**Question 2.2.19:**
The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$.

**Solution:**

**Given**

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

We require:

$$\mathbf{u}^\top \frac{\mathbf{a}+\mathbf{b}}{\|\mathbf{a}+\mathbf{b}\|} = 1 \tag{2}$$

**Sum of vectors:**

$$\mathbf{a}+\mathbf{b} = \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \tag{3}$$

**Dot product:**

$$\mathbf{u}^\top(\mathbf{a}+\mathbf{b}) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} = (2+\lambda)+6+(-2) = \lambda+6 \tag{4}$$

**Norm of the sum:**

$$\|\mathbf{a}+\mathbf{b}\| = \sqrt{(2+\lambda)^2+6^2+(-2)^2} = \sqrt{(2+\lambda)^2+40} \tag{5}$$

**Condition:**

$$\frac{\lambda+6}{\sqrt{(2+\lambda)^2+40}} = 1 \tag{6}$$

**Simplify:**

$$\lambda+6 = \sqrt{(\lambda+2)^2+40} \tag{7}$$

Squaring,

$$(\lambda+6)^2 = (\lambda+2)^2+40 \tag{8}$$

$$\lambda^2+12\lambda+36 = \lambda^2+4\lambda+44 \tag{9}$$

$$8\lambda = 8 \quad \Rightarrow \quad \lambda = 1 \tag{10}$$

**Conclusion:** The required value is

$$\boxed{\lambda = 1} \tag{11}$$

$$\boxed{a = -3} \tag{12}$$

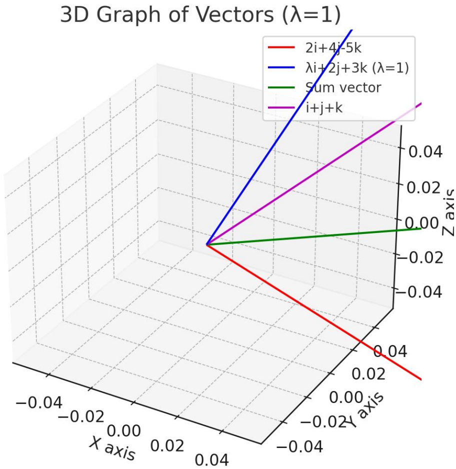
3D Graph of Vectors (λ=1)
2i+4j-5k
λi+2j+3k (λ=1)
Sum vector
i+j+k
−0.04
−0.02
0.00
0.02
0.04
X axis
Y axis
Z axis

Fig. 1