2.2.29

AI25BTECH11033–SNEHAMRUDULA

Find the angle between the two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.

Solution: The normal vectors of the planes are:

$$\mathbf{n}_1 = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \tag{0.1}$$

$$\mathbf{n}_2 = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \tag{0.2}$$

The angleθ between the planes is given by:

$$\cos\theta = \frac{|\mathbf{n}_1^\top \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} \tag{0.3}$$

Compute the dot product:

$$\mathbf{n}_1^\top \mathbf{n}_2 = (3)(2) + (-6)(2) + (2)(-2) = 6 - 12 - 4 = -10 \tag{0.4}$$

$$|\mathbf{n}_1^\top \mathbf{n}_2| = 10 \tag{0.5}$$

Compute the magnitudes:

$$\|\mathbf{n}_1\| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \tag{0.6}$$

$$\|\mathbf{n}_2\| = \sqrt{2^2 + 2^2 + (-2)^2} = \sqrt{4 + 4 + 4} = \sqrt{12} = 2\sqrt{3} \tag{0.7}$$

Thus,

$$\cos\theta = \frac{10}{7 \cdot 2\sqrt{3}} = \frac{5}{7\sqrt{3}} \tag{0.8}$$

$$\theta = \cos^{-1}\left(\frac{5}{7\sqrt{3}}\right) \tag{0.9}$$

Therefore,the angle between the planes is:

$$\boxed{\cos^{-1}\left(\frac{5}{7\sqrt{3}}\right)}$$

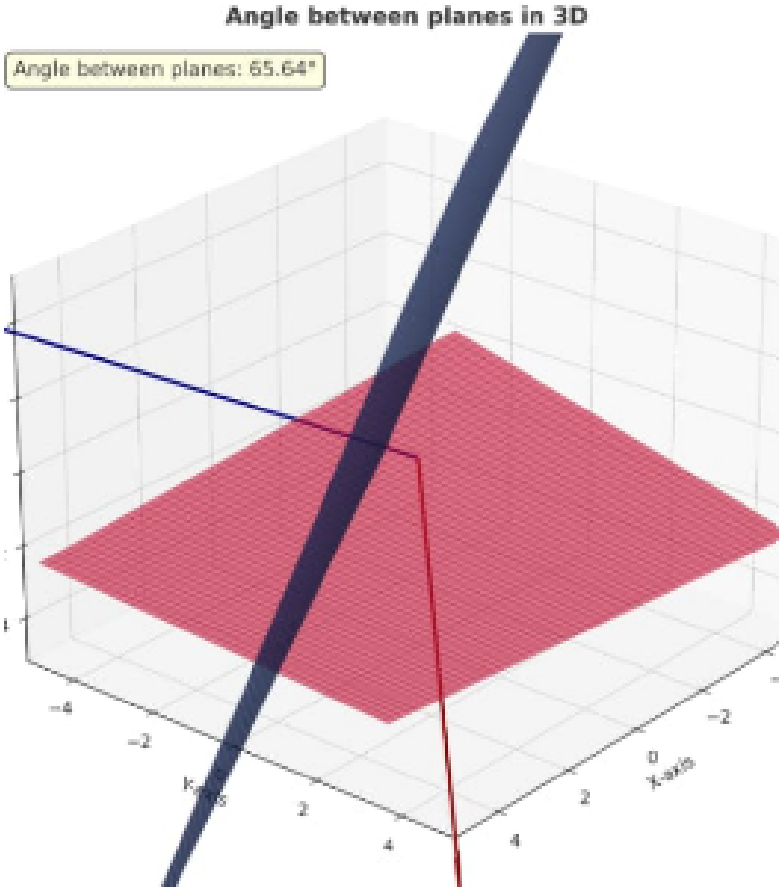
Angle between planes in 3D
Angle between planes: 65.64°
−4
−2
2
4
−2
0
2
4
X-axis

Fig. 0.1