1.5.39

Surya Sri - EE25BTECH11053

August, 2025

Question

The point R divides the line segment PQ in the ratio $3:1$ and S is the midpoint of the line segment PR. Find the position vector of S in terms of P and Q.

Variables used

Vector	Matrix
$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$	P
$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$	Q

Table: Variables Used

Solution

The position vector of S in terms of P and Q is given completely in matrix form as follows:
Let

$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$\mathbf{R} = \frac{3\mathbf{Q} + 1\mathbf{P}}{3+1} = \frac{3\mathbf{Q} + \mathbf{P}}{4} \tag{1}$$

So,

$$\mathbf{R} = \frac{1}{4} \begin{pmatrix} p_1 + 3q_1 \\ p_2 + 3q_2 \\ p_3 + 3q_3 \end{pmatrix} \tag{2}$$

Solution

Since S is the midpoint of PR,

$$\mathbf{S} = \frac{\mathbf{P} + \mathbf{R}}{2} \tag{3}$$

$$\mathbf{S} = \frac{1}{2}\left(\mathbf{P} + \frac{1}{4}(\mathbf{P} + 3\mathbf{Q})\right) \tag{4}$$

$$= \frac{1}{2}\left(\frac{4\mathbf{P} + \mathbf{P} + 3\mathbf{Q}}{4}\right) = \frac{1}{2}\left(\frac{5\mathbf{P} + 3\mathbf{Q}}{4}\right) = \frac{5\mathbf{P} + 3\mathbf{Q}}{8} \tag{5}$$

Therefore, the position vector of S is:

$$\boxed{\mathbf{S} = \frac{1}{8}\begin{pmatrix} 5p_1 + 3q_1 \\ 5p_2 + 3q_2 \\ 5p_3 + 3q_3 \end{pmatrix}} \tag{6}$$

Python

```
import matplotlib.pyplot as plt

# Step 1: Assume coordinates for P and Q
P = (0, 0)
Q = (4, 0)

# Step 2: Find coordinates of R (divides PQ in ratio 3:1)
R_x = (3 * Q[0] + 1 * P[0]) / 4
R_y = (3 * Q[1] + 1 * P[1]) / 4
R = (R_x, R_y)

# Step 3: Find coordinates of S (midpoint of PR)
S_x = (P[0] + R[0]) / 2
S_y = (P[1] + R[1]) / 2
S = (S_x, S_y)
```

Python

```
# Step 4: Plot all points
plt.figure(figsize=(6, 4))
plt.plot([P[0], Q[0]], [P[1], Q[1]], 'k-', label='PQ')
plt.plot([P[0], R[0]], [P[1], R[1]], 'g--', label='PR')
plt.scatter(*P, color='blue', label='P (0,0)')
plt.scatter(*Q, color='red', label='Q (4,0)')
plt.scatter(*R, color='orange', label='R')
plt.scatter(*S, color='purple', label='S')

# Annotate points
plt.text(P[0], P[1]+0.2, 'P', ha='center')
plt.text(Q[0], Q[1]+0.2, 'Q', ha='center')
plt.text(R[0], R[1]+0.2, 'R', ha='center')
plt.text(S[0], S[1]+0.2, 'S', ha='center')
```

Python

```
plt.legend()
plt.grid(True)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Points P, Q, R, S on Line Segment')
plt.axis('equal')
plt.savefig(graph.png)
plt.show()
```

C Code

```c
#include <stdio.h>

// For 2D vectors; change n to 3 for 3D.
#define n 2

void find_S(double P[], double Q[], double S[]) {
    // S = (5P + 3Q) / 8
    for (int i = 0; i < n; i++) {
        S[i] = (5 * P[i] + 3 * Q[i]) / 8.0;
    }
}

int main() {
    double P[n], Q[n], S[n];
```

C Code

```
    // Input coordinates for P and Q
    printf(Enter coordinates for P (x y): );
    for (int i = 0; i < n; i++) scanf(%lf, &P[i]);

    printf(Enter coordinates for Q (x y): );
    for (int i = 0; i < n; i++) scanf(%lf, &Q[i]);

    find_S(P, Q, S);

    printf(Coordinates of S: (%.3lf, %.3lf)\n, S[0], S[1]);
    return 0;
}
```

Python and C Code

```
 import subprocess

# Compile the C program
subprocess.run([gcc, points.c, -o, points])

# Run the compiled C program
result = subprocess.run([./points], capture_output=True, text=
    True)

# Print the output from the C program (solution)
print(result.stdout)
```

Graph

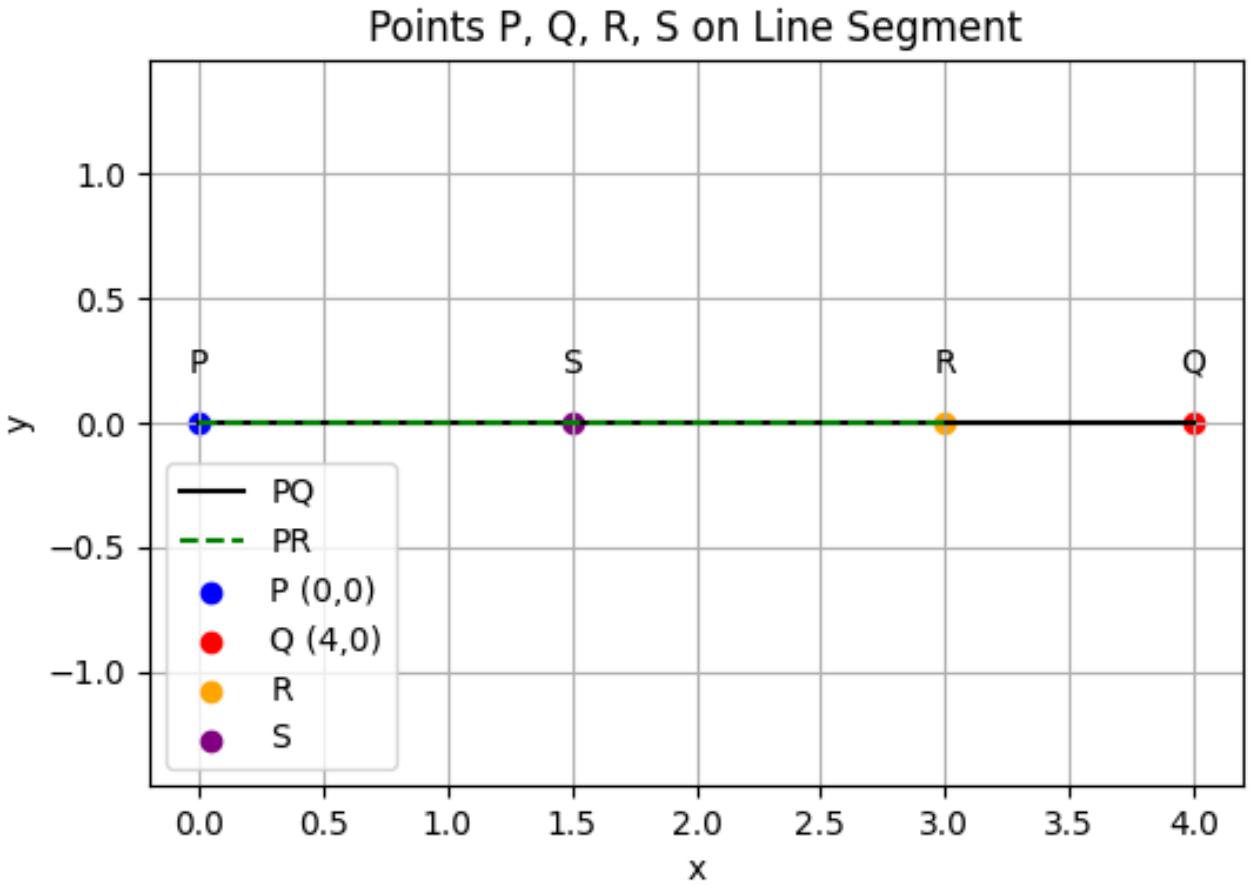

Figure: