2.2.24

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QuestionShow that the points $(1,7)$, $(4,2)$, $(-1,-1)$ and $(-4,4)$ are the vertices of a square.

Solution Given details:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{1}$$

For the points **ABCD** to represent a square:

$$||AB|| = ||BC|| = ||CD|| = ||DA|| \tag{2}$$

$$\angle BAD = \angle ABC = \angle DCA = \angle ADC = 90^\circ \tag{3}$$

Find the sides

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \ \mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \tag{4}$$

$$\mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \ \mathbf{DA} = \mathbf{A} - \mathbf{D} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{5}$$

Check side lengths

$$||AB|| = \sqrt{\mathbf{AB}^T\mathbf{AB}} = \sqrt{3^2 + (-5)^2} = \sqrt{34} \tag{6}$$

$$||BC|| = \sqrt{\mathbf{BC}^T\mathbf{BC}} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \tag{7}$$

$$||CD|| = \sqrt{\mathbf{CD}^T\mathbf{CD}} = \sqrt{(-3)^2 + 5^2} = \sqrt{34} \tag{8}$$

$$||DA|| = \sqrt{\mathbf{DA}^T\mathbf{DA}} = \sqrt{5^2 + 3^2} = \sqrt{34} \tag{9}$$

Therefore all the sides are of equal length

$$||AB|| = ||BC|| = ||CD|| = ||DA|| \tag{10}$$

Condition for right angle: For two sides to be angled at 90° the Dot product between the 2 side vectors should be 0

$$\mathbf{AB^TBC} = (3)(5) + (-5)(-3) = -15 + 15 = 0 \tag{11}$$

Therefore the sides are perpendicular to each other.

Since all the sides are equal and one the angles is 90° , all the points represent a square.

figs/sqaure.png

Fig. 0. Sqaure