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Question

Find the length of the median of the triangle with vertices **A**(0,0,6),**B**(0,4,0) and **C**(6,0,0).

Step 1: Midpoints of Opposite Sides

$$M_{BC} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2}\begin{pmatrix}4+6\\0+0\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix} \tag{1}$$

$$M_{AC} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) = \frac{1}{2}\begin{pmatrix}0+6\\6+0\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix} \tag{2}$$

$$M_{AB} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2}\begin{pmatrix}0+4\\6+0\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} \tag{3}$$

Step 2: Vectors Representing Medians

$$\mathbf{AM} = M_{BC} - A = \begin{pmatrix}5\\0\end{pmatrix} - \begin{pmatrix}0\\6\end{pmatrix} = \begin{pmatrix}5\\-6\end{pmatrix} \tag{4}$$

$$\mathbf{BM} = M_{AC} - B = \begin{pmatrix}3\\3\end{pmatrix} - \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix} \tag{5}$$

$$\mathbf{CM} = M_{AB} - C = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}6\\0\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix} \tag{6}$$

Step 3: Lengths of Medians

Using the Euclidean norm:

$$||\mathbf{v}|| = \sqrt{x^2 + y^2} \tag{7}$$

$$||\mathbf{AM}|| = \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36} = \sqrt{61} \tag{8}$$

$$||\mathbf{BM}|| = \sqrt{(-1)^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10} \tag{9}$$

$$||\mathbf{CM}|| = \sqrt{(-4)^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \tag{10}$$

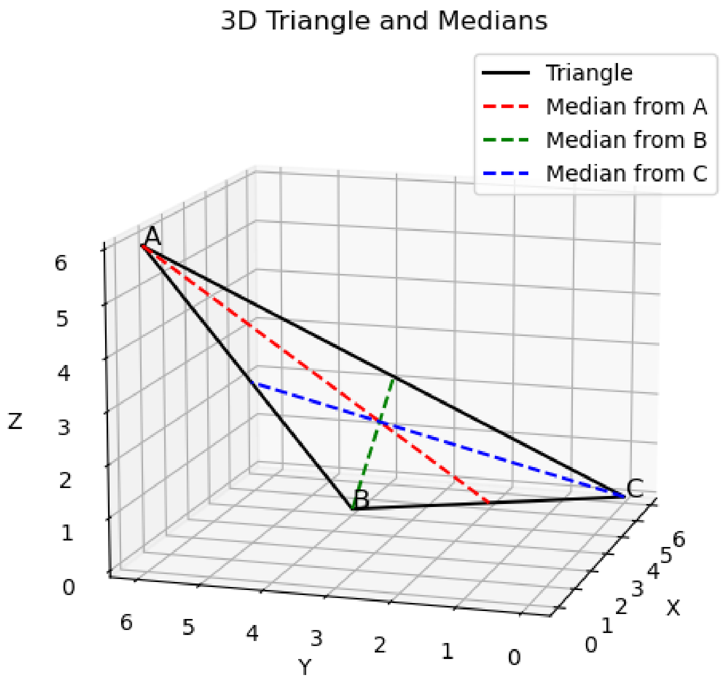

Figure 1

Final Answer

$$\boxed{||\mathbf{AM}|| = \sqrt{61}, \quad ||\mathbf{BM}|| = \sqrt{10}, \quad ||\mathbf{CM}|| = 5} \tag{11}$$