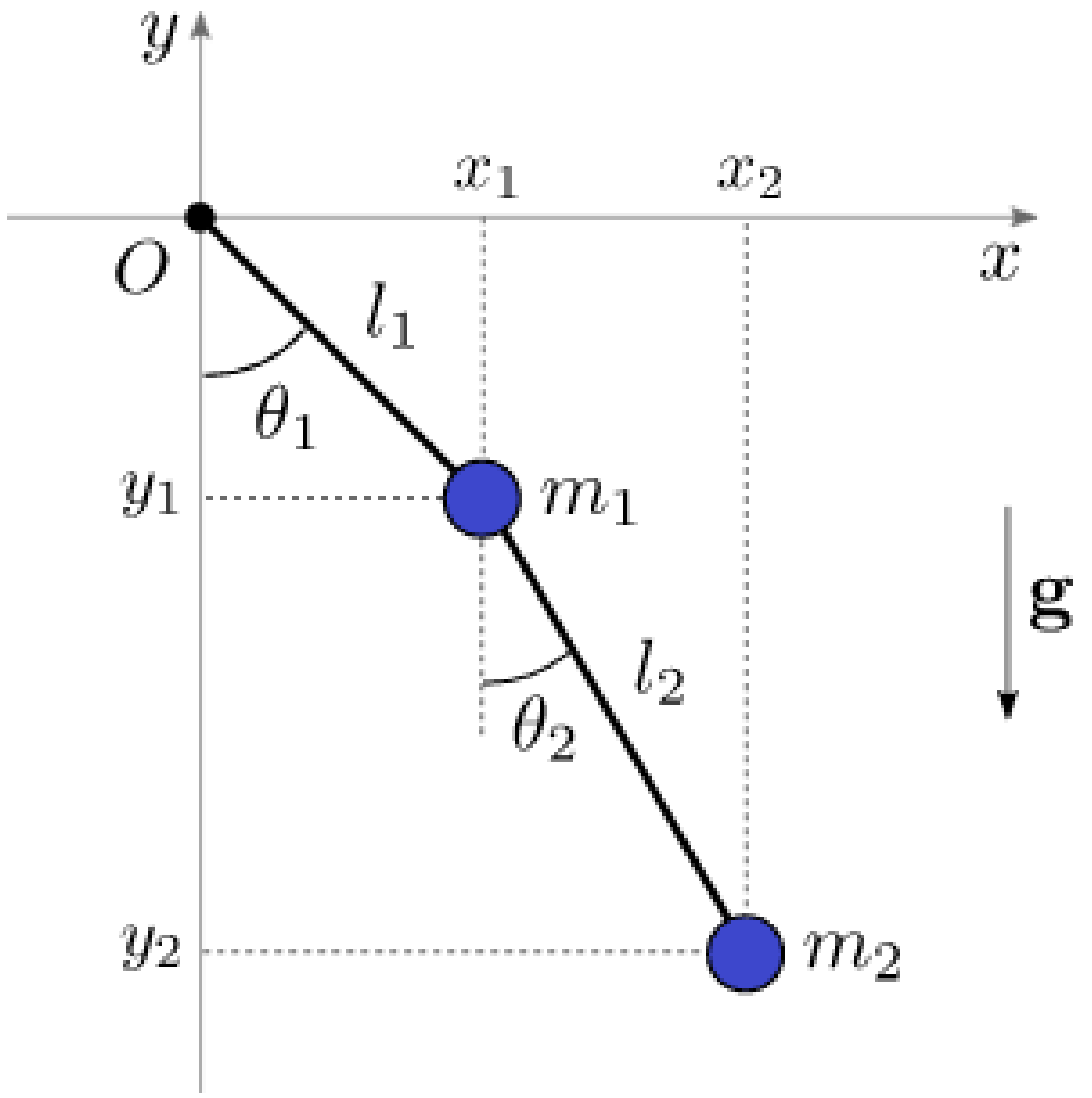
UNDER THE GUIDANCE OF PROFESSOR> "DR.KRISHNA KUMAR"

Equation of Motion Using Lagrangian



Positions

$$x_1 = l_1 \sin\theta_1$$

$$y_1 = -l_1 \cos\theta_1$$

$$x_2 = x_1 + l_2 \sin\theta_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$y_2 = y_1 - l_2 \cos\theta_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$

Equation of Motion Using Lagrangian

Velocities

$$\dot{x}_1 = l_1\dot{\theta}_1 \cos\theta_1$$

$$\dot{y}_1 = l_1\dot{\theta}_1 \sin\theta_1$$

$$\dot{x}_2 = \dot{x}_1 + l_2\dot{\theta}_2 \cos\theta_2 = l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2$$

$$\dot{y}_2 = \dot{y}_1 + l_2\dot{\theta}_2 \sin\theta_2 = l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2$$

Total Kinetic Energy of Double Pendulum

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2).$$

Equation of Motion Using Lagrangian

$$T = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2).$$

Total Potential Energy of Double Pendulum

$$U = -g\big((m_1 + m_2)l_1 \cos\theta_1 + m_2 l_2 \cos\theta_2\big).$$

Lagrangian L = T - U

$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + g(m_1 + m_2)l_1 \cos\theta_1 + g m_2 l_2 \cos\theta_2.$$

Equation of Motion Using Lagrangian

Euler Langrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,$$

For m1

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0.$$

We can determine each parts separately, giving

$$\frac{\partial L}{\partial \theta_1} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - l_1 g(m_1 + m_2) \sin\theta_1,$$

Equation of Motion Using Lagrangian

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \frac{d}{dt}\left((m_1+m_2)l_1^2\dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1-\theta_2)\right)$$
$$= (m_1+m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\left(\ddot{\theta}_2\cos(\theta_1-\theta_2) - \dot{\theta}_2(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)\right).$$

For m2

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0,$$

$$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 g \sin\theta_2 = 0$$

Equation of Motion Using Lagrangian

The two coupled differential equations are

$$(m_1+m_2)l_1\ddot{\theta}_1 + m_2 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) + m_2 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) + (m_1+m_2)g\sin\theta_1 = 0$$

$$m_2 l_2\ddot{\theta}_2 + m_2 l_1\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2 l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) + m_2 g\sin\theta_2 = 0$$

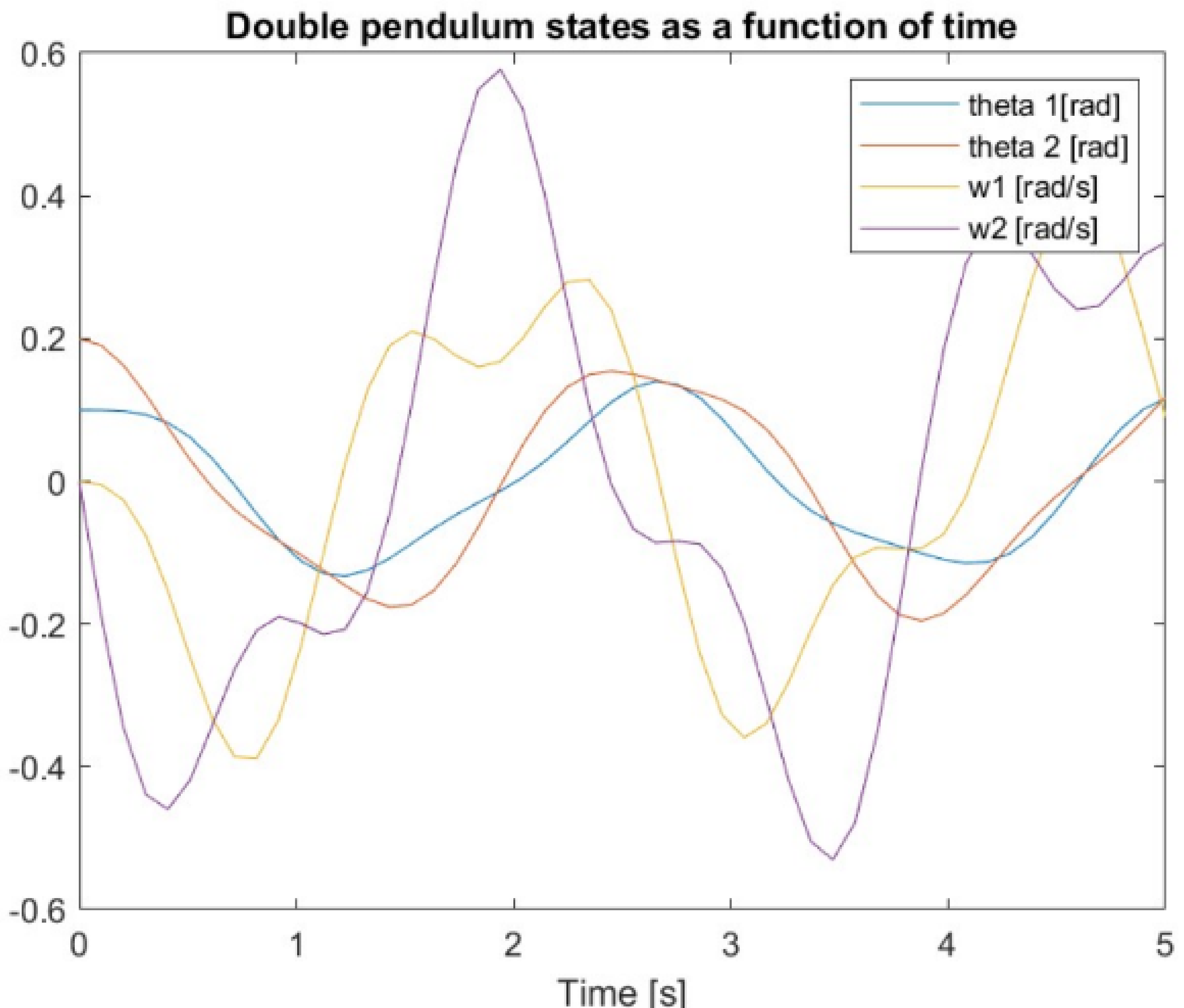
Now Solving these differential equations by help of calculator we obtain theta1 and theta 2

Equation of Motion Using Lagrangian

$$\ddot{\theta}_1 = -\frac{m_2 l_1 w_1^2 \sin(2\Delta\theta) + 2m_2 l_2 w_2^2 sin(\Delta\theta) + 2m_2 g\cos\left(\theta_2\right)\sin(\Delta\theta) + 2m_1 gsin\left(\theta_1\right)}{2l_1(m_1 + m_2 sin^2\Delta\theta)}$$

$$\ddot{\theta}_2 = \frac{m_2 l_2 w_2^2 sin(2\Delta\theta) + 2\left(m_1 + m_2\right) l_1 w_1^2 sin(\Delta\theta) + 2\left(m_1 + m_2\right) g\cos\left(\theta_1\right) sin(\Delta\theta)}{2l_2\left(m_1 + m_2 sin^2\Delta\theta\right)}$$

We have also written Matlab code for plot of theta 1, theta 2, w1 ,w2



Trajectory of Pendulum

we will show it in Matlab Simulation

Flipping of Double Pendulum

The minimum condition for inner pendulum to flip

$$\text{when } \theta_1 = \pi \text{ and } \theta_2 = 0.$$

The minimum potential energy required is

$$U_{min1} = g(l_1(m_1 + m_2) - l_2 m_2).$$

Flipping of Double Pendulum

Potential energy U must be greater than or equal to its Umin

$$U \geq U_{min1}.$$

$$l_1(m_1 + m_2)(\cos\theta_1 + 1) + l_2 m_2(\cos\theta_2 - 1) \leq 0,$$

Until and unless above is satisfied the inner pendulum cannot flip.

Flipping of Double Pendulum

The minimum condition for outer pendulum to flip

$$\theta_1 = 0 \text{ and } \theta_2 = \pi.$$

Potential energy U must be greater than or equal to its Umin

$$U \geq U_{min}$$

$$l_1(m_1 + m_2)(\cos\theta_1 - 1) + l_2 m_2(\cos\theta_2 + 1) \leq 0.$$

Flipping of Double Pendulum

Until and unless above is satisfied the outer pendulum cannot flip.

Damping of Double Pendulum

The Damping force is μ

$$\mu = k\dot{\theta},$$

the periodic external force F applied is $F(t)$

$$F(t) = F\cos\varphi t,$$

Damping of Double Pendulum

The equations of motion for damping are

For m1

$$(m_1 + m_2)l_1\ddot{\theta}_1 + m_2 l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + (m_1 + m_2)g \sin\theta_1 + k_1\dot{\theta}_1 = F_1 \cos\varphi_1$$

For m2

$$m_2 l_2\ddot{\theta}_2 + m_2 l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + m_2 g \sin\theta_2 + k_2\dot{\theta}_2 = F_2 \cos\varphi_2 t.$$

Damping of Double Pendulum

Now we assume

$$\alpha = k_1\dot{\theta}_1 - F_1 \cos \varphi_1 t,$$
$$\beta = k_2\dot{\theta}_2 - F_2 \cos \varphi_2 t,$$

After substituting and solving the differential equations using calculator we get,

Damping of Double Pendulum

$$\dot{\omega}_1 = \frac{m_2 l_1 \omega_1^2 \sin(2\Delta\theta) + 2m_2 l_2 \omega_2^2 \sin\Delta\theta + 2g m_2 \cos\theta_2 \sin\Delta\theta + 2g m_1 \sin\theta_1 + \gamma_1}{-2l_1(m_1 + m_2 \sin^2\Delta\theta)},$$

$$\dot{\omega}_2 = \frac{m_2 l_2 \omega_2^2 \sin(2\Delta\theta) + 2(m_1 + m_2) l_1 \omega_1^2 \sin\Delta\theta + 2g(m_1 + m_2)\cos\theta_1 \sin\Delta\theta + \gamma_2}{2l_2(m_1 + m_2 \sin^2\Delta\theta)}$$

where y1 and y2 represent

Damping of Double Pendulum

$$\gamma_1 = 2\alpha - 2\beta\cos\Delta\theta,$$

$$\gamma_2 = 2\alpha\cos\Delta\theta - \frac{2(m_1+m_2)}{m_2}\beta.$$