Contents

1 Big Ideas

1.0.1 Passive Sign Convention

current's tail is (+) positive and head is (-) negative
opposite direction for voltage s.t. voltage over passive element is (positive-negative) terminals

1.0.2 Common Component Formulas

Series Resistors $= R_1 + ... + R_n$ Parallel Resistors $= \frac{1}{R_1} + ... + \frac{1}{R_n}^{-1}$ Capacitors perform vice versa
*Note:*Mnemonic: CAP = Capacitors add in parallel

1.0.3 Ohm's Law

1.0.4 Kirchoff's Current Law (KCL)

1.0.5 Kirchoff's Voltage Law (KVL)

1.0.6 Omperational Amplifiers

Different omp-amp circuits allow for different relationships betweein input and output circuts
Negative feedback loop can be on terminal with supply, such that opposite terminal is connected to ground. OR loop can be connected to opposite terminal.

1.1 Types of Circuits

1.1.1 VD (Voltage Divider) Circuit

$V_{out} = \frac{R_2}{R_1+R_2} V_S$

1.1.2 CD (Current Divider) Circuit

$I_X = \frac{R_T}{R_X+R_T} I_T$
Disperses current through multiple resistors, such that ratio of all other resistors over all resistors

1.1.3 RC (Resistor Capacitor) Circuit

2 Notes

2.1 Note 1: System of Linear Equations with Guassian Elimination

2.1.1 1.7.4 Gussian Elimination w/ Augmented Matrix

basic variable = pivot column
free variable = non-basic variable

2.2 Note 2: Consistency Vectors and Matrices

2.2.1 2.1 Solutions to Systems of Linear Equations

2.2.2 Linear Algebra Terminology

consistent = ≥ 1 solution, s.t. 1 solution if and only if RREF has more pivot columns than rows without inconsistent shows below
inconsistent = no solutions, if and only if RREF has row of form $[0...0 \mid c]$

2.3 Note 8: Subspaces

2.3.1 8.2.1 Rank

Def 8.2: rank(A) = dim(columnspace(A)) = # of lin. ind. column vectors of A
Note: Range and Columnspace are equivalent

2.4 Note 9: Eigenvalues and Eigenvectors

2.4.1 9.4 Computing Eigenvalues and Determinants

Def: $(A - \lambda I)\vec{x} = 0 \implies det(A - \lambda I) = 0$
Rationale: Implication is true because we are trying to find solutions to the equation s.t. $\vec{x}$ does not equal 0, meaning columns of $(A - \lambda I)$ is linearly dependent and therfore determinant of it must be 0.
This is because by definition of a eigenvector, it can't be zero vector, therefore, we are trying to find a set of set of nonzero coefficients to multipy the columns of the Matrix to get the zero vector, which means that the matrix must be linearly independent.

Eigenvalue and Eigen Vector Computation Steps

1. Compute determinant of $(A - \lambda I)$ s.t. diagonal is subtracted by λ's, and the detererminant is in polynomial form *Note:* This polynomial is called

the "characteristic polynonial"

2. Set polynomial to 0, then compute the values of λ's, which are the roots of the polynomial

3. Plug each lambda back into the matrix and compute the solutions of the matrix, which will have at 1 free variable, s.t. it's the span of some column

2.4.2 9.6.3 Repeated Eigenvalues

Matrix is defective is the dimension of the eigenspace is less than the multiplicity of the eigenvector.
Ex: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ s.t. $\lambda = 0(mult.2)$ but eigenspace of $\lambda = 0$ is span([1, 0])

2.5 Note 11: Introduction to Circuit Analysis

2.5.1 11.3 Basic Circuit Elements

Voltage Source: V_s is always voltage at positive minus voltage at negative end

2.5.2 NVA (Node Voltage Analysis) Algorithm

1. 1. Find reference node, and set to ground
2. 2. Label nodes whose voltages are set by voltage sources
3. 3. Label remaining nodes
4. 4. Label element's currents arbitrarily, then label corresponding voltages according to passive sign covention
5. 5. Write KCL for each node whose voltage not defined
6. 6. Find expressions for the current through each element, then express all voltages in the form of node voltages
7. 7. Substitute KCL equations from step 5 with expressions from step 6, then rearrange with all node voltages to one side
8. 8. Solve the system of linear equations using matrix form *Note:*Resistors not connected to nodes in question, such as source or reference only appear once in matrix

2.6 Note 13: Resistive Touchscreen

2.6.1 13.2 Power

$$P = \frac{dE}{dt}$$

$$dE = VdQ \implies \frac{dE}{dt} = V\frac{dQ}{dt} \implies P = VI$$

*Note:*Positive Power = Dissipation vs. Negative Power = Supply, which served as impetus for PSC
*Note:*Voltmeter behaves like open circuit and Ammeter behaves like short circuit

2.7 Note 15A: Superposition and Dependent Sources

2.7.1 15.3 Superposition

A circuity can be modelled as a matrix-vector equation
$A\vec{x} = \vec{b}$ s.t. $\vec{x}$ contains unknown node potentials and currents & $\vec{b}$ contains independent voltage and current sources
Note: Since it models a real system, there must exist a solution, therefore A is invertible
When represented as $\vec{x} = A^{-1}\vec{b}$
$b_i = \alpha_1 V_{s1} + ... + \alpha m V_{sm} + \beta_1 I_{s1} + ... + \beta_n I_{sn}$
Therefore, each coefficient can be calculated separately and summer together at the end, while making every other input voltage or current source equal to 0.
Setting an input voltage to zero equates to a wire also called a short circuit or 0 resistance, and setting an input current to zero equates to no wire also known as an open circuity or or infinite resistance.

2.8 Note 15B: Equivalence

Equivalence simplifies interactions between circuits
Equivalence = same I-V relationship, which can be simulated through a source (either voltage or current) and resistor
Thevenin: Defined by I-V intersection with x-axis intercept at zero current, represented by current source and resistor Norton: Defined by I-V interesection with y-axis intercept at zero voltage, represented by voltage source and resistor

2.8.1 15.2 Thevenin Equivalent Circuit

Solve for V_{Th}, R_{Th}

1. Find V_{Th}, by measuring voltage over open circuit s.t. $V_{Th} = V_{OC}$
2. Find R_{Th}, by zero'ing all independent sources then apply test current or test voltage to find the resistance

2.8.2 15.3 Norton Equivalent Circuit

$R_{No} = R_{Th}$, and the Nortan current is measured by placing short across the terminals and measuring the current

2.8.3 15.4 Equivalence Examples

15.4.1 Series Resistors = same current flows through them
15.4.2 Parallel Resistors = voltage across them are the same
Parallel operator $|| \; s.t. x \; || \; y = \frac{xy}{x+y}$

2.9 Note 16: Introduction to Capacitor Touchscreen

Resistive Touch Screen: Uses voltage divider to map analog voltage to position
Capcitor Touch Screen: Determines if each pixel is touched or not, so it can register multiple touches

2.9.1 16.2 Capacitor

Self-Capacitance: $Q = CV_C$ s.t. V_C is voltage across capacitor and Q is charge stored in capacitor
*Note:*measured by Farads F s.t. capacitor of 1 Farad store 1 Coulomb of charge when voltage across capacitor is 1 volt

$$\frac{d}{dt}Q = C\frac{d}{dt}V_C \tag{1}$$

s.t. $\frac{dQ}{dt} = I$

$$I = C\frac{dV_C}{dt} \tag{2}$$

Therfore, current only flows when voltage of capacitor changes

Applying separation of variables, then integratin both sides, and assuming current is constant we get: $V_C(t) = \frac{I}{C}t + V_C(0)$

2.9.2 16.3 Capacitor Equivalence

$C_{eq} = \frac{I_{test}}{\frac{dV_{V_{test}}}{dt}}$

2.9.3 16.4 Capacitor Physics

$E = \frac{1}{2}CV^2$

2.10 Note 17: Capacitive Touchscreen

Touchscreen Variables

1. F (Finger), E1 (Bollow Electrode), E2 (Top Electrode)
2. C_0 (Capacitance between E1 and E2), C_{E1-F} (Capacitance between F and E1), & C_{F-E2} (capacitance between F and E2)

The equivalence capacitance for the entire circuit changes when there is and isn't a finger on top of the pixel s.t. C_0 is parallel to $C_{F-E1}\&C_{F-E2}$, so that the capacitances are added up

2.10.1 17.2 Capacitance Measurement

Easy Method: Use constant current source, measure the voltage over a capacitor after a specific unit of time to determine the capacitance
Better Method: Use reference capacitor and three switches, such that on phase 1, the voltage square and capacitor in interest is in a closed circuit while the reference capacitor is in it's separate circuit. In phase two, the capacitor in interest and reference capacitor are their separate closed circuit, which is measured to determine the capacitance of the capacitance in interest. This design allows for the reference capacitor to have 0 voltage across it before entering phase two.

Charge Sharing $Q_{\phi 1} = Q_{\phi 2}$ s.t. $Q_{\phi i} = \sum$ of equivalence charges for each capacitor in the circuit
$V_{out} = \frac{C_{eq,E1-E2}}{C_{eq,E1-E2}+C_{ref}} V_S$

2.10.2 17.3 Comparatars & Omp-amp Basics

Op-amp can transform input $(V_+ - V_-)$ into $A(V_+ - V_-)$
Given an ideal omp-amp: if $(V_+ - V_-) < 0$, then $V_{out} = V_{SS}$ and then > 0, $V_{out} = V_{DD}$

2.11 Note 17B: Charge Sharing

2.11.1 Two-Phase Switch Capacitor Circuit

goal: final all node voltages at end of 2 phases
principle: charge conservation

Algorithm

1. 1. Label all voltages of capacitors
2. 2. Draw separate circuits for each phase, then label all node voltages for each phase

3. 3. Identify "floating nodes" which are connected only to capacitor plates, op-amp inputs, or comparator inputs, s.t. no charge can flow into or out of, so we can apply "charge sharing"
4. 4. Pick each floating node from phase 2, and find the charge on each plate from phase 1. $Q = CV_C$ s.t. positive for positive plate and negative for negative plat.
5. 5. Find total charge of "floating nodes" in phase 2
6. 6. Equate each total charge from step 4 and 5, due to charge sharing
7. 7. Repeat 4-6 for each floating node

2.12 Note 18: Operational Amplifier

2.12.1 Internal Gain

$U_{out} = A(U_+ - U_-)$ s.t. A is Internal Gain
$V_{SS} < U_{out} < V_{DD}$

2.12.2 Digital to Analog Converter (DAC)

Represent DAC with Thevenin equivalent V_{TH}, R_{TH} and speaker with resistance Ω

Buffer between DAC and Speaker, s.t. voltage of DAC remains and current of speaker can fluctuate
Buffer must also triple output voltage of DAC

note: Omp-amp as comparator is pushed to Voltage of DD (drain) or SS (supply), can't fluctuate linearly.

2.12.3 Negative Feedback Loop

Def: output is sent back to input, to keep output at a fixd value.

Output is scaled by f called feedback, and added to input, which is scaled by A called gain.

Error = (input - feedback)

2.12.4 Golden Rules

1. 1. $I_+ = I_- = 0$ b/c input terminals are not a closed circuit

2. 2. $u_+ = u_-$ therefore, error as defined above equals zero. only holds with negative feedback loops and and if internal gain $A \rightarrow \inf$

   Given negative feedback and large A, we can prove rule 2.

Applying the Golden Rules, KCL, and Ohm's Law on Omp-amp results in a relationship between input and output voltage.
note: relationship between input and output voltage equivalent to ratio between R_1, R_2

2.12.5 Inverting Omp-Amplifier

feature loop is connected to input voltage terminal with resistor, while other input terminal is grounded
relationship between input and output is by scaling factor $-\frac{R_f}{R_{in}}$

2.12.6 Two Voltage Source Omp-Amplifier

$V_{out} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2$

2.13 Note 19: Golden Rules Revisted

2.13.1 19.3 Inverting Amplifier

$V_{out} = -\frac{R2}{R1}V_{in}$ s.t. R2 is placed in loop and R1 placed before V_- input terminal, while V_+ is grounded

2.13.2 19.4 Checking Negative Feedback

Steps to Check for NF

1. 1. 0 all independent sources
2. 2. "Dink" output and check if feedback $(V_+ - V_-)$ moves in opposite direction

2.13.3 19.5 Inverting vs. Non-Inverting Amplifiers

Non-inverting: $V_{out} = (1 + \frac{R1}{R2})V_{in}$ s.t. R1 and R2 create a voltage divider on the feedback loop
Inverting: Defined in 19.3

2.13.4 19.6 Artificial Neuron

Attach an "inverting amplifier" at the output of a "inverting summing amplifier" to mimic vector-vector multiplication with coeffients that are two ratios of $\frac{R_3}{R1}$ and $\frac{R3}{R2}$

2.13.5 19.7 Loading and Buffering

Loading: When voltage divder drops current alot at point between two resistors
Unity gain buffer: Op-amp that has gain of 1
Since op-amp has infinite internal resistance, when included it can prevent loading

2.14 Note 20: Op-amps Current Source and Circuit Design

2.14.1 20.1 Design Procedure

1. 1. Specification - Restate design specs which are typically verbose, and comeback to make sure all checkpoints complete
2. 2. Strategy - Use block diagram and specify (1) What you can measure & (2) What you know
3. 3. Implementation - "Pattern-matching" to solve block functionalities
4. 4. Verification - Check each block, block-block connections, and contradictions

2.15 Note 21: Localization and Inner Products

2.15.1 21.2 Positioning Systems

GPS (Global Positioning System) recieves signals from satellites, which can tell (1) Distance from satellite (2) Position of satellite, to then determine it's own distance.
Note: Intersection of 3 circles, with origin as position of satellite and distance as radius

2.15.2 21.3 Inner Product

Note: Known as "Euclidean Inner Product" by mathematicians and "Dot Product" by physicists
Notation: $\langle \vec{v}, \vec{y} \rangle$

2.15.3 21.4 Properties of IP's

1. Commutative (a.k.a. symmetric)
2. Scalar Multiplication (applies to one vector at a time)
3. Distributive over vector addition

2.15.4 21.5 Orthogonal Vectors

Note: Orthogonal if inner product is zero (a.k.a. perpendicular in 2-dimensions)

2.15.5 21.6 Special Inner Product Operations

1. 1. Sum of components: IP of a vector with vector of 1's.
2. 2. Average of components: IP of a vector with vector with equal component multiplicative inverse of dimension of the vector (1/n)
3. 3. Sum of Squares: IP of vector with itself
4. 4. Selective Sum: IP of a vector with a vector of zeros with specific indexes set to zero to specify which elements to count towards sum.

2.15.6 21.7 Norm

Note: Euclidean norm: square root of the sum of squares (a.k.a. magnitude or distance)

2.15.7 21.8 Norm Properties

1. 1. Non-negativity
2. 2. Zero vector: only vector with norm zero
3. 3. Scalar multiplication
4. 4. Triangle Inequality: Sum of two norms is greater than the norm of the sum of vectors
   $\|x + y\| \leq \|x\| \, \|y\|$

2.15.8 21.9 Interpretation of IP

Note: General unit vector: $\vec{x} = \begin{bmatrix} sin(\alpha) \\ cos(\beta) \end{bmatrix}$
Theorem: The innerproduct between two unit vectors is the cosine of the angle between them
Note: Extends to any vectors, because any vector can be turned into a vector with norm 1, by dividing by norm called **normalization**
Note: Versatile because innerproduct does not depend on the coordinate system the vectors are found in, so it can be applied to physical laws. *Note:* Geometric Interpretation: The angle between the two vectors are portrayed through the plane unique to the vectors

2.15.9 21.10 Chauchy-Schwarz Inequality

$| \langle x, y \rangle | \leq \|x\| \, \|y\|$ which relates innerproduct to magnitude
proof: Given the IP-magnitude cosine relationship, since cosine is always less than 1, the less than or equal property holds when the cosine value removed

2.15.10 21.11 p-Norm

Definition: P-root of the sum of the p-powers x^p

2.15.11 Misc: Projection

Projection of vector b onto a: $proj_{\vec{a}}(\vec{b})$

2.16 Note 22: Trilateration and Correlation

Positioning system: (1) Reciever (2) Beacon
Localization Algorithm:
(1) Find distances between a beacon and reciever through message recieved via **correlation**
(2) Compute location through distance measurements via **trilateration**

2.16.1 22.2 Trilateration

Find $\|\vec{x} - \vec{a_i}\|^2 = d_i^2$ s.t. $\vec{x}$ is the reciever, $\vec{a}_i$ is beacon i, and d_i is scalar distance of i'th beacon
Expand right hand side, then write equations s.t. powers of $\vec{x}$ (reciever) are removed
Solve subsequent linear equations s.t. 3 Beacons can find point in 2-space

2.16.2 22.3 Correlation

Signal: Message that contains information as function of time
*Note:*there exists both discrete-time and continuous-time signals s.t. DT signal can be represented as vector with 0-index

2.16.3 22.4 Cross-Correlation

Definition: $corr_{\vec{x}}(\vec{y})[k] = \sum i = -\mathrm{infinf}\vec{x}[i]\vec{y}[i-k]$ s.t. correction of y onto x given k, is the linear combination with y's values displaced k indexes behind x
Cross-correlation is a vector of all these values
*Note:*Linear cross-correlation assumes that that the values of vectors of x and y are zero where not defined, unlike circular cross-correlation
*Note:*numpy.correlate($\vec{x}, \vec{y}, 'full'$), to compute LCC, but 0 shift must be computed on your own
Interpretation: sliding inner product, defining similarity between relative signals at each shift
Properites: Neither LCC or CCC are commutative
*Note:*autocorrelation is cross-correlation on itself

2.16.4 22.5 Modeling Positioning Problem

There is a time delay τ between beacon and reciever
Use $d = v\tau$ s.t. d is distance, v is velocity signal can travel through medium, τ is time delay
$\vec{r} = \alpha\vec{s}^{\tau} \implies \vec{r}[k] = \alpha\vec{s}[k-\tau]$ s.t. r is reciever signal and s is original signal
Goal: Given N equations from N measurements of signals, and two unknowns (time delay, signal shrink factor) find time delay which is non-linearly related to measurements
Given: Reciever already knows the original signal measurement, and performs a circular cross-correlation with the recieved signal.
Solution: $\hat{\tau} = arg\max_k(circorr_{\vec{r}}(\vec{s}))[k]$ *Note:*Periodic correlation is a circular correlation that is a subset of linear correlation where the signal is repeated indefinitely with period N.

2.17 Note 23: Least Squares

Gaussian Elimination or Invertible Matrix can't solve some linear equations due to "noisy data" that causes inconsistent equations with no solutions.
Given, more equations than unknowns, can we approximate a soltuion? a.k.a "overdetermined systems"

2.17.1 Approximate Solution to Linear Systems 23.2

Minimization Problem: $\min_{\vec{x}} \vec{e} = \vec{b} - A\vec{x}$
Duality: columns of A multiplied by elements of x form the matrix-vector product Ax, same as placing each element of Ax as the dot product of row of A and x.
Note: Equivalent to find closest element in the span of columns of A to b, because of duality note above where Ax is represented as linear combination of columns and elemtnts of x, where elements of x can be anyvalue

2D Special Case: s.t. A is one vector
Claim: Best approximation of x to minimize error, is projection of b onto A, with error being orthogonal to A.
Proof: Assume a value of x s.t. error is not orthogonal, which will create a hypothenuse with respect to the original orthogonal vector. Therfore by pythagorean's theorem, the norm of the orthogonal is always less than any other vector.

Solving for x ("Orthogonal Projection"): $\langle \vec{e}, x\vec{a} \rangle = 0$
Using innerproduct laws and replace of e by def., we get $x = \frac{\langle \vec{a}, \vec{b} \rangle}{\langle \vec{a}, \vec{a} \rangle}$
General Case: Find an error vector e, s.t. it's orthogonal to all possible Ax's, which is the span of the columns of A, therefore the columnspace.

2.17.2 Theorem 23.2: Orthogonal Vector to Subspace

A vector is orthogonal to the column space of matrix A, if it is orthogonal to it's columns that form the basis for it's columnspace.
Skipped Proof
Derived: $A^T\vec{e} = \vec{0}$ which can be composed onto $\vec{e} = \vec{b} - A\vec{x}$
After multiple more steps we get: $\vec{x} = (A^TA)^{-1}A^T\vec{b}$

2.18 Note 24: Trilateraztion w/ Multiple Beacons

2.18.1 Orthogonal Matching Pursuit

Goal: Unmix multiple signals by OMP to later compute distance
Reciever records sum of all signals denoted as $\vec{y}$, which is a linear combination of each signal $\vec{s_i}$ that is scaled by it's message α and has a distinct time shit τ
Iterative Algorithm

1. Compute the circular cross-correlation between $\vec{y}$ and each $\vec{s_i}$, and find the max, which will return a specific i'th signal and it's timeday shift
2. Assume recieved signal only comprises that signal, then set up least squares to estimate the scalar α to complete the estimate s.t. $\vec{\hat{y}} = \alpha\vec{s_i}^{\tau}$

3. Compute residual error $\vec{e} = \vec{y} - \vec{\hat{y}}$

4. Repeat above steps on $\vec{e}$, because we assume the largest signal blocks the other signals; recalculate $\vec{\hat{y}}$ and it's corresponding coefficients at each iteration

2.18.2 24.6 Connection to Machine Learning

OMP is a type of "boosting" algorithm
"Gradient Boosting" algos are used with non-euclidean spaces but with principles of OMP
OMP solves N unknowns with less than N equations, using the assumptions that a lot of the unkowns will equal 0 called "sparcity"

2.19 Practice Finals Content

2.19.1 Spring 2020 Final

7D. $det(AB) = det(A)det(B)$, so multiplicativity applies
7D. $det(A^T) = det(A)$, so transpose equivalent as original 10. Least Squares cannot be applied to column rank-deficient matrices, s.t. rank ¡ # of columns