1.7.12

Rathlavath Jeevan-AI25BTECH11026

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Question

Find the value of k, if the points $P(5, 4)$, $Q(7, k)$ and $R(9, -2)$ are collinear.

Hint: Three points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are collinear if the area of the triangle formed by them is zero.

Theoretical Solution

Solution:

$$\mathbf{P} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 7 \\ k \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} 9 \\ -2 \end{pmatrix} \tag{1}$$

Collinearity via rank Three points P, Q, R are collinear iff

$$\operatorname{rank}\begin{pmatrix} \mathbf{Q}-\mathbf{P} & \mathbf{R}-\mathbf{P} \end{pmatrix} = 1. \tag{2}$$

Compute the direction columns:

$$\mathbf{Q}-\mathbf{P} = \begin{pmatrix} 7-5 \\ k-4 \end{pmatrix} = \begin{pmatrix} 2 \\ k-4 \end{pmatrix}, \qquad \mathbf{R}-\mathbf{P} = \begin{pmatrix} 9-5 \\ -2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}. \tag{3}$$

Hence the collinearity matrix is

$$M = \begin{pmatrix} 2 & 4 \\ k-4 & -6 \end{pmatrix}. \tag{4}$$

Row reduction (rank = 1)

Theoretical Solution

$$\begin{pmatrix} 2 & 4 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2-(k-4)R_1} \begin{pmatrix} 1 & 2 \\ 0 & 2(1-k) \end{pmatrix}. \tag{5}$$

For $\text{rank}(M) = 1$, the second row must be the zero row:

$$2(1-k) = 0 \quad \Rightarrow \quad k = 1. \tag{6}$$

Conclusion For $k = \boxed{1}$, the three points $P(5,4)$, $Q(7,k)$, $R(9,-2)$ are collinear.

C Code

```c
#include <stdio.h>

int main() {
    int x1 = 5, y1 = 4;
    int x2 = 7, y2; // y2 = k
    int x3 = 9, y3 = -2;
    int k;

    // Equation: x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2) = 0
    // Substituting values
    // 5(k - (-2)) + 7((-2) - 4) + 9(4 - k) = 0
    // Solve manually inside program:

    // Simplified form: -4k + 4 = 0 => k = 1
    k = 1;
```

C Code

```
    printf(The value of k is: %d\n, k);

    return 0;
}
```

Python Code

```
import matplotlib.pyplot as plt

# Given points
P = (5, 4)
R = (9, -2)

# Find k using collinearity condition
slope_PR = (R[1] - P[1]) / (R[0] - P[0])
k = slope_PR * (7 - 5) + 4
Q = (7, k)

# Plotting the points and line
plt.figure(figsize=(6,6))
plt.plot([P[0], Q[0], R[0]], [P[1], Q[1], R[1]], 'ro') # points
plt.plot([P[0], Q[0], R[0]], [P[1], Q[1], R[1]], 'b-') # line
```

Python Code

```
# Annotating points
plt.text(P[0]+0.1, P[1], fP{P})
plt.text(Q[0]+0.1, Q[1], fQ{Q})
plt.text(R[0]+0.1, R[1], fR{R})

plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True)
plt.title(Collinear Points Diagram)
plt.xlabel(x-axis)
plt.ylabel(y-axis)

# Save as image
plt.savefig(collinear_points.png, dpi=300)
plt.show()
```

Python Code

```
print(Value of k:, k)
print(Graph saved as 'collinear_points.png')
```

Plot