

1.5.39

EE24BTECH11053 - Surya Sri

Question: The point R divides the line segment PQ in the ratio $3 : 1$ and S is the midpoint of the line segment PR . Find the position vector of S in terms of P and Q .

Solution: The position vector of S in terms of P and Q is given completely in matrix form as follows: Let

$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$\mathbf{R} = \frac{3\mathbf{Q} + 1\mathbf{P}}{3 + 1} = \frac{3\mathbf{Q} + \mathbf{P}}{4} \quad (1)$$

So,

$$\mathbf{R} = \frac{1}{4} \begin{pmatrix} p_1 + 3q_1 \\ p_2 + 3q_2 \\ p_3 + 3q_3 \end{pmatrix} \quad (2)$$

Since S is the midpoint of PR ,

$$\mathbf{S} = \frac{\mathbf{P} + \mathbf{R}}{2} \quad (3)$$

$$\mathbf{S} = \frac{1}{2} \left(\mathbf{P} + \frac{1}{4}(\mathbf{P} + 3\mathbf{Q}) \right) \quad (4)$$

$$= \frac{1}{2} \left(\frac{4\mathbf{P} + \mathbf{P} + 3\mathbf{Q}}{4} \right) = \frac{1}{2} \left(\frac{5\mathbf{P} + 3\mathbf{Q}}{4} \right) = \frac{5\mathbf{P} + 3\mathbf{Q}}{8} \quad (5)$$

Therefore, the position vector of S is:

$$\mathbf{S} = \frac{1}{8} \begin{pmatrix} 5p_1 + 3q_1 \\ 5p_2 + 3q_2 \\ 5p_3 + 3q_3 \end{pmatrix} \quad (6)$$

Refer to Fig. 0

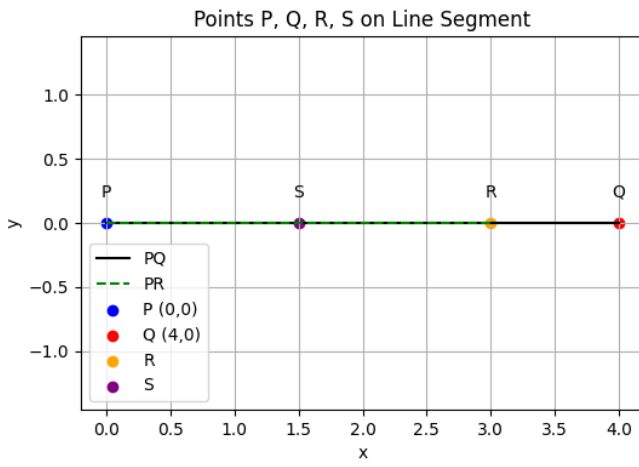


Fig. 0