2.8.2

EE25BTECH11023 - Venkata Sai

Question:
$\mathbf{A}(6,1), \mathbf{B}(8,2)$ and $\mathbf{C}(9,4)$ are three vertices of a parallelogram $ABCD$. If E is the midpoint of DC find the area of △ADE.

Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}, \ \mathbf{C} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}. \tag{1}$$

As $ABCD$ is a parallelogram with $AB \parallel CD$;

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{2}$$

$$\mathbf{D} = \mathbf{C} + \mathbf{A} - \mathbf{B} \tag{3}$$

$$\mathbf{D} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix} \tag{4}$$

$$\mathbf{D} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \tag{5}$$

Finding Area Of $\triangle ADE$:

$$\mathbf{E} = \frac{\mathbf{D} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 7 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ 4 \end{pmatrix}}{2} = \begin{pmatrix} 8 \\ \frac{7}{2} \end{pmatrix} \tag{6}$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{E} - \mathbf{A} = \begin{pmatrix} 8 \\ \frac{7}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{5}{2} \end{pmatrix} \tag{7}$$

$$\text{Area}(\triangle ADE) = \frac{1}{2} \|(\mathbf{D} - \mathbf{A}) \times (\mathbf{E} - \mathbf{A})\|. \tag{8}$$

$$= \frac{1}{2} \left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ \frac{5}{2} \end{pmatrix} \right\| = \frac{1}{2} \begin{vmatrix} 1 & 2 \\ 2 & \frac{5}{2} \end{vmatrix} = \frac{3}{4} \tag{9}$$

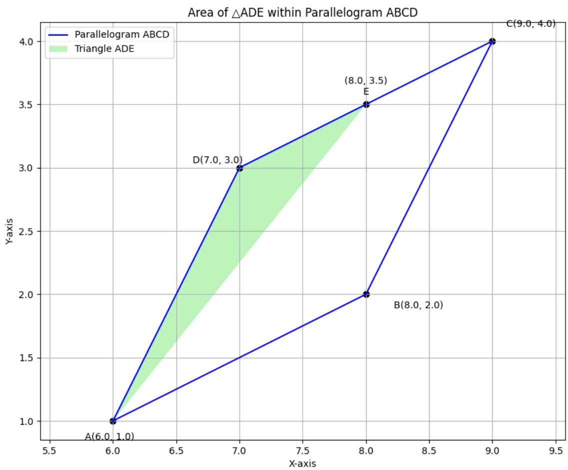
Area of △ADE within Parallelogram ABCD
Parallelogram ABCD
Triangle ADE
C(9.0, 4.0)
(8.0, 3.5)
E
D(7.0, 3.0)
B(8.0, 2.0)
A(6.0, 1.0)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
Y-axis
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
X-axis

Fig. 0.1: Area