

Matgeo-q.2.2.29

AI25BTECH11036-SNEHAMRUDULA

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Question

Find the angle between the two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.

Solution

Given: The normal vectors of the planes are:

$$\mathbf{n}_1 = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{n}_2 = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \quad (2)$$

The angle θ between the planes is given by:

$$\cos \theta = \frac{|\mathbf{n}_1^\top \mathbf{n}_2|}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (3)$$

Compute the dot product:

$$\mathbf{n}_1^\top \mathbf{n}_2 = (3)(2) + (-6)(2) + (2)(-2) = 6 - 12 - 4 = -10 \quad (4)$$

$$|\mathbf{n}_1^\top \mathbf{n}_2| = 10 \quad (5)$$

Compute the magnitudes:

$$\|\mathbf{n}_1\| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \quad (6)$$

$$\|\mathbf{n}_2\| = \sqrt{2^2 + 2^2 + (-2)^2} = \sqrt{4 + 4 + 4} = \sqrt{12} = 2\sqrt{3} \quad (7)$$

Thus,

$$\cos \theta = \frac{10}{7 \cdot 2\sqrt{3}} = \frac{5}{7\sqrt{3}} \quad (8)$$

$$\theta = \cos^{-1} \left(\frac{5}{7\sqrt{3}} \right) \quad (9)$$

Therefore, the angle between the planes is:

$$\boxed{\cos^{-1} \left(\frac{5}{7\sqrt{3}} \right)}$$

Graphical Representation

