2.4.20

EE25BTECH11025 - Ganachari Vishwambhar

Question:
Find the value of λ such that the vectors $\mathbf{a} = 2\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are orthogonal.
Solution:
Given vectors are:

$$\mathbf{a} = \begin{pmatrix} 2 \\ \lambda \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

For two vectors to be orthogonal their dot product should be equal to zero which is equal to product of transpose column matrix $\mathbf{a}$ and column matrix $\mathbf{b}$:

$$\mathbf{a}^T\mathbf{b} = 0 \tag{2}$$

$$\begin{pmatrix} 2 & \lambda & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0 \tag{3}$$

$$2 + 2\lambda + 3 = 0 \tag{4}$$

$$\lambda = \begin{pmatrix} \frac{-5}{2} \end{pmatrix} \tag{5}$$

Therefore, the final vectors are:

$$\mathbf{a} = \begin{pmatrix} 2 \\ \left(\frac{-5}{2}\right) \\ 1 \end{pmatrix} \tag{6}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{7}$$

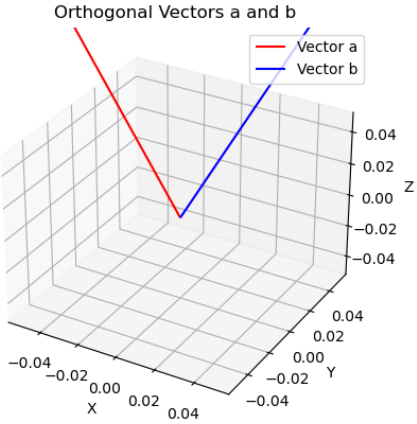

Fig. 1: Plot of orthogonal vectors **a** and **b**