2.8.6

EE25BTECH11025 - Ganachari Vishwambhar

Question:
Assuming that the straight lines work as the plane mirror for a point, find the image of the point $(1, 2)$ in the line $x - 3y + 4 = 0$.
Solution:
Translating the system by $\mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ so that the line passes through origin:

$$L = \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -4; \mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{P}_{trans} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \tag{2}$$

$$L_{trans} = \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{3}$$

Finding the normal vector:

$$\mathbf{N} = \begin{pmatrix} 1 & -3 \end{pmatrix} \tag{4}$$

Finding the unit normal vector:

$$||\mathbf{N}|| = \sqrt{1^2 + (-3)^2} = \sqrt{10} \tag{5}$$

$$\mathbf{n} = \frac{\mathbf{N}}{||\mathbf{N}||} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{6}$$

Calculating the reflection matrix R is given by the formula $R = I - 2\mathbf{n}\mathbf{n}^T$

$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2\left(\frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -3 \end{pmatrix}\right)\left(\frac{1}{\sqrt{10}} \begin{pmatrix} 1 & -3 \end{pmatrix}\right) = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{-4}{5} \end{pmatrix} \tag{7}$$

Reflecting the given point:

$$\mathbf{P'}_{trans} = R.P_{trans} = \begin{pmatrix} \frac{26}{5} \\ \frac{7}{5} \end{pmatrix} \tag{8}$$

Inverting the translation:

$$\mathbf{P'} = \mathbf{P'}_{trans} + \mathbf{A} = \begin{pmatrix} \frac{6}{5} \\ \frac{7}{5} \end{pmatrix} \tag{9}$$

Thus the final image of the given point is $\mathbf{P'} = \begin{pmatrix} \frac{6}{5} \\ \frac{7}{5} \end{pmatrix}$

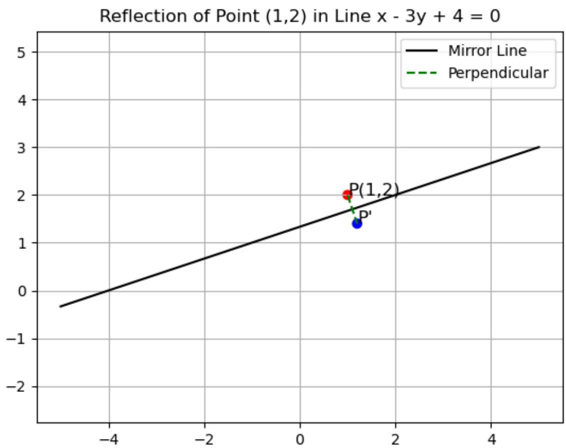

Fig. 1: Plot of the given line, point and reflected point