Problem 2.8.2

ee25btech11023-Venkata Sai

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Problem Statement

A $(6, 1)$, **B** $(8, 2)$ and **C** $(9, 4)$ are three vertices of a parallelogram $ABCD$. If E is the midpoint of DC find the area of $\triangle$ADE.

Formula

To Find Area of Triangle

$$\text{Area}(\triangle ADE) = \frac{1}{2}\|(\mathbf{D} - \mathbf{A}) \times (\mathbf{E} - \mathbf{A})\| \tag{1.1}$$

Finding the Point D

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{1.2}$$

$$\mathbf{D} = \mathbf{C} + \mathbf{A} - \mathbf{B} \tag{1.3}$$

$$\mathbf{D} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix} \tag{1.4}$$

$$\mathbf{D} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \tag{1.5}$$

Finding the midpoint of DC and Area($\triangle$ADE)

$$\mathbf{E} = \frac{\mathbf{D}+\mathbf{C}}{2} = \frac{\begin{pmatrix}7\\3\end{pmatrix}+\begin{pmatrix}9\\4\end{pmatrix}}{2} = \begin{pmatrix}8\\\frac{7}{2}\end{pmatrix} \tag{1.6}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix}7\\3\end{pmatrix}-\begin{pmatrix}6\\1\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{E}-\mathbf{A} = \begin{pmatrix}8\\\frac{7}{2}\end{pmatrix}-\begin{pmatrix}6\\1\end{pmatrix} = \begin{pmatrix}2\\\frac{5}{2}\end{pmatrix} \tag{1.7}$$

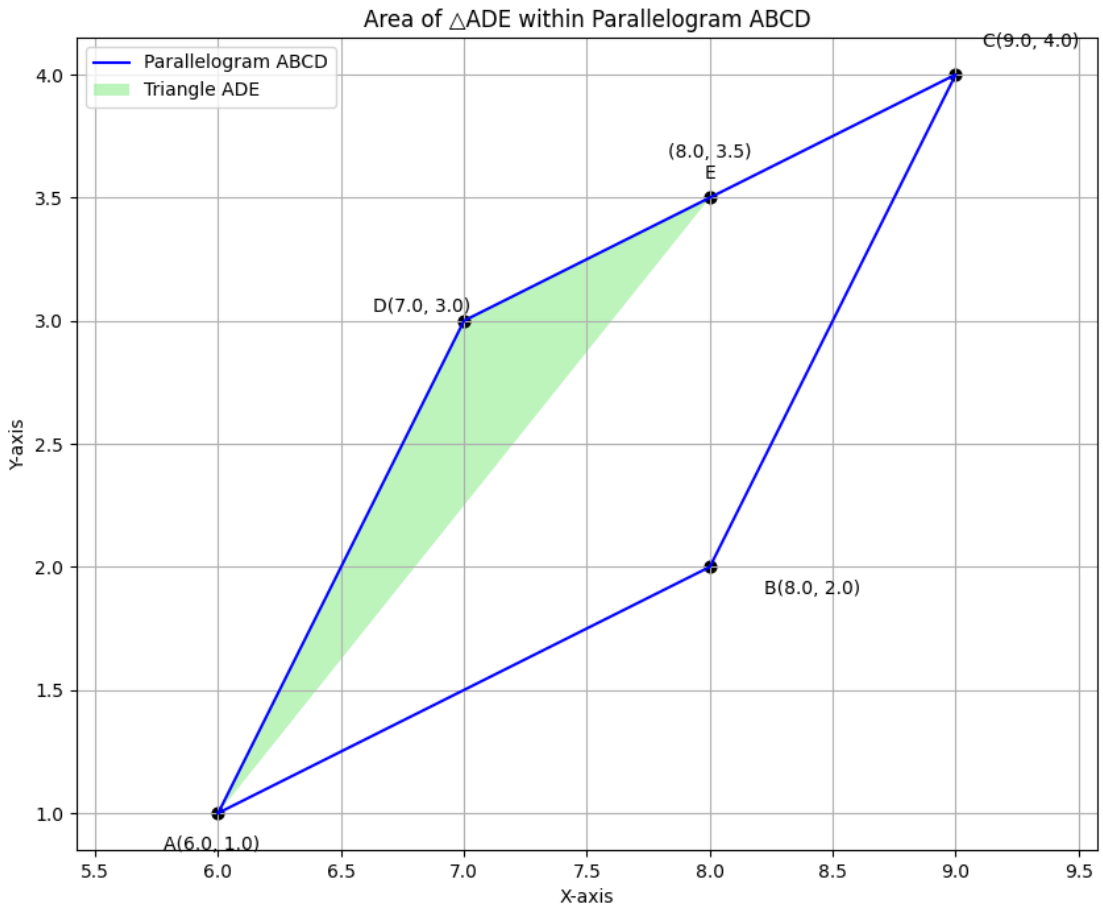
Area($\triangle$ADE)

$$\text{Area}(\triangle ADE) = \frac{1}{2}\|(\mathbf{D}-\mathbf{A})\times(\mathbf{E}-\mathbf{A})\| \tag{1.8}$$

$$= \frac{1}{2}\left\|\begin{pmatrix}1\\2\end{pmatrix}\times\begin{pmatrix}2\\\frac{5}{2}\end{pmatrix}\right\| = \frac{1}{2}\begin{vmatrix}1 & 2\\2 & \frac{5}{2}\end{vmatrix} = \frac{3}{4} \tag{1.9}$$

Hence the Area of triangle is $\frac{3}{4}$ units.

Plot


Area of △ADE within Parallelogram ABCD
Parallelogram ABCD
Triangle ADE
C(9.0, 4.0)
(8.0, 3.5)
E
D(7.0, 3.0)
B(8.0, 2.0)
A(6.0, 1.0)
Y-axis
X-axis
4.0
3.5
3.0
2.5
2.0
1.5
1.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5

Figure

C Code for finding points

```c
void calculate_parallelogram_coords(double* out_coords) {
    // Given vertices
    double Ax = 6.0, Ay = 1.0;
    double Bx = 8.0, By = 2.0;
    double Cx = 9.0, Cy = 4.0;

    double Dx = Ax - Bx + Cx;
    double Dy = Ay - By + Cy;

    double Ex = (Dx + Cx) / 2.0;
    double Ey = (Dy + Cy) / 2.0;

    out_coords[0] = Ax; out_coords[1] = Ay;
    out_coords[2] = Bx; out_coords[3] = By;
    out_coords[4] = Cx; out_coords[5] = Cy;
    out_coords[6] = Dx; out_coords[7] = Dy;
    out_coords[8] = Ex; out_coords[9] = Ey;
}
```

Calling C Function

```
import ctypes
import numpy as np

def get_all_points():

    # Load the compiled C shared library
    lib = ctypes.CDLL('./coord.so')
    lib.calculate_parallelogram_coords.argtypes = [ctypes.POINTER
        (ctypes.c_double)]

    out_coords_c = double_array_10()

    # Call the C function, passing the C array by reference
    lib.calculate_parallelogram_coords(out_coords_c)

    # Convert the C array result back into a NumPy array
    # Reshape it to 5x2 (5 points, each with x and y)
    all_points = np.array(out_coords_c).reshape(5, 2)
    return all_points
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL

import sys
import numpy as np
import matplotlib.pyplot as plt
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
# --- Import from our C Interface Module ---
from call import get_all_points

from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen
```

Python Code for Plotting

```
points=get_all_points()
A, B, C, D, E = points

print(f"Midpoint E coordinates: ({E[0]:.1f}, {E[1]:.1f})")
# Calculate and print the area of triangle ADE
area_ADE = 0.5 * np.abs(A[0]*(D[1] - E[1]) + D[0]*(E[1] - A[1]) +
     E[0]*(A[1] - D[1]))
print(f"The area of Triangle ADE is: {area_ADE}")

# --- Plotting ---
fig, ax = plt.subplots(figsize=(10, 8))

# Draw and fill the shapes
ax.plot(np.vstack([A, B, C, D, A])[:, 0], np.vstack([A, B, C, D,
    A])[:, 1], 'b-', label='Parallelogram ABCD')
ax.fill(np.vstack([A, D, E, A])[:, 0], np.vstack([A, D, E, A])[:,
     1], 'lightgreen', alpha=0.6, label='Triangle ADE')
```

Python Code for Plotting

```
# Draw the vertices
ax.scatter(points[:, 0], points[:, 1], color='black', s=40)

# Add annotations with specific offsets
ax.annotate(f'A({A[0]:.1f}, {A[1]:.1f})', xy=A, xytext=(-30, -20)
    , textcoords='offset points')
ax.annotate(f'B({B[0]:.1f}, {B[1]:.1f})', xy=B, xytext=(30, -15),
     textcoords='offset points')
ax.annotate(f'C({C[0]:.1f}, {C[1]:.1f})', xy=C, xytext=(15, 15),
    textcoords='offset points')
ax.annotate(f'D({D[0]:.1f}, {D[1]:.1f})', xy=D, xytext=(-50, 5),
    textcoords='offset points')
ax.annotate(f'({E[0]:.1f}, {E[1]:.1f})\nE', xy=E, xytext=(0, 8),
    textcoords='offset points', ha='center', va='bottom')
```

Python Code for Plotting

```
# --- Final Formatting ---
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_title('Area of Triangle ADE within Parallelogram ABCD')
ax.grid(True)
ax.axis('equal')
ax.legend()
plt.show()
plt.savefig('../figs/fig1.png')
```