① MFML Assignment NO. 1

- 2023AC05339
- Sec-3

Q-1

Ans. Let A Be the Invertible lower triangular matrix.

So, By the property of Inverse

$$AA^{-1} = I \quad \text{--- ①}$$

consider $A^{-1} = B$

As we know $I_{ij} = \begin{cases} 1, & \text{if } j=i \\ 0, & \text{if } j \neq i \end{cases}$

$\therefore [A_{i1}, A_{i2}, A_{i3} \cdots A_{in}] * \begin{bmatrix} B_{1k} \\ B_{2k} \\ B_{3k} \\ \vdots \\ B_{nk} \end{bmatrix} = \begin{cases} 1, & \text{if } k=i \\ 0, & \text{if } k \neq i \end{cases}$

$$\Rightarrow \sum_{j=1}^{n} A_{ij} B_{jk} = \begin{cases} 1, & \text{if } i=k \\ 0, & \text{if } i \neq k \end{cases} \quad \text{--- ②}$$

from ② for $i<k$ we have

$$\sum_{j=1}^{n} A_{ij} B_{jk} = 0 \quad \text{--- ③}$$

But As A is lower triangular matrix
$A_{ij} \neq 0$ for $i<j$.

So for equation ③ to be 0,
~~$B_{jk} = 0$~~. $B_{jk} = 0$

As $B_{jk} = 0$
all element above the diagonal are $\neq 0$. — (4)

Also As we know, For matrix to be invertible,
diagonal element should $\neq 0$ — (5)

That also implies
Inverse of matrix also have non-zero diagonal elements. — (6)

From equation (4) & (6) we prove that B is lower triangular matrix

Hence we proved Inverse of lower triangular matrix is lower triangular matrix.

③

Q.2

Ans.

let A be the square matrix with integer element $\longrightarrow$ ①

$\therefore$ from given $\det(A) = nk$ — ②

let $\lambda_1, \lambda_2 \dots \lambda_m$ be the eigenvalues of matrix A.

where n is also part of those value.

As we know.

$\det(A)$ = product of eigenvalues of A

$\therefore \det(A) = \lambda_1 \times \lambda_2 \times \lambda_3 \cdots n \times \lambda_m$

$= n \times \lambda_1 \times \lambda_2 \times \lambda_3 \cdots \lambda_m$

since all element is integer for matrix A. so determinent is also integer.

$\therefore n \times (\lambda_i)$ where $i = 1 \cdots m$.

is also integer.

$\Rightarrow$ As n is already given integer λ_i must be integer

from equation ②

$\det(A) = nk = n\lambda_i$

$nk = \lambda_i n$

$k = \lambda_i$

As λ_i is integer k is also integer

Hence proved

(4)

Q.3

Ans

Given: A is square matrix whose inverse exists.

→ B is matrix such that AB can be defined

From given
$$AA^{-1} = I \quad (\text{As } A^{-1} \text{ exists})$$

B can be written
$$B = A^{-1}AB \Rightarrow \text{rank}(B) = \text{rank}(A^{-1}AB) \quad -①$$

As the rank of product of 2 matrices is less than or equal to rank of either factor

$\therefore \quad \text{rank}(B) \geq \text{rank}(AB)$

$\text{rank}(A) \geq \text{rank}(AB) \quad ②$

As A is Invertible matrix ∴
$\text{rank}(A) = n.$

If B is choosen in such way that (AB) Exists.

Then Rank of $(AB) = n$. $-③$

and if B is choosen in such way the AB Does not exist then

$\text{rank}(AB)$ is not defined ④

From ③ & ④

$\text{rank}(AB) = \text{rank}(A)$, if AB exists

$\text{rank}(AB)$ canot defined, If AB not exists.

Q.4
Ans Given:

$$A^2 = 8I \quad , a = 1 \quad , b = -1$$

(a) As we know, if λ is eigenvalue of vector A, then
there is eigenvecto v such that

$$AV = \lambda V \quad \Rightarrow \text{eigenvalue equation.} \quad -(1)$$

multiply both side by A

$$\Rightarrow \quad A^2 V = \lambda V A$$
$$= \lambda (AV)$$
$$= \lambda(\lambda V) \quad - \text{from equation (1)}$$
$$A^2 V = \lambda^2 V \quad - (2)$$

from Given substitute value of A^2 in eq (2)

$$(8I) V = \lambda^2 V$$
$$8\cancel{V} = \lambda^2 \cancel{V}$$
$$\lambda^2 = 8$$
$$\lambda = \pm 2\sqrt{2}$$

(b) from (a) $\lambda_{max} = 2\sqrt{2}$
$\lambda_{min} = -2\sqrt{2}$

$$\therefore \quad \frac{\lambda_{max}}{\lambda_{min}} = \frac{2\sqrt{2}}{-2\sqrt{2}} = -1$$

Q.5

Ans (a) Assume $x+y+z=0$

consider any vector in span{x,y} as linear combination of x & y, i.e.

$V_{xy} = ax+by$ where $a, \& b$ are scalars.

since $x+y+z=0 \Rightarrow x=-(y+z)$ — (1)

$V_{xy} = ax+by = -(a+b)^{x} z = (-a+b)y - az$

$V_{xy} = (-a+b)y - az$ — (2)

$\therefore$ any vector in span (x, y) can be written as linear combination of y and z.

$\therefore$ span(x,y) = span(y,z) — (3)

Similarly vector from span(x,z) can written as $V_{xz} = ax + bz$

where $a \& b$ are scalars.

$\therefore V_{xz} = ax + bz$

replace z from equ (1)

$V_{xz} = ax + b(-x-y)$

$V_{xz} = x(a-b) - by$ — (4)

$\therefore$ any vector from x, z span can be written as linear combination of x & y

$\therefore$ span(x,y) = span(x,z) — (5)

(7)

therefore from equ (3) & equ (5)
we can write as
$$\underline{\text{span}(x,y) = \text{span}(y,z) = \text{span}(x,z)}$$

(b)

(1) Suppose we have vector space V and the vectors $v_1, v_2, \ldots v_n$ span V.

This means that any vector W ~~can~~ from vector space V can be expressed as linear combination of these vectors

$\therefore$ let take W as

$$W = a_1 v_1 + a_2 v_2 - a_2 v_1 + a_3 v_3 - a_3 v_1 \ldots a_n v_n a$$

where $a_1, a_2, a_3 \ldots a_n$ are scalar coefficient

rewrite the W as below

$$W = a_1 v_1 + a_2(v_2 - v_1) + a_3(v_3 - v_1) \cdots a_n(v_n - v_1)$$

This show vector W can expressed as linear combination of vectors
$$v_1, v_2 - v_1, v_3 - v_1 \ldots v_n - v_1$$
therefore the vectors $v_1, v_2 - v_1, v_3 - v_1 \ldots v_n - v_1$ also span the vector space V.

(2) Suppose, there exist scalars $b_1, b_2, b_3 \ldots b_n$

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Such that,

$$b_1 v_1 + b_2(v_2 - v_1) + b_3(v_3 - v_1) \cdots b_n(v_n - v_1) = 0$$

Rearranging the term we get

$$(b_1 + b_2 + b_3 \cdots b_n)v_1 + b_2 v_2 + b_3 v_3 \cdots + b_n v_n = 0$$

Since $v_1, v_2, v_3 \cdots v_n$ are linearly independent this implies that

$$\Rightarrow b_1 + b_2 + b_3 \cdots b_n = 0$$
$$b_2 = 0$$
$$b_3 = 0$$
$$b_n = 0$$

$\therefore$ only solution to equation

$$b_1 = b_2 = b_3 = \cdots = b_n = 0.$$

which mean that the vectors $v_1, v_2 - v_1, v_3 - v_1 \cdots v_n - v_1$ are also linearly independent