2.2.19

RATHLAVATH JEEVAN -AI25BTECH11026

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Question

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Theoretical Solution

Solution:
Given:

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

We require:

$$\mathbf{u}^\top \frac{\mathbf{a}+\mathbf{b}}{\|\mathbf{a}+\mathbf{b}\|} = 1 \tag{2}$$

Sum of vectors:

$$\mathbf{a}+\mathbf{b} = \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \tag{3}$$

Dot product:

$$\mathbf{u}^\top(\mathbf{a}+\mathbf{b}) = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}2+\lambda \\ 6 \\ -2\end{pmatrix} = (2+\lambda)+6+(-2) = \lambda+6 \tag{4}$$

Norm of the sum:

$$\|\mathbf{a}+\mathbf{b}\| = \sqrt{(2+\lambda)^2+6^2+(-2)^2} = \sqrt{(2+\lambda)^2+40} \tag{5}$$

Condition:

$$\frac{\lambda+6}{\sqrt{(2+\lambda)^2+40}} = 1 \tag{6}$$

Theoretical Solution

Simplify:

$$\lambda + 6 = \sqrt{(\lambda + 2)^2 + 40} \tag{7}$$

Squaring,

$$(\lambda + 6)^2 = (\lambda + 2)^2 + 40 \tag{8}$$

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \tag{9}$$

$$8\lambda = 8 \quad \Rightarrow \quad \lambda = 1 \tag{10}$$

Conclusion: The required value is

$$\boxed{\lambda = 1} \tag{11}$$

$$\boxed{a = -3} \tag{12}$$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    double lambda1, lambda2;

    // Equation: (lambda + 6) / sqrt((lambda+2)^2 + 40) = 1
    // Square both sides => (lambda+6)^2 = (lambda+2)^2 + 40

    // Expanding manually:
    // lambda^2 + 12lambda + 36 = lambda^2 + 4lambda + 44
    // => 8lambda = 8
    // => lambda = 1

    lambda1 = 1;
```

C Code

```
    printf(The value of lambda is: %.2f\n, lambda1);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Vectors
v1 = np.array([2, 4, -5])
v2 = np.array([1, 2, 3]) # lambdai + 2j + 3k

# lambda = 1 solution
lam = 1
v_sum = np.array([2+lam, 6, -2])

# Vector (i + j + k)
u = np.array([1, 1, 1])

# Create A4 figure (8.27 * 11.7 inches)
fig = plt.figure(figsize=(8.27, 11.7))
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```
# Plot vectors
ax.quiver(0, 0, 0, v1[0], v1[1], v1[2], color='r', label='2i+4j-5
    k')
ax.quiver(0, 0, 0, lam*v2[0], lam*v2[1], lam*v2[2], color='b',
    label='lambdai+2j+3k (lambda=1)')
ax.quiver(0, 0, 0, v_sum[0], v_sum[1], v_sum[2], color='g', label
    ='Sum vector')
ax.quiver(0, 0, 0, u[0], u[1], u[2], color='m', label='i+j+k')

# Labels and title
ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
ax.set_title('3D Graph of Vectors (lambda=1)', fontsize=14)
ax.legend()
```

Python Code

```
# Save as A4 size PDF and PNG
plt.savefig(vector_solution_lambda_A4.png, dpi=300, bbox_inches='
    tight')
plt.savefig(vector_solution_lambda_A4.pdf, bbox_inches='tight')

plt.show()

print(Saved as vector_solution_lambda_A4.png and
    vector_solution_lambda_A4.pdf)
```

Plot

beamer/figs/matg3.jpeg