The Circulation Model

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WindKessel Model

https://www.youtube.com/watch?v=bTFCnuh9lDM

Single Compartment WindKessel Model

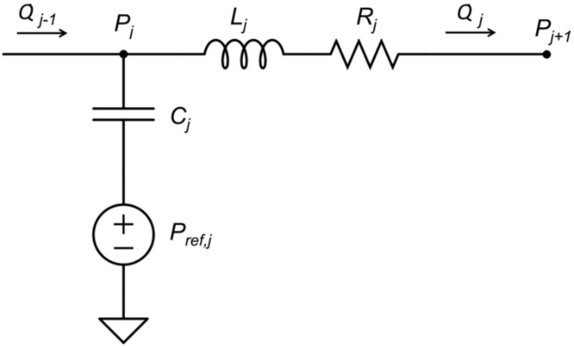


Fig. 3. Single-compartment, windkessel-type model. Q, outgoing blood flow rate; R, resistance; C, compliance; L, inertance; j, j + 1, j − 1, compartment index; P_{ref}, extravascular pressure reference (atmospheric pressure or P_{pl}, depending on the location of j).

Hydraulic Resistance

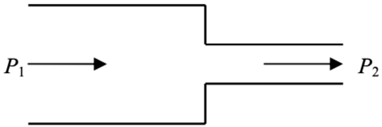


Figure: Hydraulic Resistance

Schematic diagram of cardiovascular system

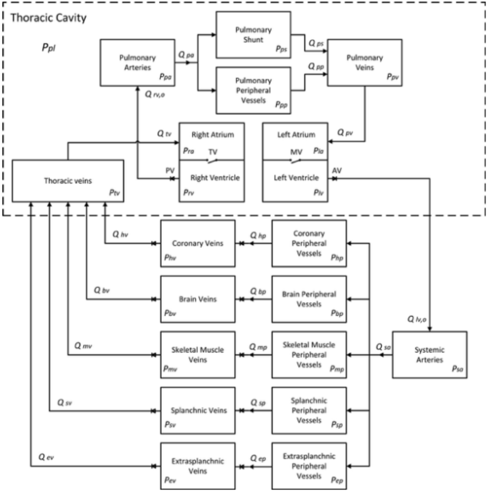


Fig. 2. Schematic diagram of the cardiovascular system model. P, pressure; Q, blood flow; MV, mitral valve; AV, aortic valve; TV, tricuspid valve; PV, pulmonary valve. Subscripts: la, left atrium; lv, left ventricle; lv, o, left-ventricle output; sa, systemic arteries; sp, splanchnic peripheral compartment; sv, splanchnic veins; ep, extrasplanchnic peripheral compartment; ev, extrasplanchnic veins; mp, skeletal muscle peripheral compartment; mv, skeletal muscle veins; bp, brain peripheral compartment; bv, brain veins; hp, coronary peripheral compartment; hv, coronary veins; tv, thoracic veins; ra, right atrium; rv, right ventricle; rv, o, right-ventricle output; pa, pulmonary artery; pp, pulmonary peripheral circulation; ps, pulmonary shunt; pv, pulmonary veins; pl, pleural space.

Linear Equations

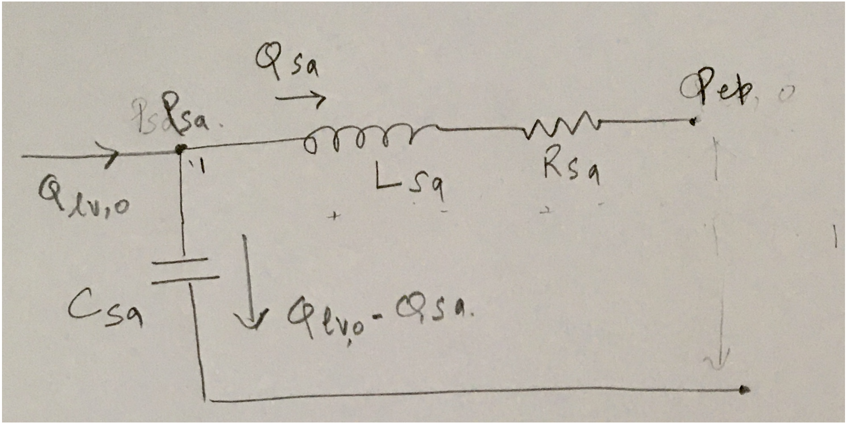
$$C_{sa} \cdot \frac{dP_{sa}}{dt} = Q_{lv,o} - Q_{sa} \tag{A1}$$

$$L_{sa} \cdot \frac{dQ_{sa}}{dt} = P_{sa} - P_{ep} - R_{sa} \cdot Q_{sa} \tag{A2}$$

$$V_{sa} = C_{sa} \cdot P_{sa} + V_{u,sa} \tag{A3}$$

$$C_{p,eq} \cdot \frac{dP_{ep}}{dt} = Q_{sa} - \sum_j Q_{jp} \tag{A4}$$

$$P_{ep} = P_{sp} = P_{mp} = P_{bp} = P_{hp} \tag{A5}$$


Q sa
Psa.
Pep, o
Qlv,o
Lsa
Rsa
Csa
Qlv,o - Qsa.

Apply KVL

$$L_{sa}\frac{dQ_{sa}}{dt} + Q_{sa}R_{sa} = P_{sa} - P_{ep}$$

$$q = cV$$

$$I = c\frac{dV}{dt}.$$

$$C_{sa}\frac{dP_{sa}}{dt} = Q_{lv,o} - Q_{sa}.$$

$$C_{sa} \cdot \frac{dP_{sa}}{dt} = Q_{lv,o} - Q_{sa}$$

$$L_{sa} \cdot \frac{dQ_{sa}}{dt} = P_{sa} - P_{ep} - R_{sa} \cdot Q_{sa}$$

$$P_{ep} = P_{sp} = P_{mp} = P_{bp} = P_{hp}$$

Linearity

Degree of differential equation is 1.

Superposition

Superposition

$$L_{sa}\frac{dQ}{dt} + Q R_{sa} = P$$

Q 1st input $\longrightarrow Q_1$ o/p $\longrightarrow P_1$

2nd i/p $\longrightarrow Q_2$ o/p $\longrightarrow P_2$

$$L_{sa}\frac{dQ_1}{dt} + Q_1 R_{sa} = P_1 \quad \text{— (1)}$$

$$L_{sa}\frac{dQ_2}{dt} + Q_2 R_{sa} = P_2 \quad \text{— (2)}$$

Superposition

$$\boxed{L_{sa} \frac{d}{dt}(Q_1 + Q_2) + R_{sa}(Q_1 + Q_2) = P_1 + P_2}$$

Equation of Pressure and Flow

$$V_j = \underbrace{C_j \cdot P_{tm,j}}_{V_{e,j}} + V_{u,j} \qquad (1)$$

The volume of each of these compartment is computed as sum of unstressed component V_{uj} and excess volume component V_{ej} which is associated with the increase in transmural pressure.

Non linear collapsible PV relationship

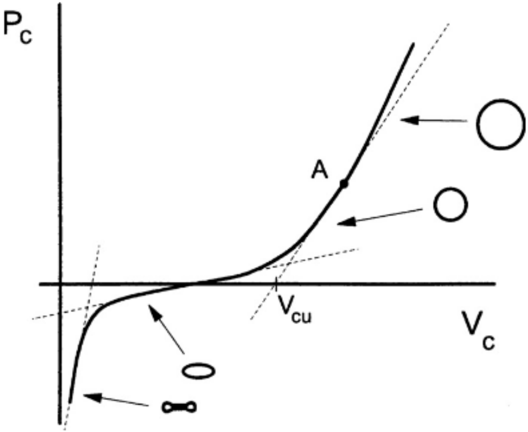


Fig. 4. Typical pressure–volume (PV) relationship of a blood vessel. P_c, transmural pressure; V_c, volume; V_{cu}, unstressed volume; A, normal operating point along the PV curve. Reproduced with permission from Timmons (65).

Non linear collapsible PV relationship

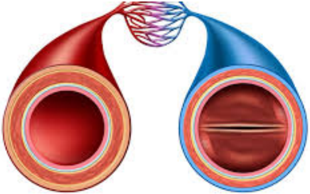


Figure: Blood vessel

Venous system

Blood returns to heart.

Intravascular

Inside the blood vessel

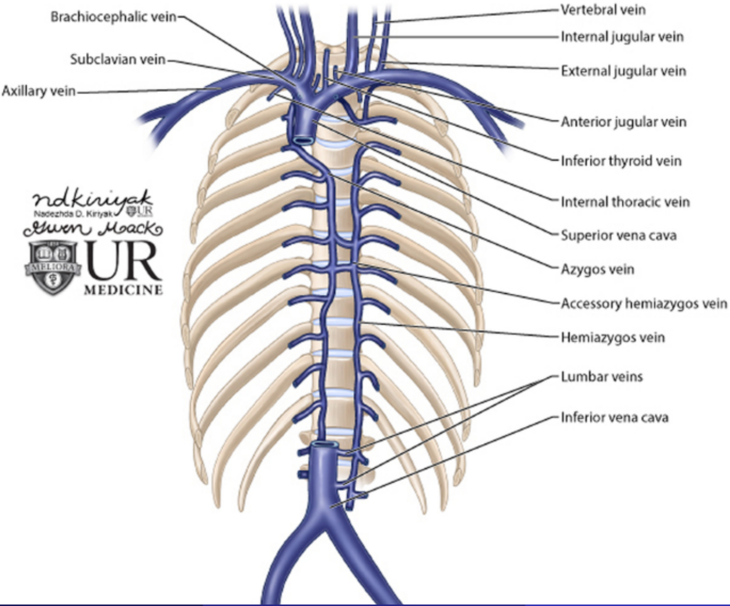
venous circuilation

Intravascular pressure is low and extravascular pressure is high. Hence, veins can collapse. That's why we get a non linear relation.

Thoracic vein

Due to high intrathoracic pressure the thoracic veins can collapse.

Thoracic vein



PV Relationship of Thoracic Vein Compartment

$$P_{tm,tv} = \begin{cases} D_1 + K_1 \cdot (V_{tv} - V_{u,tv}) - \psi & V_{tv} \geq V_{u,tv} \\ D_2 + K_2 \cdot e^{\frac{V_{tv}}{V_{tv,min}}} - \psi & V_{tv} < V_{u,tv} \end{cases} \qquad (2)$$
$$\text{with } \psi = K_{xp} \Big/ \left(e^{\frac{V_{tv}}{K_{xv}}} - 1 \right)$$

Thoracic vein compartment is modeled via a non linear collapsible PV relationship.

Table

Table 2. *Parameters of the thoracic veins (Eqs. 2 and 3)*

Pressure–Volume Relationship	Resistance
$D_1 = 0.3855$ mmHg (9)	$K_R = 0.001$ mmHg $\cdot$ s $\cdot$ ml^{-1} (9)
$K_1 = 0.15$ mmHg/ml (9)	$V_{tv,max} = 350$ ml (9)
$V_{u,tv} = 130$ ml (9)	$R_{tv,0} = 0.025$ mmHg $\cdot$ s $\cdot$ ml^{-1} (9)
$D_2 = -5$ mmHg (9)	
$K_2 = 0.4$ mmHg (9)	
$V_{tv,min} = 50$ ml (9)	
$K_{xp} = 2$ mmHg (49)	
$K_{xv} = 8$ ml (49)	

Code

Non linear collapsible PV relationship

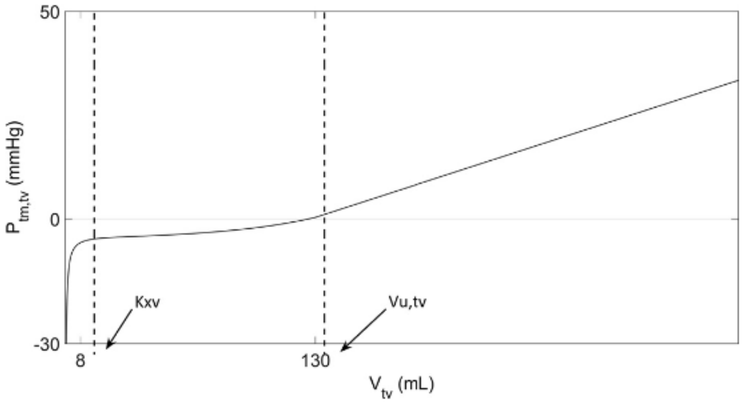


Fig. 5. PV relationship of the thoracic veins compartment according to *Eq. 2*. $P_{tm,tv}$, transmural pressure; V_{tv}, volume; $V_{u,tv}$, unstressed volume; K_{xv}, volume below which ψ becomes dominant.

Resistance of Thoracic Vein Compartment

$$R_{tv} = K_R \cdot \left(\frac{V_{tv,max}}{V_{tv}}\right)^2 + R_{tv,0} \qquad (3)$$

The resistance of the thoracic veins compartment varies as a function of the volume.

Assumption

Gravity on the cardiovascular system has not been taken into account.

P_{atm} has been assumed to be **Zero**

The End