Network Flow - A study

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1 The Problem

We have an abstraction of traffic flow in the form of flow networks. We have a source that generate traffic, a sink that absorb the traffic and edges with capacities that transmit the traffic.

1.1 Flow Networks

The traffic is refereed to as the flow, an abstract entity. A flow network is a directed graph $G = (V, E)$ with the following features.

- Each edge e has a capacity, a non-negative number c_e.
- Single *source* node $s \in V$
- Single *sink* node $t \in V$

Assumptions

- No edge enter the source and no edge leaves the sink.
- Atleast one edge is incident to each node.
- All capacities are integers.

1.2 Defining Flow

$s - t$ flow is a function f, mapping each edge to a non-negative real number $f : E \to R^+$ satisfying the following.

1. (Capacity constraint) For each $e \in E$, $0 \leq f(e) \leq c_e$.
2. (Conservation constraint) For each node v other than s and t

$$\sum_{e\ into\ v} f(e) = \sum_{e\ out\ of\ v} f(e)$$

The value of flow f is the flow generated at the source Notation : if $S \subseteq V$, then $f^{out}(S) = \sum_{e\ out\ of\ S} f(e)$ and $f^{in}(S) = \sum_{e\ into\ S} f(e)$

$$v(f) = f^{out}(s)$$