

# CS F222 Discrete Structures for Computer Science

## Assignment

### Students:

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### Question:

54. Let  $H = \{[0], [2], [4], [6]\}$ . Write a program that computes the right cosets of  $H$  in  $\mathbb{Z}_8$ .

### Real Life Applications:

#### 1. Lagrange's Theorem:

- One of the most important applications of right cosets is in Lagrange's theorem, which states that for any finite group  $G$  and its subgroup  $H$ , the order of  $H$  divides the order of  $G$ . This theorem is fundamental in understanding the structure of finite groups.

#### 2. Group Quotients:

- Right cosets play a crucial role in defining factor groups or quotient groups. The set of right cosets of a subgroup  $H$  in a group  $G$  forms a group, known as the factor group or quotient group  $G/H$ . This construction helps to study the structure of  $G$  by focusing on the cosets of  $H$ .

#### 3. Cayley's Theorem:

- Cayley's theorem states that every group is isomorphic to a subgroup of a symmetric group. The right cosets are used in the proof of Cayley's theorem, where permutations are associated with group elements.

#### 4. Normal Subgroups:

- If the right cosets of a subgroup coincide with the left cosets (i.e.,  $aH=Ha$  for all  $a \in G$ ), then the subgroup is termed a normal subgroup. Normal subgroups are essential in quotient group constructions and have applications in various areas of mathematics and physics.

#### 5. Cryptographic Applications:

- In certain cryptographic protocols, group theory concepts, including right cosets, are employed for designing secure systems. For example, in public-key cryptography, the difficulty of solving problems related to certain groups forms the basis for security.

#### 6. Error-Correcting Codes:

- Group theory, including the study of cosets, is applied in coding theory for designing error-correcting codes. The mathematical structure of groups provides a framework for creating efficient and reliable error-correcting codes.

#### 7. Permutation Groups:

- In permutation group theory, right cosets are used to study the actions of groups on sets. The concept of cosets helps in understanding the orbits and stabilizers of group actions.

#### 8. Automorphism Groups:

- Right cosets are involved in the study of automorphism groups. Understanding the cosets allows for the analysis of symmetries and transformations within a group.