Question 1.7.12:
Find the value of k, if the points $P(5,4)$, $Q(7,k)$ and $R(9,-2)$ are collinear.

Hint: Three points $P(x_1,y_1)$, $Q(x_2,y_2)$, $R(x_3,y_3)$ are collinear if the area of the triangle formed by them is zero.

Solution

QUESTION

Find the value of a, if the distance between the points $A\begin{pmatrix}-3\\-14\end{pmatrix}$ and $B\begin{pmatrix}a\\-5\end{pmatrix}$ is 9 units.

SOLUTION

$$\mathbf{P}=\begin{pmatrix}5\\4\end{pmatrix},\qquad \mathbf{Q}=\begin{pmatrix}7\\k\end{pmatrix},\qquad \mathbf{R}=\begin{pmatrix}9\\-2\end{pmatrix} \tag{1}$$

Collinearity via rank Three points P,Q,R are collinear iff

$$\operatorname{rank}\begin{pmatrix}\mathbf{Q}-\mathbf{P} & \mathbf{R}-\mathbf{P}\end{pmatrix}=1. \tag{2}$$

Compute the direction columns:

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}7-5\\k-4\end{pmatrix}=\begin{pmatrix}2\\k-4\end{pmatrix},\qquad \mathbf{R}-\mathbf{P}=\begin{pmatrix}9-5\\-2-4\end{pmatrix}=\begin{pmatrix}4\\-6\end{pmatrix}. \tag{3}$$

Hence the collinearity matrix is

$$M=\begin{pmatrix}2 & 4\\ k-4 & -6\end{pmatrix}. \tag{4}$$

Row reduction (rank = 1)

$$\begin{pmatrix}2 & 4\\ k-4 & -6\end{pmatrix}\xrightarrow{R_1\leftarrow\frac{1}{2}R_1}\begin{pmatrix}1 & 2\\ k-4 & -6\end{pmatrix}\xrightarrow{R_2\leftarrow R_2-(k-4)R_1}\begin{pmatrix}1 & 2\\ 0 & 2(1-k)\end{pmatrix}. \tag{5}$$

For $\operatorname{rank}(M)=1$, the second row must be the zero row:

$$2(1-k)=0\quad\Rightarrow\quad k=1. \tag{6}$$

Conclusion For $k=\boxed{1}$, the three points $P(5,4)$, $Q(7,k)$, $R(9,-2)$ are collinear.

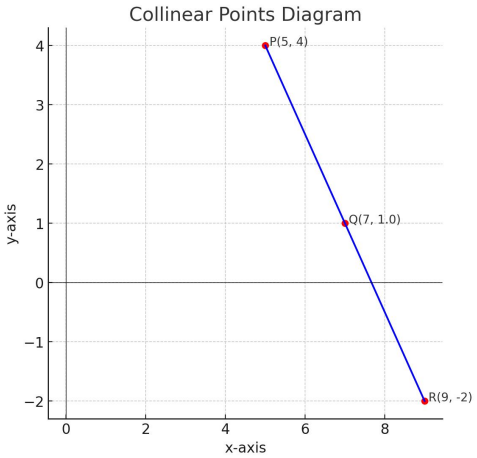
Collinear Points Diagram
P(5, 4)
Q(7, 1.0)
R(9, -2)
y-axis
x-axis
4
3
2
1
0
−1
−2
0
2
4
6
8

Fig. 1