Car Price Prediction Project

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Managerial Decision Analysis

Team Members:

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REGRESSION PROJECT:

Dataset:

Source: https://www.kaggle.com/hellbuoy/car-price-prediction

Descriptions of Data:

Variables Used:

Total Observations/Sample Size = 159.

$\hat{Y}_i$ = Price of Cars ($), this is the Dependent/Response Variable.

$\hat{X}_{1i}$ = Horsepower (hp), this is the First Independent/Explanatory Variable.

$\hat{X}_{2i}$ = Peak RPM (RPM), this is the Second Independent/Explanatory Variable.

$\hat{X}_{3i}$ = No. of Doors, this is the Third Independent/Explanatory Variable. This is the Binary variable with two levels.

Two Door = 1 (67 Observations, 42% of the total).

Four Door = 0 (92 Observations, 58% of the total).

(Binary Variable satisfies the 35/15 condition).

Case: We are required to model the price of cars with the available independent variables. It will be used by the management to understand how exactly the prices vary with the independent variables. They can accordingly manipulate the design of the cars, the business strategy etc. to meet certain price levels. Further, the model will be a good way for management to understand the pricing dynamics of a new market.

Dataset Attached Below: (Refer: *Sheet1*)

CarPrice Data.xlsx

Hypothesis Test for Regression:

H₀: *ß₀ = 0* | H₀: *ß₁ = 0*
H₁: *ß₀ ≠ 0* | H₁: *ß₁ ≠ 0*

H₀: *ß₂ = 0* | H₀: *ß₃ = 0*
H₁: *ß₂ ≠ 0* | H₁: *ß₃ ≠ 0*

=> *ß₀ = Y-Intercept Coefficient*
=> *ß₁ = Slope of First Coefficient*
=> *ß₂ = Slope of Second Coefficient*
=> *ß₃ = Slope of Third Coefficient*

Regression Analysis done on the following model combinations:

1. (y, X_1), (y, X_2), (y, X_3)
2. (y, X_1, X_2), (y, X_1, X_3), (y, X_2, X_3)
3. (y, X_1, X_2, X_3)
4. (y, X_1, X_2, X_1X_2)
5. (y, X_1, X_1^2) and (y, X_2, X_2^2)

Regression equations used:

MULTIPLE REGRESSION MODEL WITH *k* INDEPENDENT VARIABLES

$$Y_i = \beta_0 + \beta_1X_{1i} + \beta_2X_{2i} + \beta_3X_{3i} + \dots + \beta_kX_{ki} + \varepsilon_i \quad \textbf{(14.1)}$$

where

β_0 = Y intercept
β_1 = slope of Y with variable X_1, holding variables $X_2, X_3, \dots, X_k$ constant
β_2 = slope of Y with variable X_2, holding variables $X_1, X_3, \dots, X_k$ constant
β_3 = slope of Y with variable X_3, holding variables $X_1, X_2, \dots, X_k$ constant
⋮
β_k = slope of Y with variable X_k holding variables $X_1, X_2, X_3, \dots, X_{k-1}$ constant
ε_i = random error in Y for observation i

SIMPLE LINEAR REGRESSION MODEL

$$Y_i = \beta_0 + \beta_1X_i + \varepsilon_i \quad \textbf{(13.1)}$$

where

β_0 = Y intercept for the population
β_1 = slope for the population
ε_i = random error in Y for observation i
Y_i = dependent variable for observation i
X_i = independent variable for observation i

R, R-Studio and Minitab were used in conducting analysis for this project. We have used Three Independent Variables for this project with one being a Binary/Categorical Variable with two Factor-Levels. Following are the analysis results:

Case_1. (y, X_1)

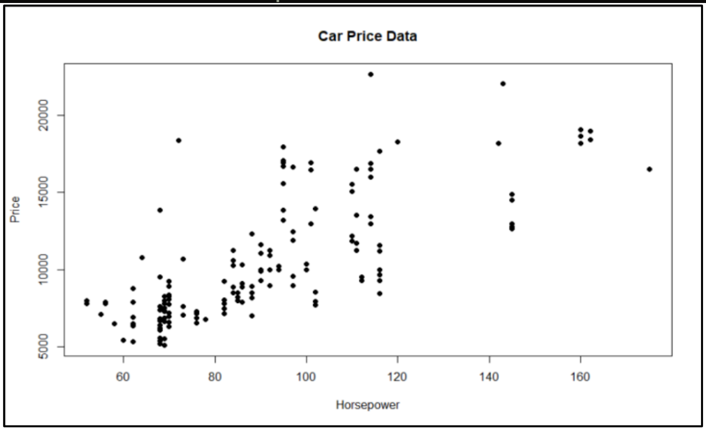
Analysis Results:

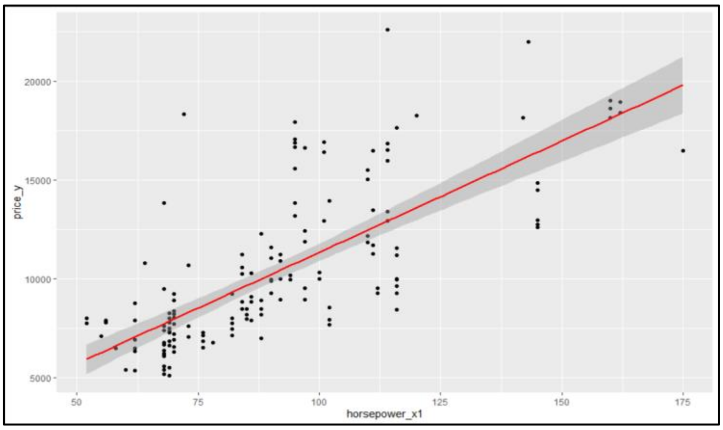
a) Setting up the Structure of R Objects:

```
> price$doornumber_x3 = as.factor(price$doornumber_x3)
> str(price)
'data.frame':	159 obs. of  7 variables:
 $ carname       : chr  "alfa-romero giulia" "alfa-romero stelvio" "audi 100 ls" "bmw 320
i" ...
 $ doors         : chr  "two" "two" "four" "two" ...
 $ doornumber_x3 : Factor w/ 2 levels "0","1": 2 2 1 2 1 2 1 2 2 2 ...
 $ cylindernumber: chr  "four" "four" "four" "four" ...
 $ horsepower_x1 : num  111 111 102 101 101 70 70 68 68 102 ...
 $ peakrpm_x2    : num  5000 5000 5500 5800 5800 5400 5400 5500 5500 5500 ...
 $ price_y       : num  13495 16500 13950 16430 16925 ...
```

b) Scatter Plot:

```
> # 1.1 (y,x1)
> # Basic scatterplot of the data.
```





c) Simple Regression:

```
> # 1.1 (y,x1)
> # Conducting a Simple regression (lm="Linear Model") on the data.
> price1.1=lm(price_y~horsepower_x1)
> summary(price1.1)

Call:
lm(formula = price_y ~ horsepower_x1)

Residuals:
    Min      1Q  Median      3Q     Max 
-4708.9 -1549.4  -630.8   949.3 10151.1 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)     68.308    773.310   0.088     0.93    
horsepower_x1  112.841      8.219  13.730   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2652 on 157 degrees of freedom
Multiple R-squared:  0.5456,	Adjusted R-squared:  0.5427 
F-statistic: 188.5 on 1 and 157 DF,  p-value: < 2.2e-16

> confint(price1.1)
                   2.5 %    97.5 %
(Intercept)   -1459.12471 1595.7407
horsepower_x1    96.60805  129.0742
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{1i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = Slope for the sample

$\hat{X}_{1i}$ = Independent variable (horsepower_x1)

R-squared: 0.5456, Adjusted R-squared: 0.5427

d) Observation:

1. This looks like a linear distribution from the scatter plot above.
2. R-squared (Coefficient of determination) is 54.56%, which means about 54.56% of the variance in price is explained by the horsepower. Regression equation as follows,

Regression Equation

price_y = 68 + 112.84 horsepower_X1

Estimated Regression Model for this case $[\hat{Y}_i = 68.3 + 112.84\hat{X}_{1i}]$

3. Interpretation of the Regression Equation:

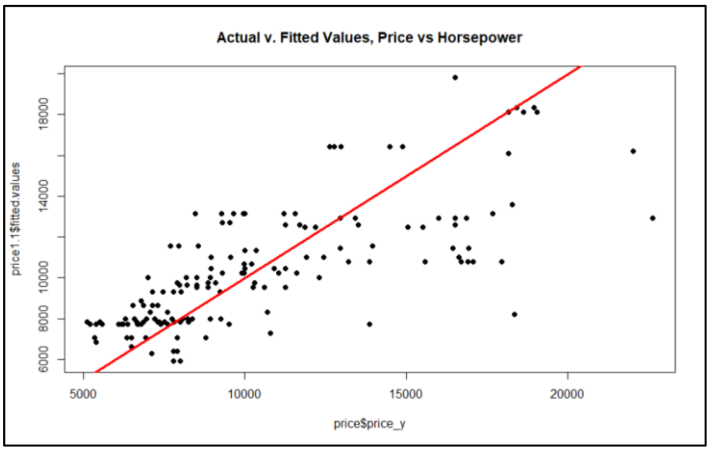
ß - Coefficient	Independent Variable Change	Net Effect
$ß_0 = 68.3$	No change in horsepower.	The predicted mean price increases by $68.3.
$ß_1 = 112.84$	Horsepower increases by 1hp.	The predicted mean price increases by $112.84.

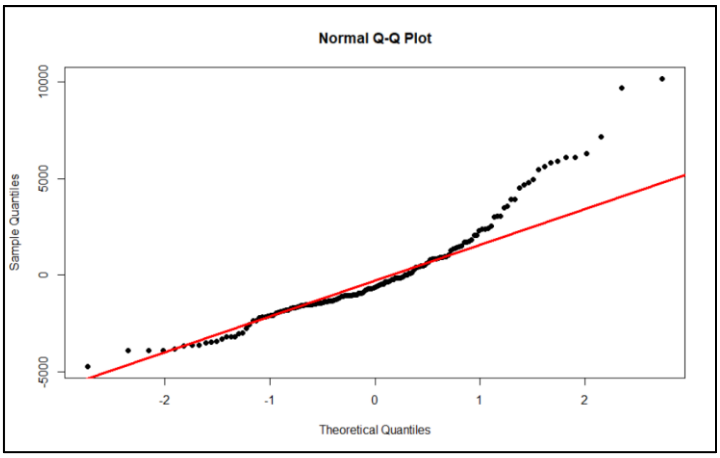
4. p-value for Y-Intercept $ß_0$ is 0.93 (Must be below 0.05), other p-value interpretations as follows:

p-value	$ß_0 = 0.93$	$ß_1 < 2e-16$
Independent Variables	Fail to Reject Null, Not Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant	

e) L.I.N.E Assumptions:

Linearity & Normality:





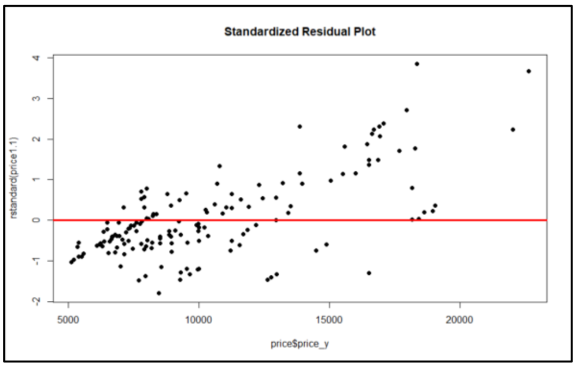
Independence of errors:

```
> # Independence of the observations.
> dwtest(lm(price_y~horsepower_x1))
        Durbin-Watson test
data:  lm(price_y ~ horsepower_x1)
DW = 0.97404, p-value = 2.787e-11
alternative hypothesis: true autocorrelation is greater than 0
> chisq.test(horsepower_x1,price_y, correct = FALSE)
        Pearson's Chi-squared test
data:  horsepower_x1 and price_y
X-squared = 5749.2, df = 5472, p-value = 0.004495
```

From Durbin-Watson test, DW=0.97, p-value = 2.787e-11, residuals are autocorrelated.

From Chi-squared test, p-value = 0.004495, means the variables are related.

Equality of Variances:

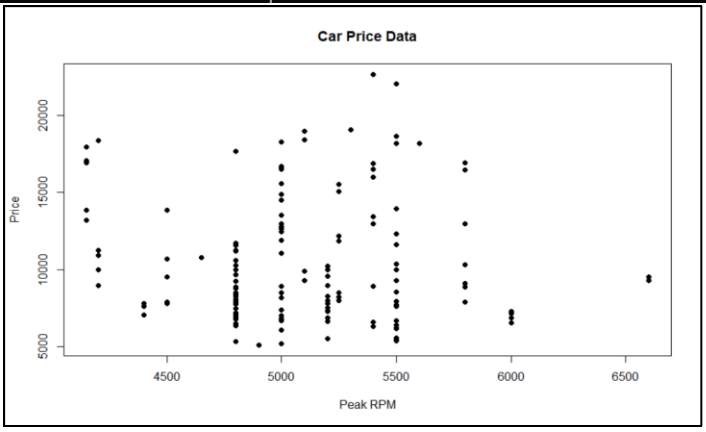


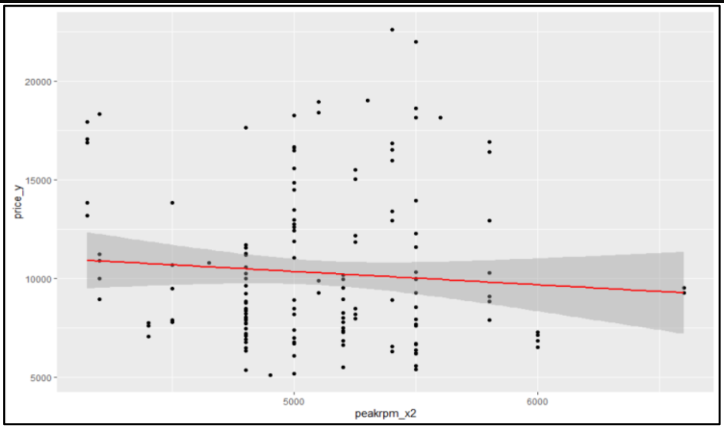
Case_2. (y, X_2)

Analysis Results:

a) Scatter Plot:

```
> # 1.2 (y,x2)
> # Basic scatterplot of the data.
```





b) Simple Regression:

```
> # 1.2 (y,x2)
> # Conducting a Simple regression (lm="Linear Model") on the data.
> price1.2=lm(price_y~peakrpm_x2)
> summary(price1.2)

Call:
lm(formula = price_y ~ peakrpm_x2)

Residuals:
   Min     1Q Median     3Q    Max 
 -5311  -2895  -1239   2171  12533 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 13736.3099  3475.8807   3.952 0.000117 ***
peakrpm_x2     -0.6749     0.6771  -0.997 0.320438    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3922 on 157 degrees of freedom
Multiple R-squared:  0.006288, Adjusted R-squared:  -4.156e-05 
F-statistic: 0.9934 on 1 and 157 DF,  p-value: 0.3204

> confint(price1.2)
                  2.5 %       97.5 %
(Intercept) 6870.787931 2.060183e+04
peakrpm_x2    -2.012273 6.625268e-01
```

$\hat{\boldsymbol{Y}}_{\boldsymbol{i}} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1}\hat{\boldsymbol{X}}_{\boldsymbol{2i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = Slope for the sample

$\hat{X}_{2i}$ = Independent variable (peakrpm_x2)

R-squared: 0.006288, Adjusted R-squared: -4.156e-05

Regression Equation

price_y = 13736 - 0.675 peakrpm_X2

c) Observation:

1. This looks like a curved distribution from the scatter plot above. R-squared (Coefficient of determination) is 0.63%, which means about 0.63% of the variance in price is explained by the peak RPM. Regression equation, $\mathit{Estimated\ Regression\ Model\ for\ this\ case}\ [\hat{Y}_i = 13736.3 - 0.675\hat{X}_{2i}]$

2. Interpretation of the Regression Equation:

ß - Coefficient	Independent Variable Change	Net Effect
$ß_0$ = 13736.3	No change in peak RPM.	The predicted mean price increases by $13736.3.
$ß_1$ = -0.675	Peak RPM increases by 1RPM.	The predicted mean price decreases by $0.675.

Case_3. (y, X_3)

Analysis Results:

a) Simple Regression:

```
> # 1.3 (y,x3)
> # Conducting a Simple regression (lm="Linear Model") on the data.
> price1.3=lm(price_y~doornumber_x3)
> summary(price1.3)

Call:
lm(formula = price_y ~ doornumber_x3)

Residuals:
   Min     1Q Median     3Q    Max
 -4672  -2982  -1382   2076  12578

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     10901.5      403.1  27.043   <2e-16 ***
doornumber_x31  -1461.2      621.0  -2.353   0.0199 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3866 on 157 degrees of freedom
Multiple R-squared:  0.03406,	Adjusted R-squared:  0.02791
F-statistic: 5.537 on 1 and 157 DF,  p-value: 0.01986

> confint(price1.3)
                  2.5 %     97.5 %
(Intercept)    10105.27 11697.7017
doornumber_x31 -2687.78  -234.6353
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{3i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = Slope for the sample

$\hat{X}_{3i}$ = Independent variable (doornumber_x3, Two Door = 1)

R-squared: 0.03406, Adjusted R-squared: 0.02791

b) Observation:

1. R-squared (Coefficient of determination) is 3.4%, which means about 3.4% of the variance in price is explained by the No. of doors. Regression equation as follows,

Regression Equation

price_y = 10901 - 1461 doornumber_X3

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = 10901.5 - 1461.2\hat{X}_{3i}]$

2. Interpretation of the Regression Equation:

ß - Coefficient	**Independent Variable Change**	**Net Effect**
$ß_0$ = 10901.5	No change in No. of doors.	The predicted mean price increases by $10901.5.
$ß_1$ = -1461.2	No. of doors increases by one.	The predicted mean price decreases by $1461.2.

3. p-value interpretations as follows:

p-value	$ß_0$ < 2e-16	$ß_1$ = 0.0199
Independent Variables	Reject the Null, Highly Significant	Reject the Null, Significant
Overall p-value = 0.01986	Reject the Null, Significant	

Case_4. (y, X_1, X_2)

Analysis Results:

a) Multiple Regression:

```
> # 2.1 (y,x1,x2)
> # Multiple regression for Horsepower and Peak RPM.
> price2.1 = lm(price_y~horsepower_x1+peakrpm_x2,data = price)
> summary(price2.1)

Call:
lm(formula = price_y ~ horsepower_x1 + peakrpm_x2, data = price)

Residuals:
    Min      1Q  Median      3Q     Max
-5303.3 -1471.9  -433.3   858.7 10053.8

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)   7785.9168  2305.0893   3.378 0.000923 ***
horsepower_x1  116.7813     8.0103  14.579  < 2e-16 ***
peakrpm_x2      -1.5792     0.4463  -3.539 0.000530 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2560 on 156 degrees of freedom
Multiple R-squared:  0.5794,	Adjusted R-squared:  0.574
F-statistic: 107.4 on 2 and 156 DF,  p-value: < 2.2e-16

> confint(price2.1)
                    2.5 %       97.5 %
(Intercept)   3232.702662 12339.130975
horsepower_x1  100.958615   132.604000
peakrpm_x2      -2.460725    -0.697713
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{1i} + \beta_2 \hat{X}_{2i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

$ß_0$ = Y-Intercept for the sample

$ß_1$ = First Slope for the sample

$ß_2$ = Second Slope for the sample

$\hat{X}_{1i}$ = First Independent variable (horsepower_x1)

$\hat{X}_{2i}$ = Second Independent variable (peakrpm_x2)

R-squared: 0.5794, Adjusted R-squared: 0.574

b) Observation:

1. R-squared (Coefficient of determination) is 57.94%, which means about 57.94% of the variance in price is explained by both horsepower and peak RPM. Regression equation as follows,

Regression Equation

price_y = 7786 + 116.78 horsepower_X1 - 1.579 peakrpm_X2

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = 7785.9 + 116.78\hat{X}_{1i} - 1.579\hat{X}_{2i}]$

2. Interpretation of the Regression Equation:

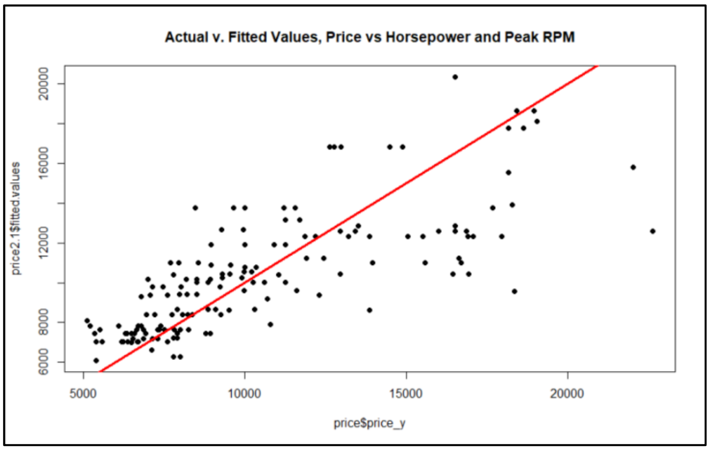
ß - Coefficient	Independent Variable Change	Net Effect
$ß_0$ = 7785.9	No change in the other two variables horsepower and peak RPM.	The predicted mean price increases by \$7785.9.
$ß_1$ = 116.78	Horsepower increases by 1hp.	The predicted mean price increases by \$116.78, holding peak RPM constant.
$ß_2$ = -1.579	Peak RPM increases by 1RPM.	The predicted mean price decreases by \$1.579, holding horsepower constant.

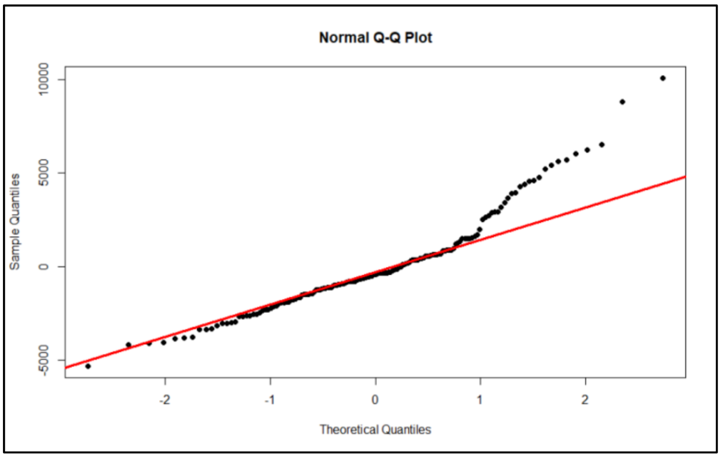
3. p-value interpretations as follows:

p-value	$\beta_0 = 0.000923$	$\beta_1 < 2e\text{-}16$	$\beta_2 = 0.000530$
Independent Variables	Reject the Null, Significant	Reject the Null, Highly Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant		

c) L.I.N.E Assumptions:

Linearity & Normality:



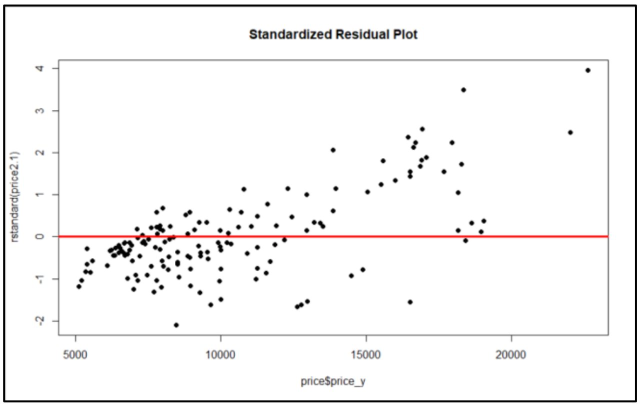


Independence of errors:

```
> # Independence of the observations.
> dwtest(price2.1)
        Durbin-Watson test
data:  price2.1
DW = 1.0392, p-value = 2.928e-10
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=1.0392, p-value = 2.928e-10, residuals are autocorrelated.

Equality of Variances:



Case_5. (y, X_1, X_3)

Analysis Results:

a) Multiple Regression:

```
> # 2.2 (y,x1,x3)
> # Multiple regression for Horsepower and No. of Doors.
> price2.2 = lm(price_y~horsepower_x1+doornumber_x3,data = price)
> summary(price2.2)

Call:
lm(formula = price_y ~ horsepower_x1 + doornumber_x3, data = price)

Residuals:
   Min     1Q Median     3Q    Max 
 -4708  -1771   -536   1017   9420 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)      640.96     739.52   0.867    0.387    
horsepower_x1    115.05       7.76  14.825  < 2e-16 ***
doornumber_x31 -1833.17     402.17  -4.558 1.04e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2499 on 156 degrees of freedom
Multiple R-squared:  0.599,	Adjusted R-squared:  0.5939 
F-statistic: 116.5 on 2 and 156 DF,  p-value: < 2.2e-16

> confint(price2.2)
                    2.5 %     97.5 %
(Intercept)    -819.80306  2101.7204
horsepower_x1    99.71919   130.3767
doornumber_x31 -2627.57285 -1038.7717
```

$\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{1i} + \beta_2 \hat{X}_{3i}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

$\hat{X}_{1i}$ = First Independent variable (horsepower_x1)

$\hat{X}_{3i}$ = Third Independent variable (doornumber_x3, Two Door = 1)

R-squared: 0.599, Adjusted R-squared: 0.5939

b) Observation:

1. R-squared (Coefficient of determination) is 59.9%, which means about 59.9% of the variance in price is explained by both horsepower and No. of doors. Regression equation as follows,

Regression Equation

price_y = 641 + 115.05 horsepower_X1 - 1833 doornumber_X3

Estimated Regression Model for this case $[\hat{Y}_i = 640.96 + 115.05\hat{X}_{1i} - 1833.17\hat{X}_{3i}]$

2. Interpretation of the Regression Equation:

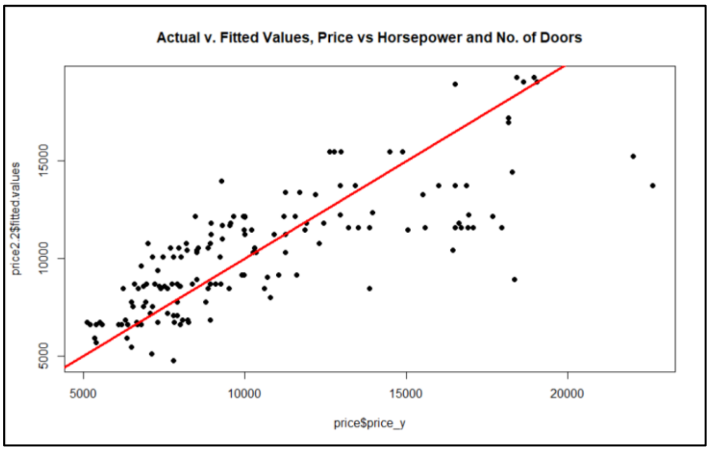
ß - Coefficient	Independent Variable Change	Net Effect
β_0 = 640.96	No change in the other two variables horsepower and No. of doors.	The predicted mean price increases by $640.96.
β_1 = 115.05	Horsepower increases by 1hp.	The predicted mean price increases by $115.05, holding No. of doors constant.
β_2 = -1833.17	No. of doors increases by one.	The predicted mean price decreases by $1833.17, holding horsepower constant.

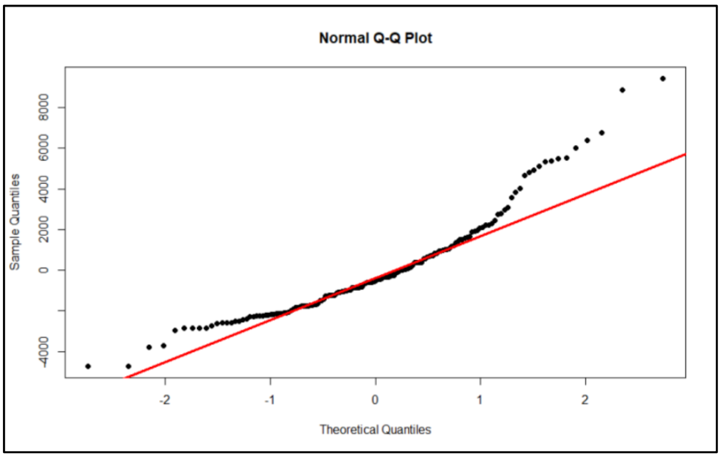
3. p-value interpretations as follows:

p-value	$\beta_0 = 0.387$	$\beta_1 < 2e-16$	$\beta_2 = 1.04e-05$
Independent Variables	Fail to Reject the Null, Not Significant	Reject the Null, Highly Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant		

c) L.I.N.E Assumptions:

Linearity & Normality:



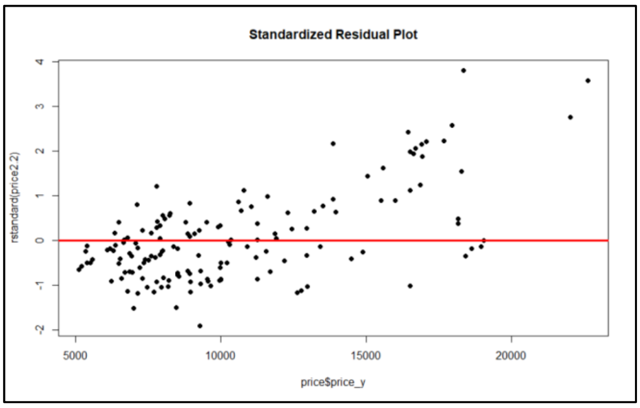


Independence of errors:

```
> # Independence of the observations.
> dwtest(price2.2)
        Durbin-Watson test
data:  price2.2
DW = 1.2012, p-value = 1.341e-07
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=1.2012, p-value = 1.341e-07, residuals are autocorrelated.

Equality of Variances:



Case_6. (y, X_2, X_3)

Analysis Results:

a) Multiple Regression:

```
> # 2.3 (y,x2,x3)
> # Multiple regression for Peak RPM and No. of Doors.
> price2.3 = lm(price_y~peakrpm_x2+doornumber_x3,data = price)
> summary(price2.3)

Call:
lm(formula = price_y ~ peakrpm_x2 + doornumber_x3, data = price)

Residuals:
   Min     1Q Median     3Q    Max 
 -4508  -3034  -1352   2081  12672 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)    12650.9818  3470.9899   3.645 0.000364 ***
peakrpm_x2        -0.3480     0.6858  -0.507 0.612531    
doornumber_x31 -1390.4147   637.9030  -2.180 0.030782 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3876 on 156 degrees of freedom
Multiple R-squared:  0.03566,  Adjusted R-squared:  0.02329 
F-statistic: 2.884 on 2 and 156 DF,  p-value: 0.05889

> confint(price2.3)
                      2.5 %       97.5 %
(Intercept)     5794.778872 19507.184819
peakrpm_x2        -1.702564     1.006547
doornumber_x31 -2650.456457  -130.372847
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1\hat{X}_{2i} + \beta_2\hat{X}_{3i}}$

$\hat{Y}_i$ = Dependent variable (price)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

$\hat{X}_{2i}$ = Second Independent variable (peakrpm_x2)

$\hat{X}_{3i}$ = Third Independent variable (doornumber_x3, Two Door = 1)

R-squared: 0.03566, Adjusted R-squared: 0.02329

b) Observation:

1. R-squared (Coefficient of determination) is 3.56%, which means about 3.56% of the variance in price is explained by both peak RPM and No. of doors. Regression equation as follows,

Regression Equation

price_y = 12651 - 0.348 peakrpm_X2 - 1390 doornumber_X3

Estimated Regression Model for this case $[\hat{Y}_i = 12650.98 - 0.348\hat{X}_{2i} - 1390.41\hat{X}_{3i}]$

2. Interpretation of the Regression Equation:

ß - Coefficient	Independent Variable Change	Net Effect
$ß_0$ = 12650.98	No change in the other two variables peak RPM and No. of doors.	The predicted mean price increases by $12650.98.
$ß_1$ = -0.348	Peak RPM increases by 1RPM.	The predicted mean price decreases by $0.348, holding No. of doors constant.
$ß_2$ = -1390.41	No. of doors increases by one.	The predicted mean price decreases by $1390.41, holding peak RPM constant.

Case_7. (y, X_1, X_2, X_3)

Analysis Results:

a) Full Multiple Regression:

```
> # Q3 (y,x1,x2,x3)
> # Multiple regression for Horsepower, Peak RPM and No .of Doors.
> price3 = lm(price_y~horsepower_x1+peakrpm_x2+doornumber_x3,data = price)
> summary(price3)

Call:
lm(formula = price_y ~ horsepower_x1 + peakrpm_x2 + doornumber_x3,
    data = price)

Residuals:
    Min      1Q  Median      3Q     Max
-4290.8 -1605.8  -432.5   990.2  9252.5

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     6489.3409  2227.9719   2.913 0.004113 **
horsepower_x1    117.7856     7.6624  15.372  < 2e-16 ***
peakrpm_x2        -1.2119     0.4366  -2.776 0.006187 **
doornumber_x31 -1595.4907   402.9991  -3.959 0.000114 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2447 on 155 degrees of freedom
Multiple R-squared:  0.618,	Adjusted R-squared:  0.6106
F-statistic: 83.59 on 3 and 155 DF,  p-value: < 2.2e-16

> confint(price3)
                    2.5 %        97.5 %
(Intercept)     2088.233982 10890.4478845
horsepower_x1    102.649285   132.9218330
peakrpm_x2        -2.074419    -0.3494229
doornumber_x31 -2391.569911  -799.4113977
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{1i} + \beta_2 \hat{X}_{2i} + \beta_3 \hat{X}_{3i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

β_3 = Third Slope for the sample

$\hat{X}_{1i}$ = First Independent variable (horsepower_x1)

$\hat{X}_{2i}$ = Second Independent variable (peakrpm_x2)

$\hat{X}_{3i}$ = Third Independent variable (doornumber_x3, Two Door = 1)

R-squared: 0.618, Adjusted R-squared: 0.6106

b) Observation:

1. R-squared (Coefficient of determination) is 61.8%, which means about 61.8% of the variance in price is explained by the independent variable's horsepower, peak RPM and No. of doors. Regression equation as follows,

Regression Equation

price_y = 6489 + 117.79 horsepower_X1 - 1.212 peakrpm_X2 - 1595 doornumber_X3

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = 6489.34 + 117.78\hat{X}_{1i} - 1.212\hat{X}_{2i} - 1595.49\hat{X}_{3i}]$

2. Interpretation of the Regression Equation:

ß - Coefficient	Independent Variable Change	Net Effect
$ß_0$ = 6489.34	No change in the other three variables horsepower, peak RPM and No. of doors.	The predicted mean price increases by $6489.34
$ß_1$ = 117.78	Horsepower increases by 1hp.	The predicted mean price increases by $117.78 holding peak RPM and No. of doors constant

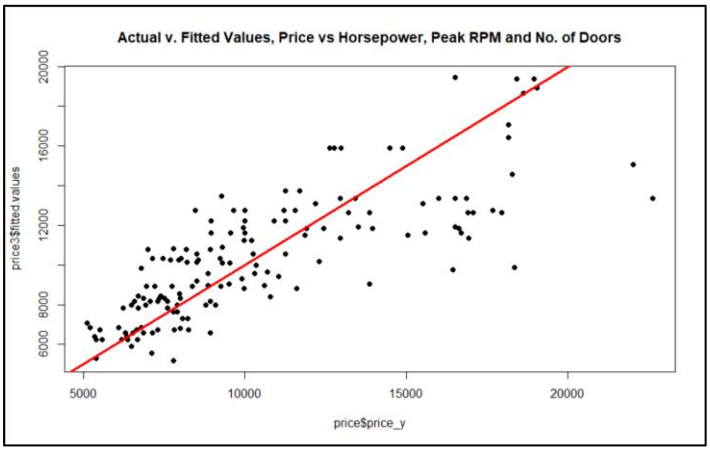
β_2 = -1.212	Peak RPM increases by 1RPM.	The predicted mean price decreases by \$1.212 holding horsepower and No. of doors constant
β_3 = -1595.49	No. of doors increases by one.	The predicted mean price decreases by \$1595.49 holding horsepower and peak RPM constant

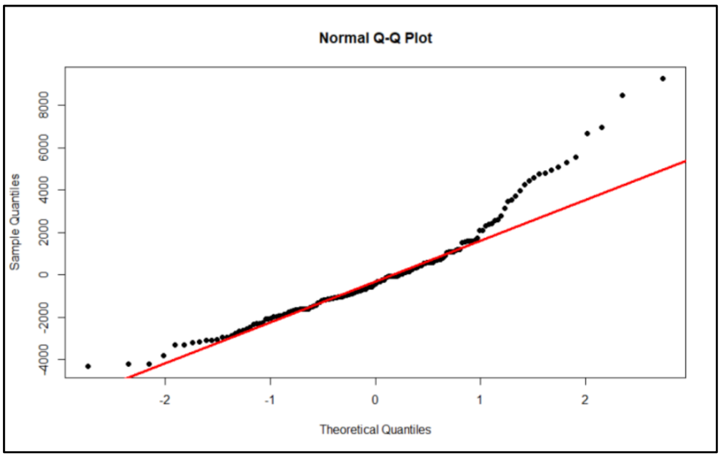
3. p-value interpretations as follows:

p-value	β_0 = 0.004113	β_1 < 2e-16	β_2 = 0.006187	β_3 = 0.000114
Independent Variables	Reject the Null, Significant	Reject the Null, Highly Significant	Reject the Null, Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant			

c) L.I.N.E Assumptions:

Linearity & Normality:



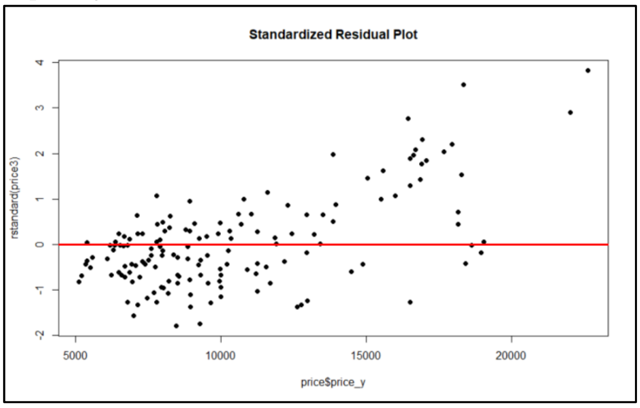


Independence of errors:

```
> # Independence of the observations.
> dwtest(price3)
           Durbin-Watson test
data:  price3
DW = 1.2107, p-value = 1.426e-07
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=1.2107, p-value = 1.426e-07, residuals are autocorrelated.

Equality of Variances:



Case_8. (y, X_1, X_2, X_1X_2)

Analysis Results:

a) Multiple Regression Using Interaction Term:

```
> # Q4 (y,x1,x2,x1x2)
> # Multiple regression for Horsepower and Peak RPM with interactions
> price4=lm(price_y~horsepower_x1+peakrpm_x2+I(horsepower_x1*peakrpm_x2),
+            data=price)
> summary(price4)

Call:
lm(formula = price_y ~ horsepower_x1 + peakrpm_x2 + I(horsepower_x1 *
    peakrpm_x2), data = price)

Residuals:
    Min      1Q  Median      3Q     Max
-4624.4 -1514.2  -395.2   888.5  9756.1

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
(Intercept)                    3.682e+04  1.093e+04   3.369 0.000951 ***
horsepower_x1                 -2.090e+02  1.202e+02  -1.738 0.084118 .
peakrpm_x2                    -7.245e+00  2.132e+00  -3.399 0.000861 ***
I(horsepower_x1 * peakrpm_x2)  6.334e-02  2.332e-02   2.716 0.007365 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2509 on 155 degrees of freedom
Multiple R-squared:  0.5985,	Adjusted R-squared:  0.5907
F-statistic: 77.01 on 3 and 155 DF,  p-value: < 2.2e-16

> confint(price4)
                                      2.5 %        97.5 %
(Intercept)                    1.523529e+04 58412.3568710
horsepower_x1                 -4.464565e+02    28.4837584
peakrpm_x2                    -1.145519e+01    -3.0339920
I(horsepower_x1 * peakrpm_x2)  1.726574e-02     0.1094084
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1\hat{X}_{1i} + \beta_2\hat{X}_{2i} + \beta_3\hat{X}_{1i}\hat{X}_{2i}}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

β_3 = Third Slope for the sample with the interaction term

$\hat{X}_{1i}$ = First Independent variable (horsepower_x1)

$\hat{X}_{2i}$ = Second Independent variable (peakrpm_x2)

$\hat{X}_{1i}\hat{X}_{2i}$ = Third Independent variable (Interaction term: horsepower_x1*peakrpm_x2)

R-squared: 0.5985, Adjusted R-squared: 0.5907

b) Observation:

1. R-squared (Coefficient of determination) is 59.85%, which means about 59.85% of the variance in price is explained by the independent variable’s horsepower, peak RPM and the Interaction term.

$Estimated\ Regression\ Model\ for\ this\ case$:

$[\hat{Y}_i = (3.682e+04) - (2.090e+02)\hat{X}_{1i} - (7.245e+00)\hat{X}_{2i} + (6.334e-02)\hat{X}_{1i}\hat{X}_{2i}]$

2. Interpretation of the Regression Equation:

ß - Coefficient	Independent Variable Change	Net Effect
$ß_0$ = 3.682e+04	No change in the other two variables horsepower and peak RPM.	The predicted mean price increases by $3.682e+04.
$ß_1$ = -2.090e+02	Horsepower increases by 1hp.	The predicted mean price changes by $[(6.334e-02)$\hat{X}_{2i}$-(2.090e+02)] with effect of horsepower on price is different for different values of peak RPM.

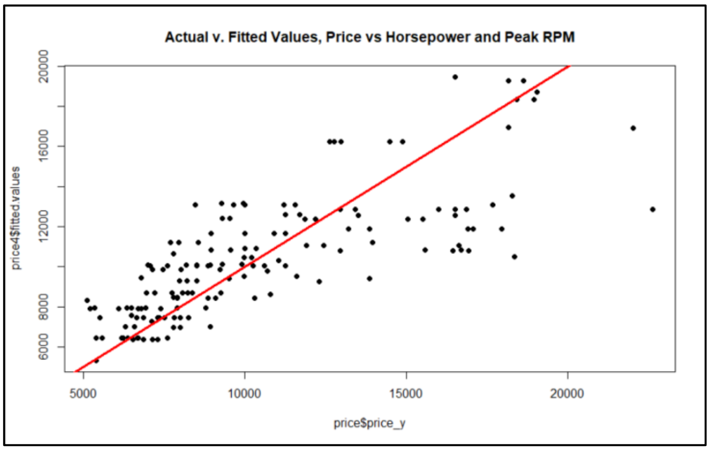
ß$_2$ = -7.245e+00	Peak RPM increases by 1RPM.	The predicted mean price changes by $[(6.334e-02)$\hat{X}_{1i}$-(7.245e+00)] with effect of peak RPM on price is different for different values of horsepower.

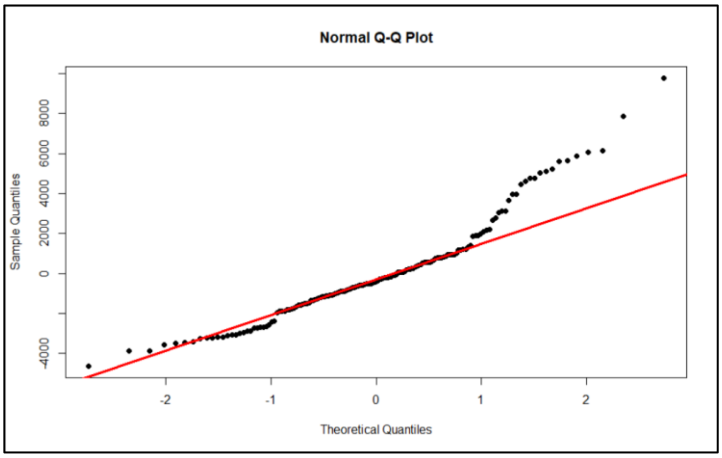
3. p-value interpretations as follows:

p-value	ß$_0$ = 0.000951	ß$_1$ = 0.084118	ß$_2$ = 0.000861	ß$_3$ = 0.007365
Independent Variables	Reject the Null, Significant	Fail to Reject the Null, Not Significant	Reject the Null, Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant			

c) L.I.N.E Assumptions:

Linearity & Normality:



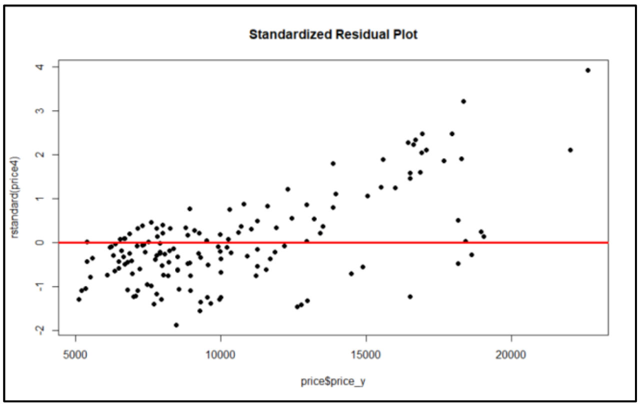


Independence of errors:

```
> # Independence of the observations.
> dwtest(price4)
        Durbin-Watson test
data:  price4
DW = 0.95068, p-value = 5.47e-12
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=0.95068, p-value = 5.47e-12, residuals are autocorrelated.

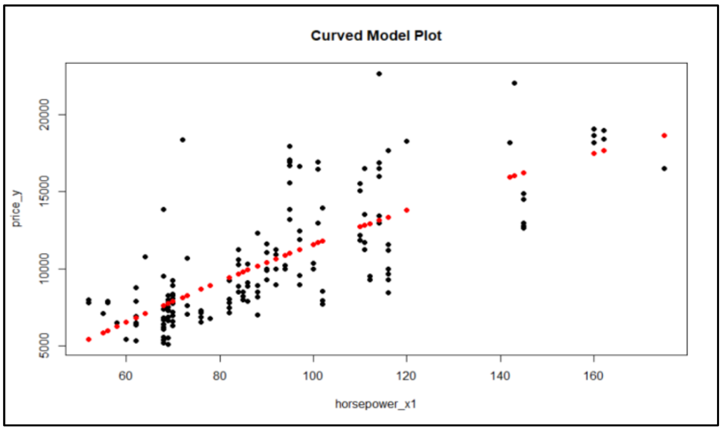
Equality of Variances:

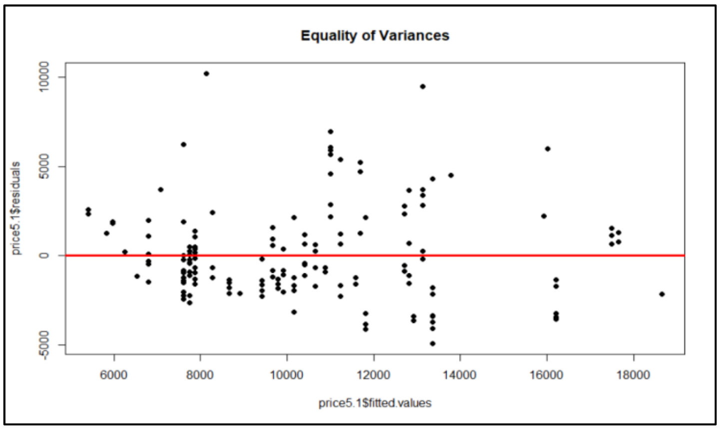


Case_9. (y, X_1, X_1^2)

Analysis Results:

a) Curved and Equality of Variances Plots:





b) Simple Regression for Non-Linearity Test:

```
> # 5.1 (y,x1,x1^2)
> # Simple regression with correcting for the non-linearity for Horsepower.
> price5.1=lm(price_y~horsepower_x1+I(horsepower_x1^2),data=price)
> summary(price5.1)

Call:
lm(formula = price_y ~ horsepower_x1 + I(horsepower_x1^2), data = price)

Residuals:
    Min      1Q  Median      3Q     Max
-4904.6 -1591.3  -665.5  1199.9 10206.0

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)        -2717.9476  2561.4304  -1.061  0.29028
horsepower_x1        170.7897    51.4511   3.319  0.00112 **
I(horsepower_x1^2)    -0.2779     0.2436  -1.141  0.25566
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2649 on 156 degrees of freedom
Multiple R-squared:  0.5494,	Adjusted R-squared:  0.5436
F-statistic: 95.09 on 2 and 156 DF,  p-value: < 2.2e-16
```

$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{1i} + \beta_2 \hat{X}_{1i}{}^2}$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

$\hat{X}_{1i}$ = First Independent variable (horsepower_x1)

$\hat{X}_{1i}{}^2$ = Second Independent variable corrected for Non-Linearity (horsepower_x1^2)

R-squared: 0.5494, Adjusted R-squared: 0.5436

c) Observation:

1. R-squared (Coefficient of determination) is 54.94%, which means about 54.94% of the variance in price is explained by the independent variable's horsepower and horsepower^2. Regression equation as follows,

Regression Equation

price_y = -2718 + 170.8 horsepower_X1 - 0.278 horsepower_X1_2

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = -2717.94 + 170.78\hat{X}_{1i} - 0.277\hat{X}_{1i}{}^2$

2. Interpretation of the Regression Equation:

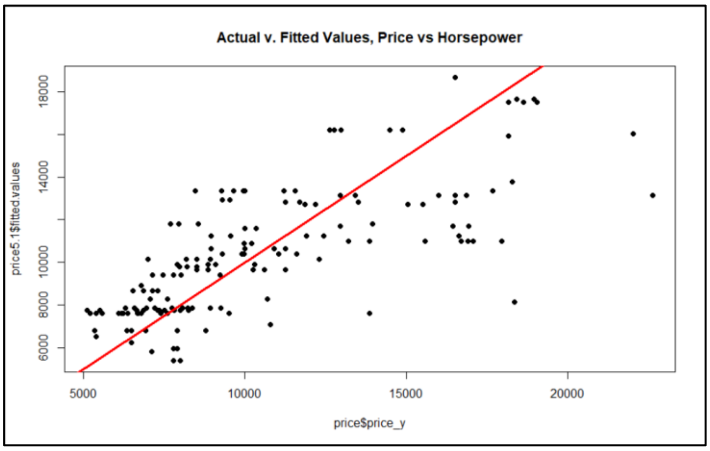
ß - Coefficient	**Independent Variable Change**	**Net Effect**
$ß_0$ = -2717.94	No change in the horsepower.	The predicted mean price decreases by $2717.94.
$ß_1$ = 170.78	Horsepower increases by 1hp.	The predicted mean price increases by $170.50.

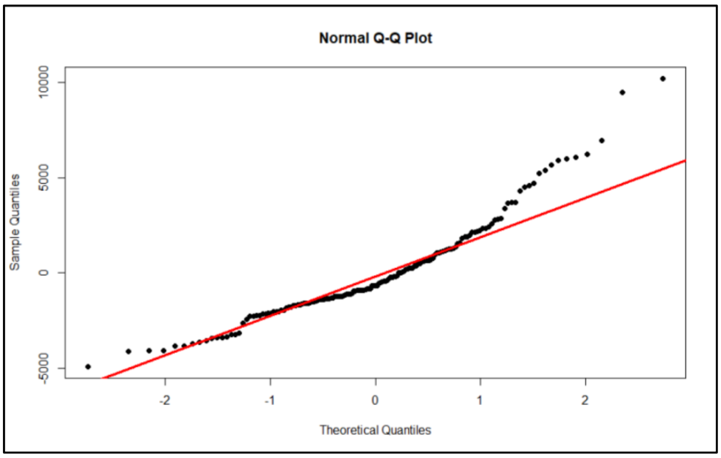
3. p-value interpretations as follows:

p-value	$ß_0$ = 0.29028	$ß_1$ = 0.00112	$ß_2$ = 0.25566
Independent Variables	Fail to Reject the Null, Not Significant	Reject the Null, Significant	Fail to Reject the Null, Not Significant
Overall p-value < 2.2e-16	Reject the Null, Significant		

d) L.I.N.E Assumptions:

Linearity & Normality:



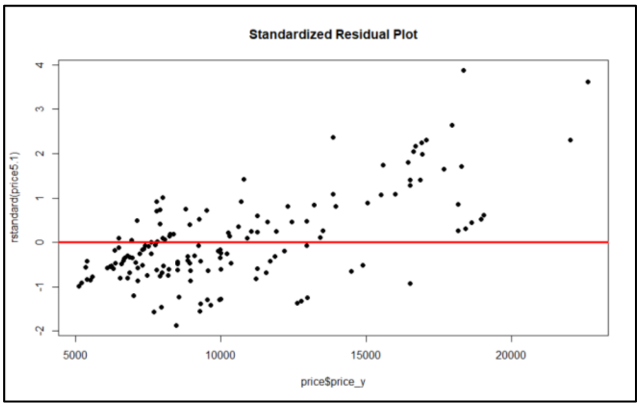


Independence of errors:

```
> # Independence of the observations.
> dwtest(price5.1)
        Durbin-Watson test
data:  price5.1
DW = 0.97814, p-value = 2.741e-11
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=0.97814, p-value = 2.741e-11, residuals are autocorrelated.

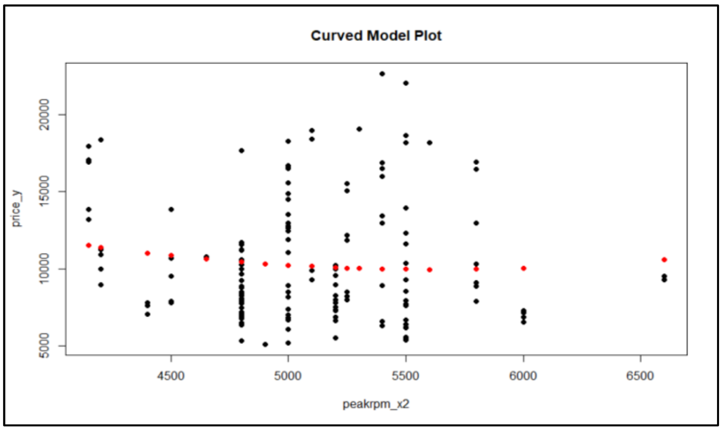
Equality of Variances:

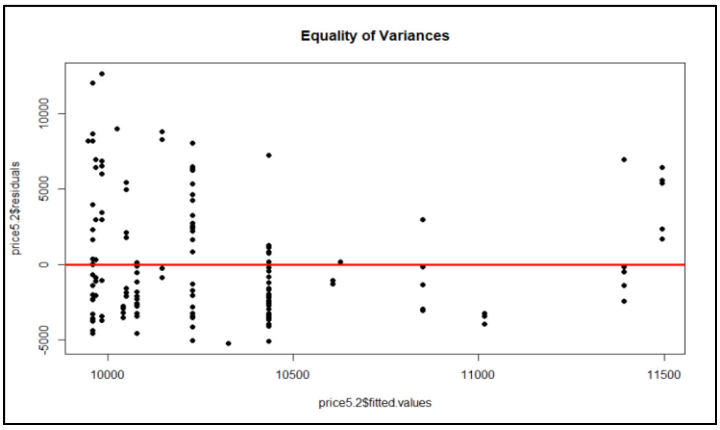


Case_10. (y, X_2, X_2^2)

Analysis Results:

a) Curved and Equality of Variances Plots:





b) Simple Regression for Non-Linearity Test:

```
> # 5.2 (y,x2,x2^2)
> # Simple regression with correcting for the non-linearity for Peak RPM.
> price5.2=lm(price_y~peakrpm_x2+I(peakrpm_x2^2),data=price)
> summary(price5.2)

Call:
lm(formula = price_y ~ peakrpm_x2 + I(peakrpm_x2^2), data = price)

Residuals:
   Min     1Q Median     3Q    Max 
 -5207  -2933  -1308   2166  12640 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)      3.229e+04  2.493e+04   1.295    0.197
peakrpm_x2      -7.934e+00  9.681e+00  -0.820    0.414
I(peakrpm_x2^2)  7.044e-04  9.370e-04   0.752    0.453

Residual standard error: 3927 on 156 degrees of freedom
Multiple R-squared:  0.009874,	Adjusted R-squared:  -0.00282 
F-statistic: 0.7779 on 2 and 156 DF,  p-value: 0.4612
```

$$\boldsymbol{\hat{Y}_i = \beta_0 + \beta_1 \hat{X}_{2i} + \beta_2 \hat{X}_{2i}{}^2}$$

$\hat{Y}_i$ = Dependent variable (price_y)

β_0 = Y-Intercept for the sample

β_1 = First Slope for the sample

β_2 = Second Slope for the sample

$\hat{X}_{2i}$ = First Independent variable (peakrpm_x2)

$\hat{X}_{2i}{}^2$ = Second Independent variable corrected for Non-Linearity (peakrpm_x2^2)

R-squared: 0.009874, Adjusted R-squared: -0.00282

c) Observation:

1. R-squared (Coefficient of determination) is 0.9%, which means about 0.9% of the variance in price is explained by the independent variable's peak RPM and peak RPM^2. Regression equation as follows,

Regression Equation

price_y = 32290 - 7.93 peakrpm_X2 + 0.000704 peakrpm_X2_2

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = 32290 - 7.93\hat{X}_{2i} + 0.000704\hat{X}_{2i}{}^2$

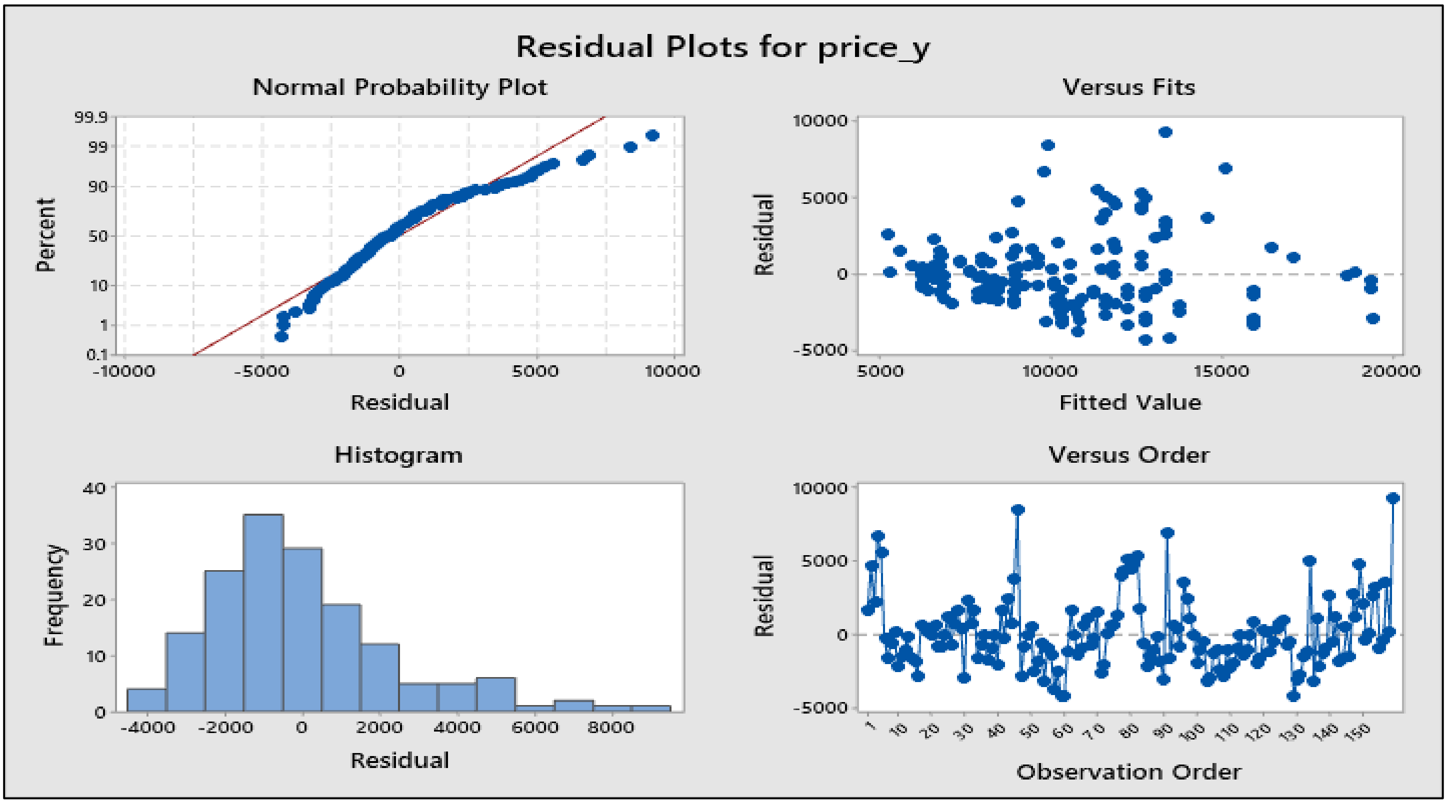
2. Rejecting this case due to high p-values and low R-squared.

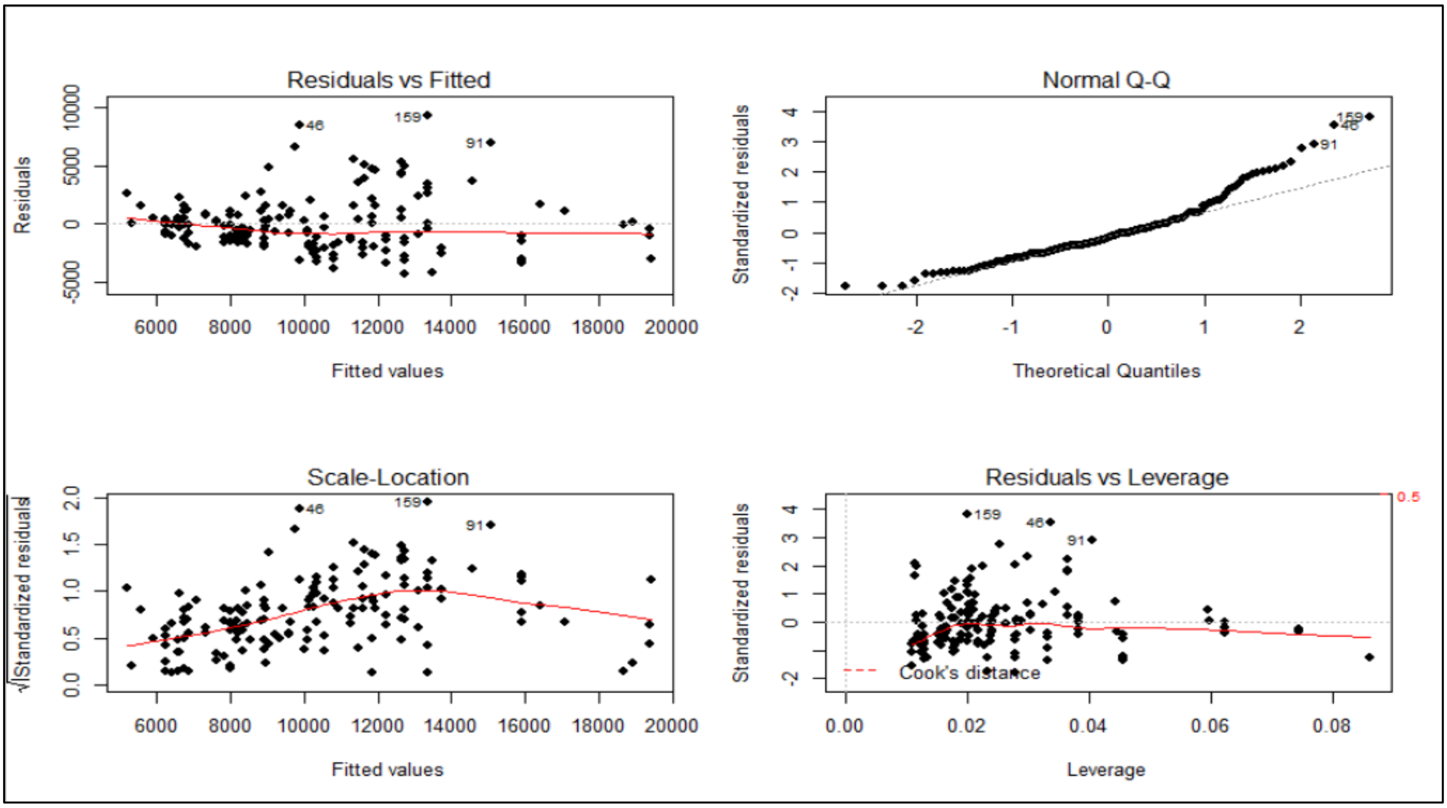
BEST FIT:

Case_7. (y, X_1, X_2, X_3)

a) Justification for selecting this model as "Best Fit":

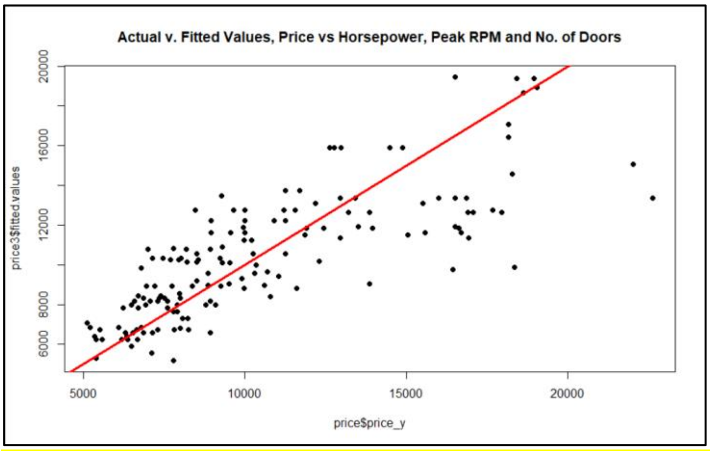
1. Out of the 10 models we tested for regression, we found this model to be the best fit. We choose this model based on the highest R-squared (61.8%) and Adjusted R-squared (61.06%) which means about 61% of the variance in price is explained by the independent variable's horsepower, peak RPM and No. of doors.
2. This model also has the best distributed residuals which is also one of the factors for choosing this model. From the Versus Fits graph below, we can see that most of our data points are close to zero and there is no upward or downward bias in the plot.

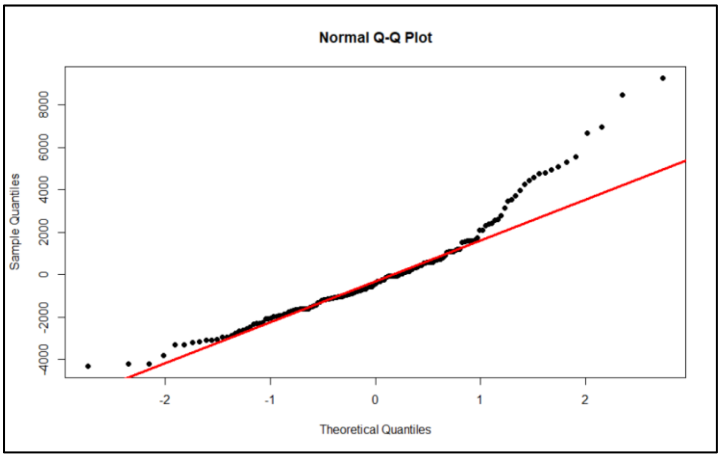




b) L.I.N.E Assumptions:

Linearity & Normality:





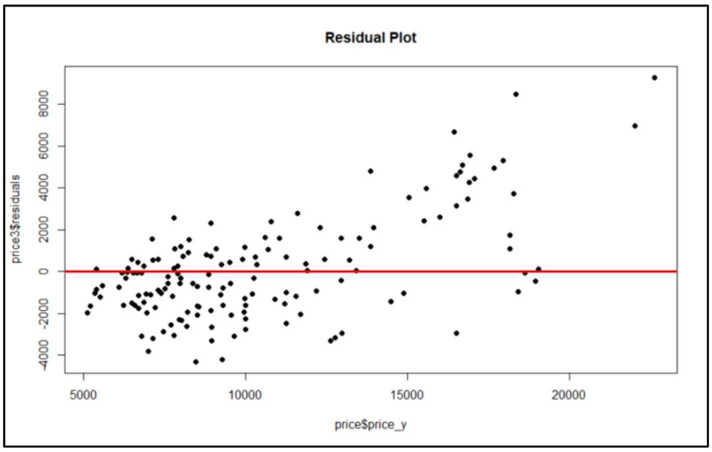
1. This model does voilate the Linearity assumption, as its curved at the end. But since the R-squared is the highest in this case, we chose this model above others.
2. From the histogram and Q-Q plots above, the model doesn't deviate much from the Normality plot so we say it satisfies the Normality assumption.

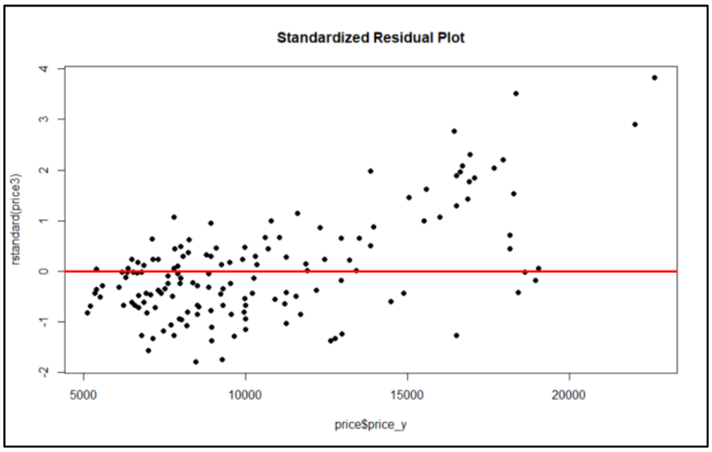
Independence of errors:

```
> # Independence of the observations.
> dwtest(price3)
           Durbin-Watson test
data:  price3
DW = 1.2107, p-value = 1.426e-07
alternative hypothesis: true autocorrelation is greater than 0
```

From Durbin-Watson test, DW=1.2107, p-value = 1.426e-07, residuals are positively autocorrelated which means we have autocorrelation of residuals. This means that our model violates Independence of errors assumption.

Equality of Variances:





Here, we do not see the residuals "dots" fanning out in any triangular fashion, so we can say Equality of Variance assumption is met.

c) Interpretations of models slope and intercept coefficients:

```
Residuals:
    Min      1Q  Median      3Q     Max
-4290.8 -1605.8  -432.5   990.2  9252.5
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     6489.3409  2227.9719   2.913 0.004113 **
horsepower_x1    117.7856     7.6624  15.372  < 2e-16 ***
peakrpm_x2        -1.2119     0.4366  -2.776 0.006187 **
doornumber_x31 -1595.4907   402.9991  -3.959 0.000114 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2447 on 155 degrees of freedom
Multiple R-squared:  0.618,    Adjusted R-squared:  0.6106
F-statistic: 83.59 on 3 and 155 DF,  p-value: < 2.2e-16
```

$\hat{Y}_i = \beta_0 + \beta_1\hat{X}_{1i} + \beta_2\hat{X}_{2i} + \beta_3\hat{X}_{3i}$

$\hat{Y}_i$ = Price for a car in dollars (Dependent/Response variable (price_y))

β_0 = Y-Intercept for the sample

β_1 = Slope of price with horsepower_x1, holding peakrpm_x2 and doornumber_x3 constant (First Regression Coefficient for the sample)

β_2 = Slope of price with peakrpm_x2, holding horsepower_x1 and doornumber_x3 constant (Second Regression Coefficient for the sample)

β_3 = Slope of price with doornumber_x3, holding horsepower_x1 and peakrpm_x2 constant (Third Regression Coefficient for the sample)

$\hat{X}_{1i}$ = Horsepower in hp (First Independent/Explanatory/Predictor variable (horsepower_x1))

$\hat{X}_{2i}$ = Peak RPM in RPM (Second Independent/Explanatory/Predictor variable (peakrpm_x2))

$\hat{X}_{3i}$ = No. of Doors on the car (Third Independent/Explanatory/Predictor variable (doornumber_x3, Two Door = 1))

$Estimated\ Regression\ Model\ for\ this\ case\ [\hat{Y}_i = 6489.34 + 117.78\hat{X}_{1i} - 1.212\hat{X}_{2i} - 1595.49\hat{X}_{3i}]$

1. Interpretation of the Regression Equation:

ß - Coefficient	**Independent Variable Change**	**Net Effect**
$ß_0$ = 6489.34	No change in the other three variables horsepower, peak RPM and No. of doors.	The predicted mean price increases by $6489.34.
$ß_1$ = 117.78	Horsepower increases by 1hp.	The predicted mean price increases by $117.78 holding peak RPM and No. of doors constant.
$ß_2$ = -1.212	Peak RPM increases by 1RPM.	The predicted mean price decreases by $1.212 holding horsepower and No. of doors constant.
$ß_3$ = -1595.49	No. of doors increases by one.	The predicted mean price decreases by $1595.49 holding horsepower and peak RPM constant.

2. p-value interpretations as follows:

p-value	$ß_0$ = 0.004113	$ß_1$ < 2e-16	$ß_2$ = 0.006187	$ß_3$ = 0.000114
Independent Variables	Reject the Null, Significant	Reject the Null, Highly Significant	Reject the Null, Significant	Reject the Null, Significant
Overall p-value < 2.2e-16	Reject the Null, Highly Significant			

d) Estimation and prediction intervals:

Regression Equation

price_y = 6489 + 117.79 horsepower_X1 - 1.212 peakrpm_X2 - 1595 doornumber_X3

$[\hat{Y}_i = 6489.34 + 117.78\hat{X}_{1i} - 1.212\hat{X}_{2i} - 1595.49\hat{X}_{3i}]$

Business Study: Let's say the company is going to introduce a new model and they want to predict the pricing, the new model specifications are as below:

Estimate_1: (High End Sports Model)

Horsepower: 200HP

Peak RPM: 7000RPM

No. of Doors: Two

Plugging-in the values in the Regression equation,

$\hat{Y}_i = 6489.34 + 117.78(200) - 1.212(7000) - 1595.49(1)$

$\hat{Y}_i = \$19965.85$

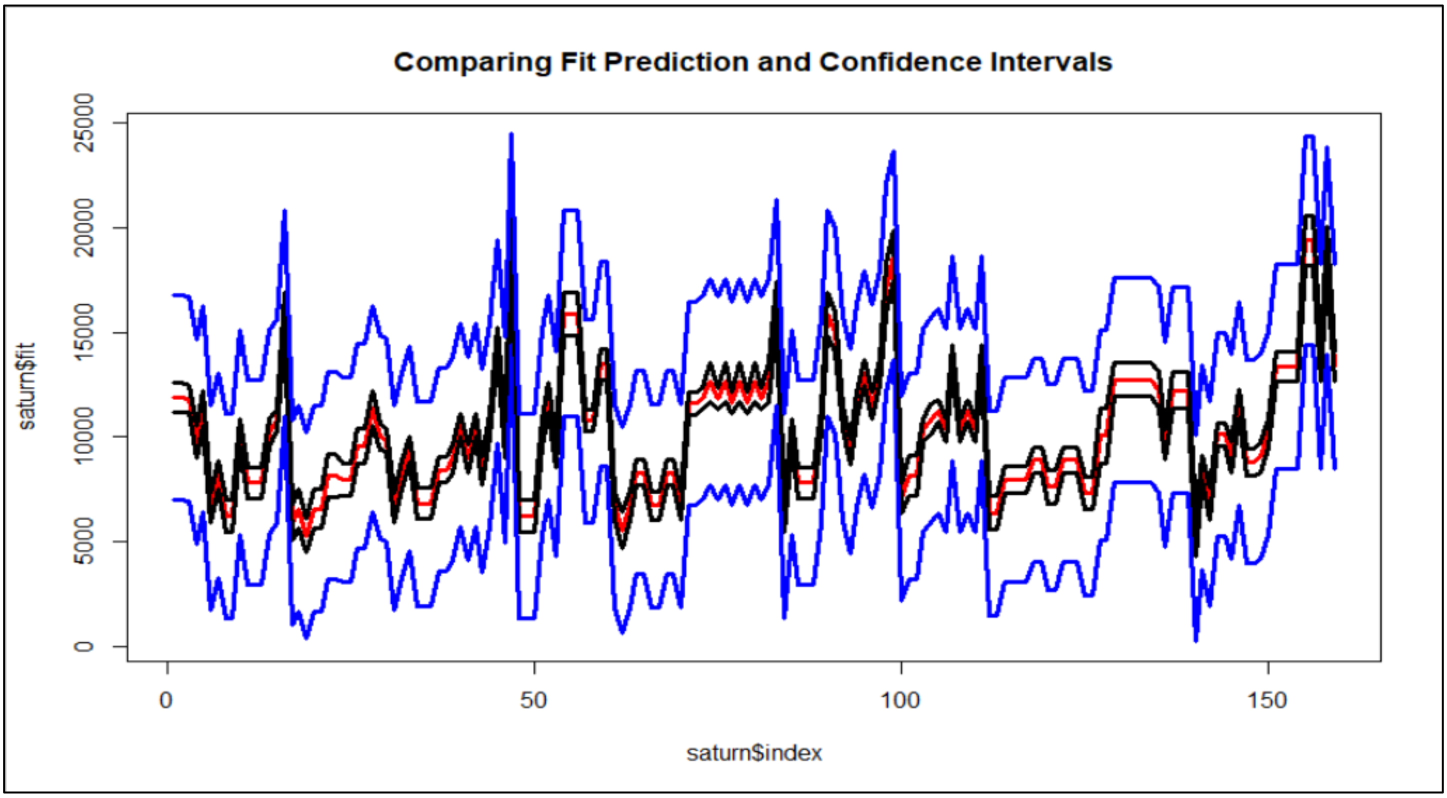
So, the predicted price of the new model is **$19965.85.**

Estimate_2: (Cheapest Model)
Horsepower: 50HP
Peak RPM: 3500RPM
No. of Doors: Two
Plugging-in the values in the Regression equation,
$\hat{Y}_i = 6489.34 + 117.78(50) - 1.212(3500) - 1595.49(1)$
$\hat{Y}_i = \$6540.85$
So, the predicted price of the new model is **$6540.85.**

Prediction Interval as follows:

```
> # Prediction Interval
> sun1=predict(price3,price,interval = "predict")
> max(sun1)
[1] 24484.85
> min(sun1)
[1] 285.138
```

Comparing Fit Prediction and Confidence Intervals:



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