1.10.11

EE25BTECH11053 - Surya Sri

Question:
Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\mathbf{a} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\mathbf{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Solution:

$$\mathbf{a} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Resultant Vector is,

$$\mathbf{R} = \mathbf{a} + \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \tag{1}$$

Magnitude of Resultant vector is,

$$\|R\| = \sqrt{3^2 + 1^2 + 0^2} = \sqrt{9 + 1} = \sqrt{10} \tag{2}$$

Let the desired vector be,

$$\|k\mathbf{R}\| = 5 \tag{3}$$

$$\|k\| \sqrt{10} = 5 \tag{4}$$

$$\implies k = \frac{5}{\sqrt{10}} \tag{5}$$

$$\mathbf{v} = k \cdot \mathbf{R} = \frac{5}{\sqrt{10}} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{15}{\sqrt{10}} \\ \frac{5}{\sqrt{10}} \\ 0 \end{bmatrix} \tag{6}$$

Therefore,

$$\mathbf{v} = \begin{bmatrix} \frac{15}{\sqrt{10}} \\ \frac{5}{\sqrt{10}} \\ 0 \end{bmatrix} \tag{7}$$

is a matrix with magnitude 5 parallel to the resultant vector.

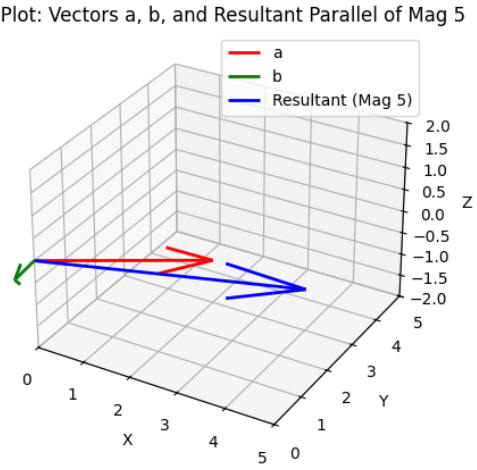
3D Plot: Vectors a, b, and Resultant Parallel of Mag 5
a
b
Resultant (Mag 5)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Z
0
1
2
3
4
5
X
0
1
2
3
4
5
Y

Fig. 0