1.2.24

AI25BTECH11016-Varun

Question:

Rain is falling vertically with a speed of 35 m/s. Wind starts blowing after some time with a speed of 12 m/s in the east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

Solution:

$\mathbf{v}_{r/w}$ be the velocity of rain relative to the wind (downward),

$\mathbf{v}_{r/w} = \begin{pmatrix} 0 \\ -35 \end{pmatrix}$

$\mathbf{v}_w$ be the velocity of wind relative to ground

$\mathbf{v}_w = \begin{pmatrix} -12 \\ 0 \end{pmatrix}$

$\mathbf{v}_r$ be the velocity of rain relative to ground Take $+x$ to the east and $+y$ upward.

Resultant rain velocity (relative to ground).

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} \tag{1}$$

Angle from vertical (downward). Let θ be the angle between $\mathbf{v}_r$ and the downward vertical unit vector

$$\hat{u}_{\downarrow} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

.

Using the dot product,

$$\cos\theta = \frac{\mathbf{v}_r \cdot \hat{u}_{\downarrow}}{\|\mathbf{v}_r\|} \tag{2}$$

$$\cos\theta = \frac{v_{r/w}}{\sqrt{v_{r/w}^2 + v_w^2}} \tag{3}$$

Equation 3 is obtained by substituting Equation 1 in Equation 2

Substitute given values.

For this problem, the speed of rain w.r.t wind is $v_{r/w} = 35$ m/s

the wind speed is $v_w = 12$ m/s (east to west).

$$\cos\theta = \frac{35}{\sqrt{35^2 + 12^2}} \tag{4}$$

$$\cos\theta = \frac{35}{37} \tag{5}$$

$$\theta = \cos^{-1}\left(\frac{35}{37}\right) \tag{6}$$

Rain falls at an angle of $\theta = \cos^{-1}\left(\frac{35}{37}\right)$ west of vertical downward

The rain's velocity component is toward the west and downward, so the umbrella must be kept in the opposite direction:

Hence,

Umbrella direction should be at an angle $\cos^{-1}\left(\frac{35}{37}\right)$ east of vertical upward.

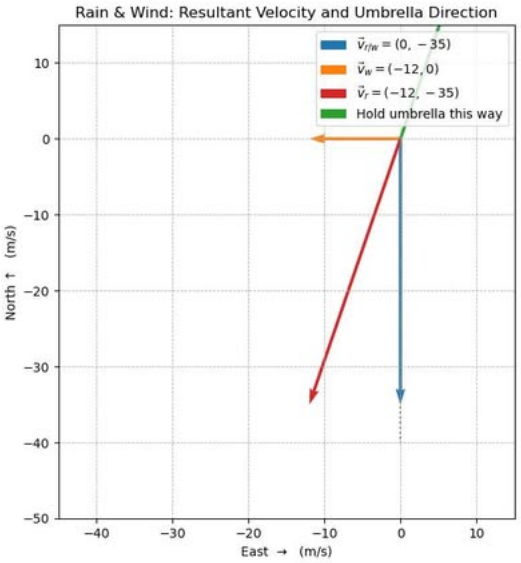

Fig. 0.1