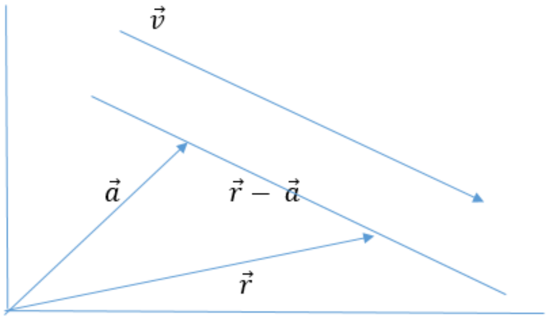
Guru Academy and Labs, Ch11 Three Dimensional Geometry – Formula Sheet

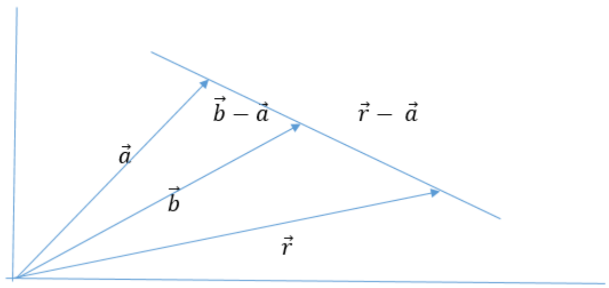
One point One parallel vector:



$\vec{r} - \vec{a} = t\,\vec{v}$

$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

Two points:

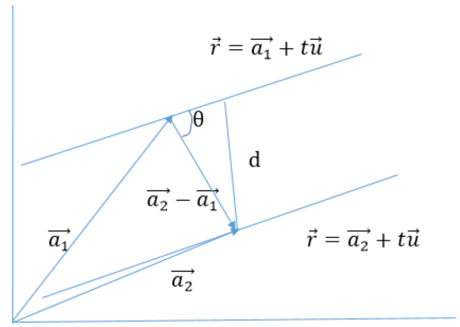


$\vec{r} - \vec{a} = t\,(\vec{b} - \vec{a})$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$(\vec{r} - \vec{a})$ x $(\vec{b} - \vec{a}) = 0$

Perpendicular distance between parallel lines:

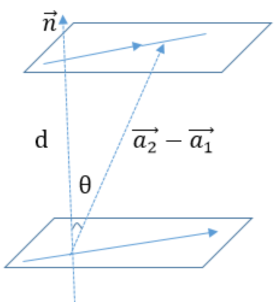


Here angle between $\vec{a_2} - \vec{a_1}$ and $\vec{u}$ is taken as θ,

since $\vec{u}$ is known.

d = $(a_2 - a_1)\sin\theta.\frac{|\vec{u}|}{|\vec{u}|} = \frac{\vec{u}\ X\ (\vec{a_2}-\vec{a_1})}{|\vec{u}|}$

Distance between skewed lines:



Here angle between $\vec{a_2} - \vec{a_1}$ and $\vec{n}$ is taken as θ,

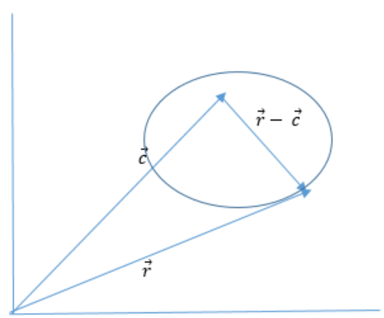
since $\vec{n}$ is known. Note $\vec{n}$ is normal vector not unit vector.

d=$|\vec{a_2} - \vec{a_1}|\cos\theta\,\frac{|\vec{n}|}{|\vec{n}|} = \frac{\vec{n}.(\vec{a_2}-\vec{a_1})}{|\vec{n}|} = \frac{(\vec{a_2}-\vec{a_1}).(\vec{u}\ X\ \vec{v})}{|\vec{u}\ X\ \vec{v}|}$

For intersection d = 0, $[\vec{a_2} - \vec{a_1}, \vec{u}, \vec{v}] = 0$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Centre and radius given:



$|\vec{r} - \vec{c}| = \vec{a}$

$(x - c_1)^2 + (y - c_2)^2 + (z - c_3)^2 = a^2$

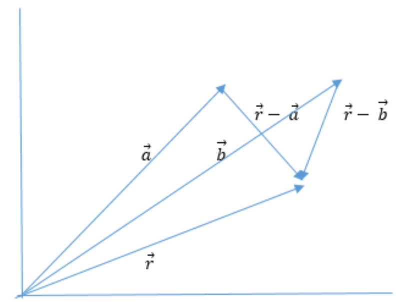
General equation

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

$(-u,-v,-w)$ = centre

$\sqrt{u^2 + v^2 + w^2 - d}$ = radius

Two end points of a diameter given:



$(\vec{r} - \vec{a}).(\vec{r} - \vec{b}) = 0$

(Because angle subtended is 90°

from the ends of the diameter)

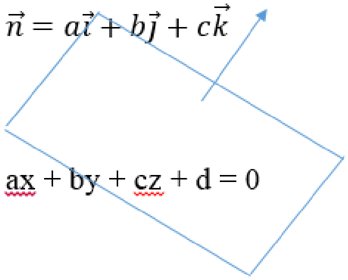
$(x - x_1)(x - x_2) +$

$(y - y_1)(y - y_2) +$

$(z - z_1)(z - z_2) = 0$

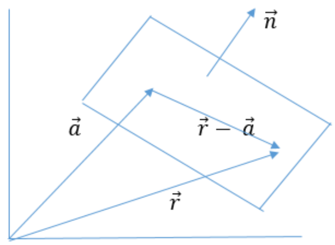
How is a plane represented?

$\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$



ax + by + cz + d = 0

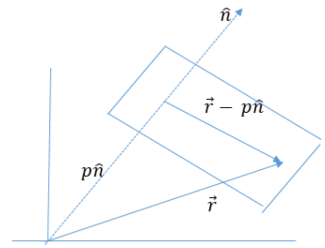
One point, One normal vector:



$(\vec{r} - \vec{a}).\vec{n} = 0$

$(x-x_1)a + (y-y_1)b + (z-z_1)\ c \quad = 0$

Perpendicular distance of a plane from origin p given, normal vector $\hat{n}$ given

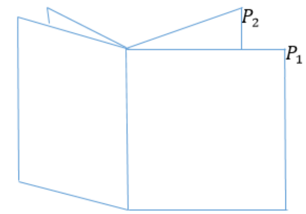


$(\vec{r} - p\,\hat{n}).\ \hat{n} = 0$

$\vec{r}\,.\,\hat{n} \quad = p$

$xl + ym + zn = p$

Intersection of two planes:



$\vec{r}\,.\,\vec{n_1} \quad = q_1$

$\vec{r}\,.\,\vec{n_2} \quad = q_2$

There are infinite planes, passing through the intersection of the two planes.

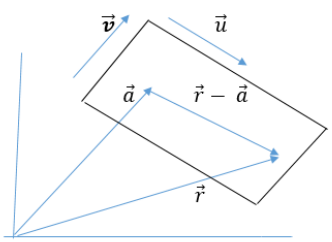
$P_1 + \lambda\, P_2 = 0$

$(\vec{r}\,.\,\vec{n_1} - q_1) + \lambda(\vec{r}\,.\,\vec{n_2} - q_2) = 0$

$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

If one point lying on the plane is given, substitute the point in the above λ equation to find the value of λ.

One point, Two parallel vectors:



$\vec{r} - \vec{a} = t\,\vec{u} + s\vec{v}$

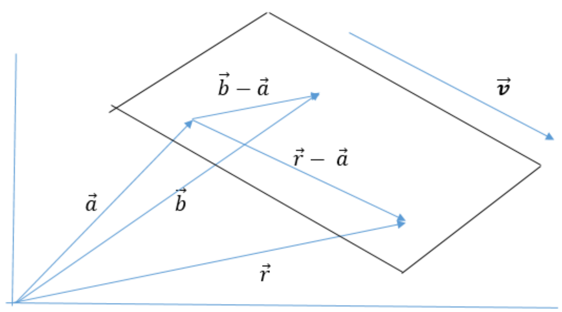
$(\vec{r} - \vec{a}).\ (\vec{u} \times \vec{v}) = 0$

$[\vec{r} - \vec{a},\ \vec{u}, \vec{v}] = 0$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

For two lines contained in the plane take one point from one of the line take two parallel vectors from the lines.

Two points, one parallel vector:



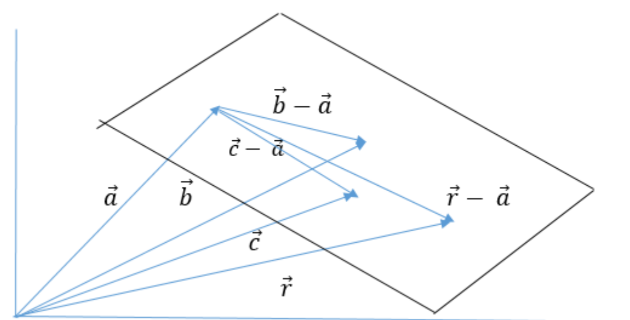
$\vec{r} - \vec{a} = t\,(\vec{b} - \vec{a}) + s\vec{v}$

$[\vec{r} - \vec{a}\,, \vec{b} - \vec{a},\ \vec{v}\,] = 0$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0$$

For two parallel lines contained in the plane, take 2 points from each of the line, then take one parallel vector.

Three points:



$\vec{r} - \vec{a} = t\,(\vec{b} - \vec{a}) + s(\vec{c} - \vec{a})$

$[\vec{r} - \vec{a}, \vec{b} - \vec{a},\ \vec{c} - \vec{a}] = 0$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Perpendicular distance between plane Ax+By+Cz+D and a point (x_1, y_1):

$$\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2+B^2+C^2}}\right|$$

Distance between two parallel planes:

$$\left|\frac{D_1 - D_2}{\sqrt{A^2+B^2+C^2}}\right|$$

Angle between two planes:

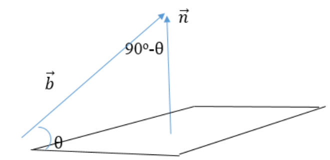
$$\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}$$

Angle between two lines:

$$\cos\theta = \frac{\vec{u}.\vec{v}}{|\vec{u}||\vec{v}|}$$

Angle between plane and a line:

$$\cos(90-\theta) = \sin\theta = \frac{\overrightarrow{n_1}.\vec{b}}{|\overrightarrow{n_1}||\vec{b}|}$$



2D equations in 3D:

$x^2 + y^2 = a^2$

is circle equation in 2D,

but a cylinder in 3D parallel to Z axis.

Z-axis line $x^2 + y^2 = 0$,

Y-axis line $z^2 + x^2 = 0$,

X-axis line $y^2 + z^2 = 0$.

Direction Cosines DCs and Direction Ratios DRs:

If α, β, γ are the angles a line makes with the X, Y, Z axes then $\cos\alpha = l$, $\cos\beta = m$, $\cos\gamma = n$, so l, m, n are the DCs. $l^2 + m^2 + n^2 = 1$. DCs basically gives the direction in which a line is lying.

If two points are given (x_1, y_1, z_1), (x_2, y_2, z_2) then $\langle x_1 - x_2,\ y_1 - y_2,\ z_1 - z_2\rangle$ is the DRs. This also basically gives the direction in which the line is lying, but it is not the same as DCs.

DCs can be obtained by,

$$\left\langle \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}}, \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}}, \frac{z_1-z_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}} \right\rangle$$