1.7.5

AI25BTECH11019 - MENAVATH SAI SANJANA

Question:
Find the value of p for which the points (-5,1),(1,p),(4,-2) are collinear
Solution:

Let the points be

Point	Name
$\begin{pmatrix} -5 \\ 1 \end{pmatrix}$	**A**
$\begin{pmatrix} 1 \\ p \end{pmatrix}$	**B**
$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	**C**

TABLE 0: Variables Used

The difference vectors are

$$(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 6 \\ p - 1 \end{pmatrix}, \tag{0.1}$$

$$(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 9 \\ -3 \end{pmatrix}. \tag{0.2}$$

Thus, $M^T = (\mathbf{B} - \mathbf{A} \ \ \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 6 & p - 1 \\ 9 & -3 \end{pmatrix}$.

Apply row operations to convert M^T into upper triangular form.

$$\begin{pmatrix} 6 & p - 1 \\ 9 & -3 \end{pmatrix} \xrightarrow{R_2 \to R_2 - \frac{3}{2}R_1} \begin{pmatrix} 6 & p - 1 \\ 0 & -\frac{3}{2}(p + 1) \end{pmatrix}. \tag{0.3}$$

For collinearity, rank(M^T) = 1. This happens when the second row is zero: $-\frac{3}{2}(p+1) = 0$.

$p = -1$

Hence, the three points A, B, C are collinear when $p = -1$.

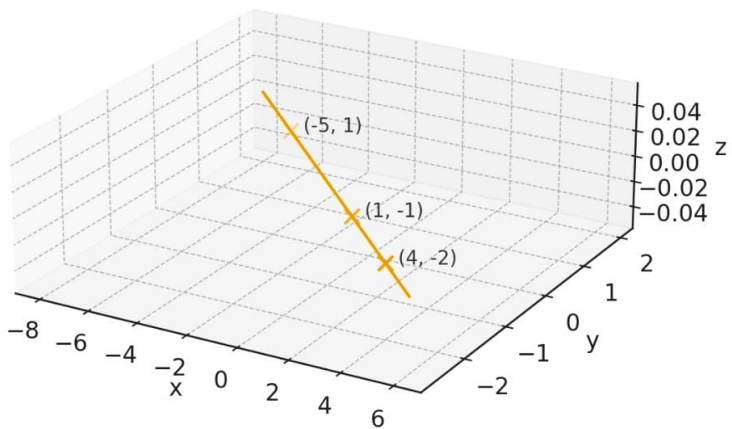Collinearity in 3D view (points lie on line in z=0 plane)

Fig. 0.1