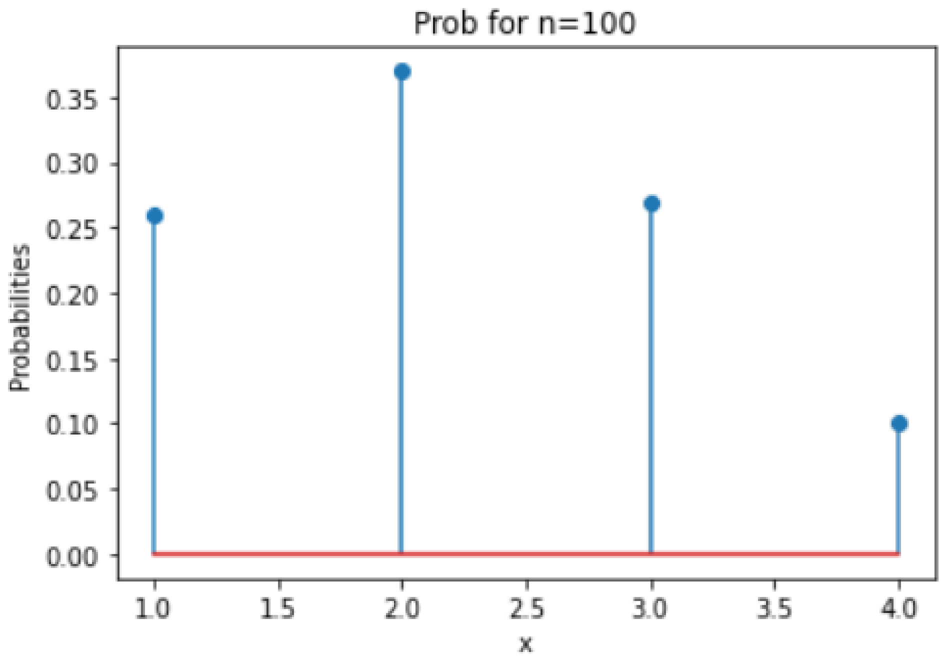
Reinforcement Learning

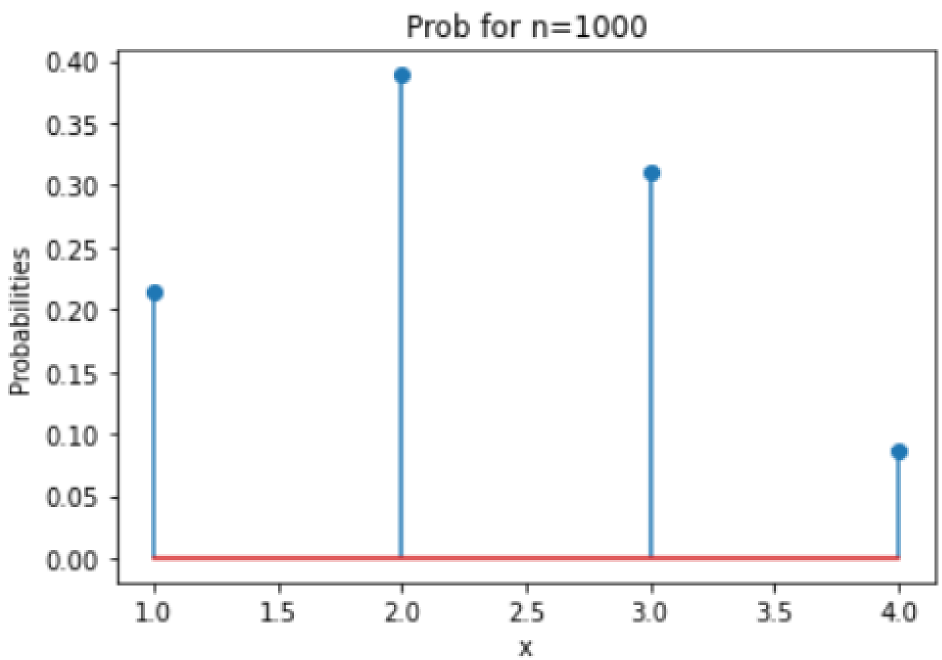
Lab Assignment 1

190020007-190020021-190020039

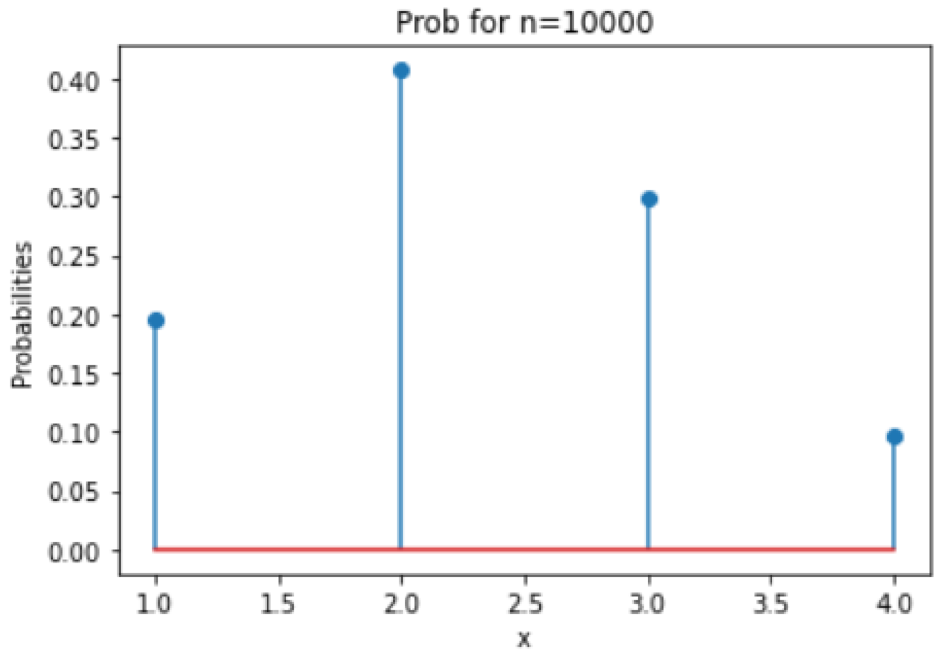
1. For the first part of the first question, we used the in-built library to generate the required distribution and then found the fraction of points for the corresponding x's. Then we plotted a discrete plot of the fraction of points against x which followed the required probabilities and the outcome became more and more appropriate with increase in the number of points in the sample. The plots for the same are attached below.



Prob for n=100
Probabilities
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
1.0
1.5
2.0
2.5
3.0
3.5
4.0

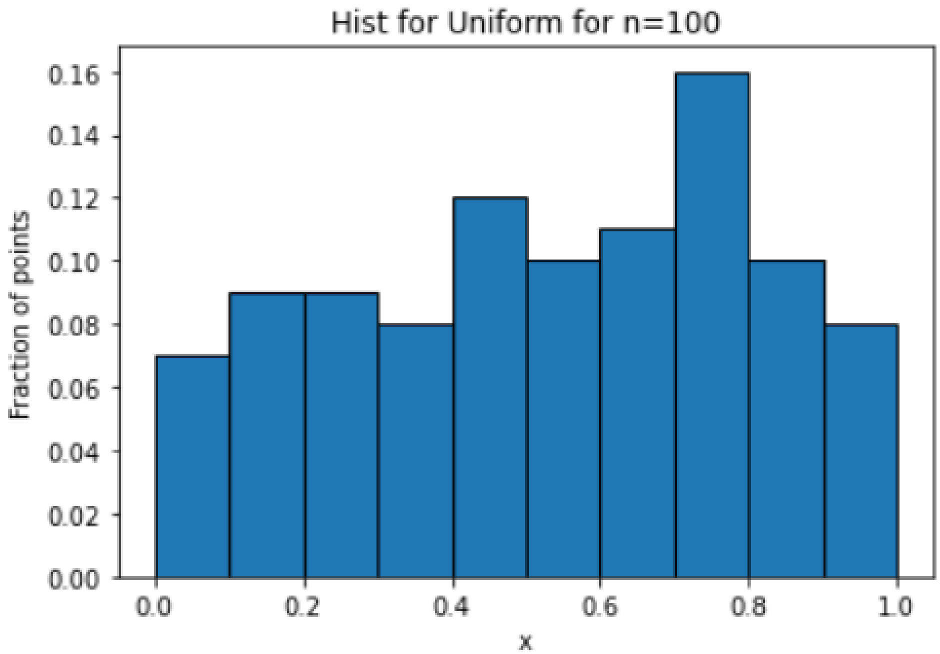


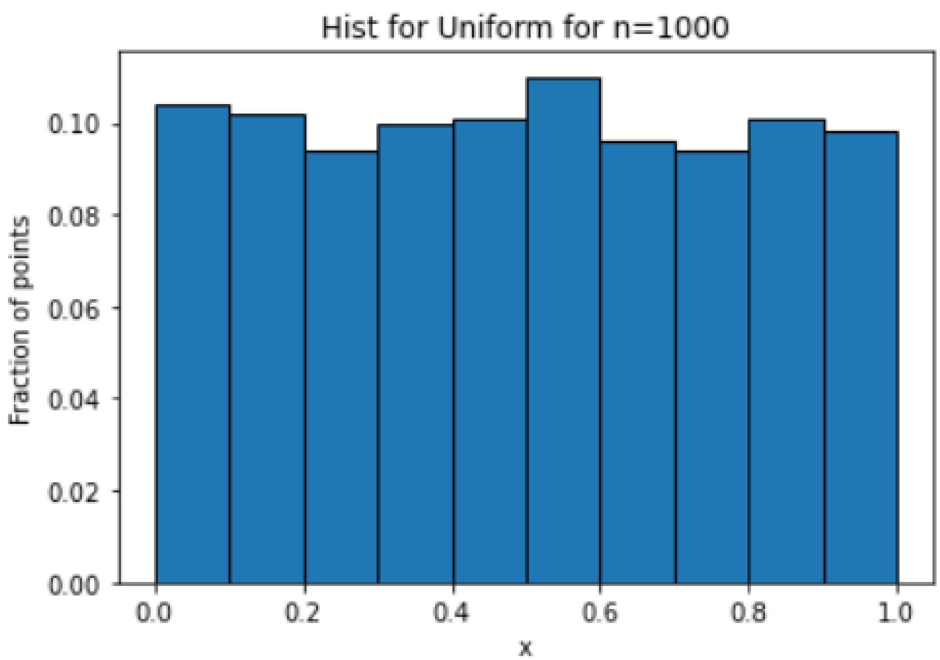
Prob for n=1000
Probabilities
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
1.0
1.5
2.0
2.5
3.0
3.5
4.0

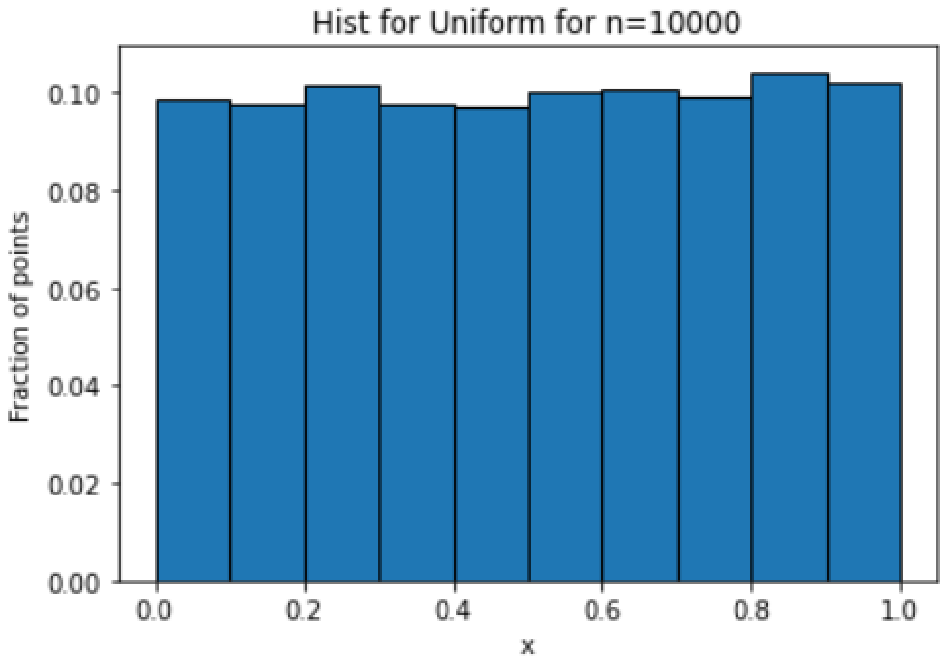


For the rest of the parts, we again generated points using the in-built libraries, then used the histogram function to find the fraction of points in ranges of the domain x (bins) and plotted the histogram of the fraction of points vs the bins. It followed the required distribution for all the cases and the outcome became more and more appropriate with an increase in the number of points.

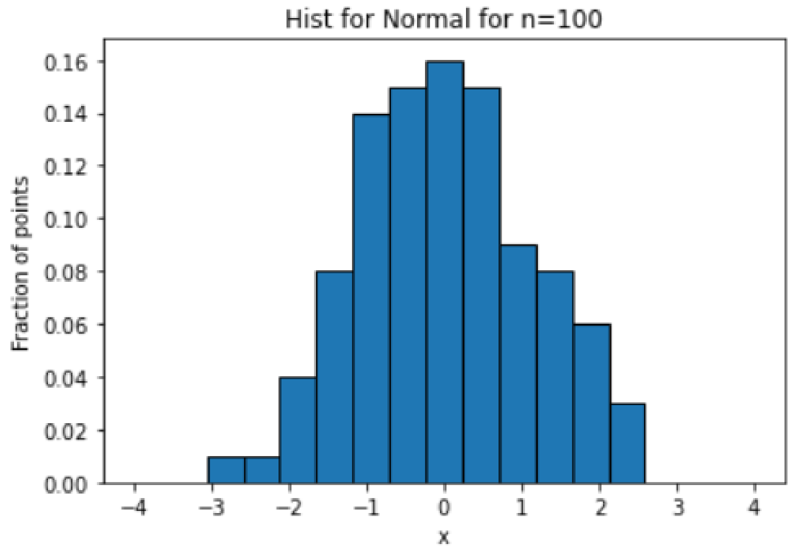
Uniform distribution: As visible, the plot approaches a uniform distribution with an increase in number of points taken in the sample. This is true since histogram of the fraction of points or probabilities should follow a uniform type of plot for a sample from uniform distribution.

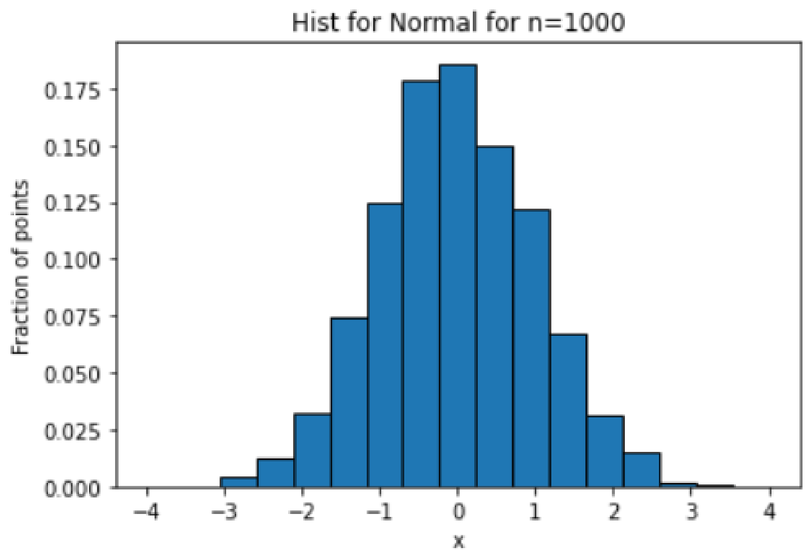

Hist for Uniform for n=100
Fraction of points
x
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.0
0.2
0.4
0.6
0.8
1.0

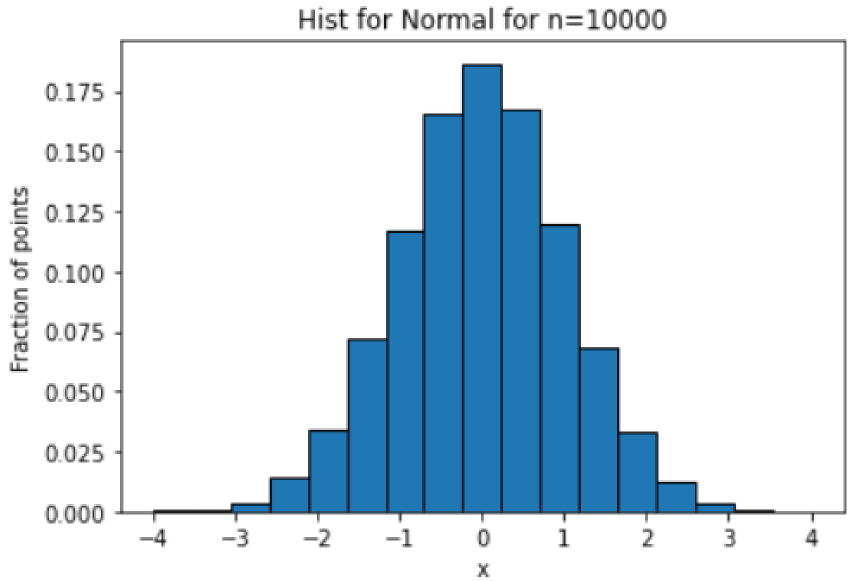

Hist for Uniform for n=1000
Fraction of points
x
0.00
0.02
0.04
0.06
0.08
0.10
0.0
0.2
0.4
0.6
0.8
1.0



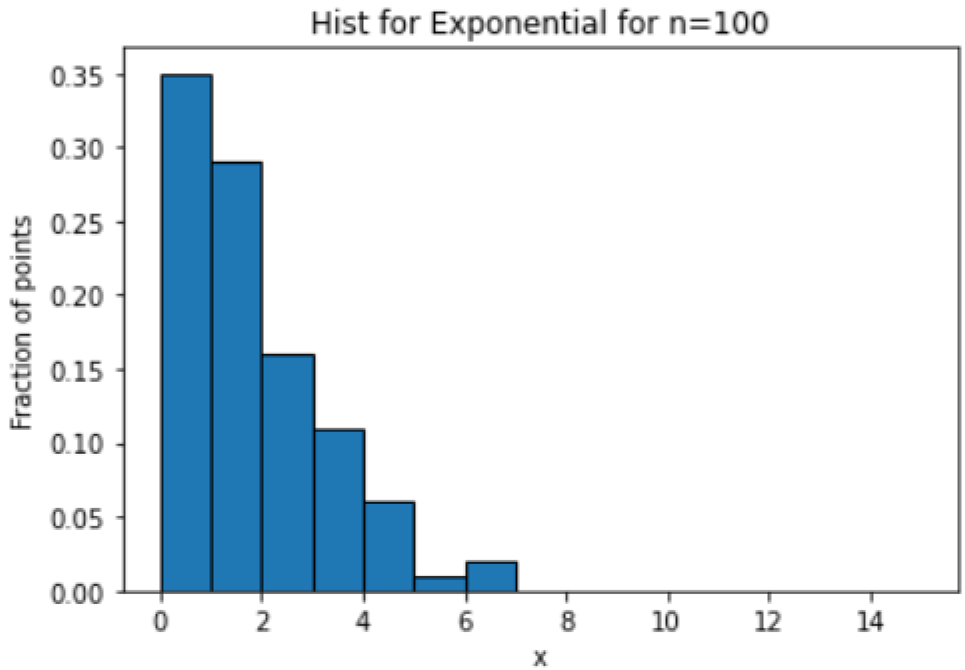
Standard Normal Distribution: As visible, the plot approaches a standard normal distribution with an increase in number of points taken in the sample. This is true since histogram of the fraction of points or probabilities should follow standard normal type of plot for a sample from standard normal distribution.

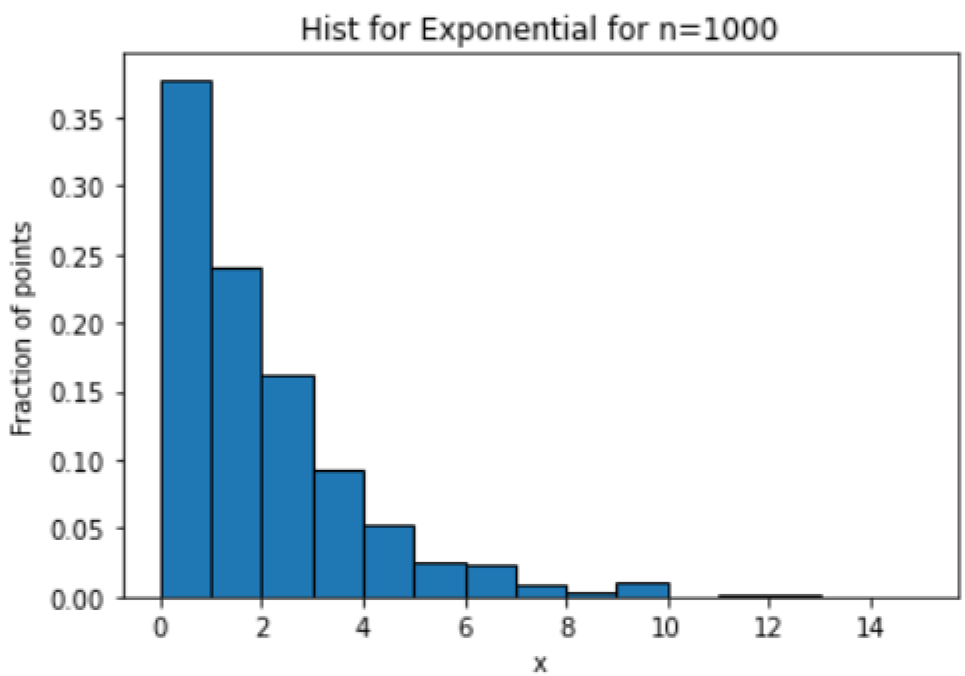

Hist for Normal for n=100
Fraction of points
x
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−4
−3
−2
−1
0
1
2
3
4

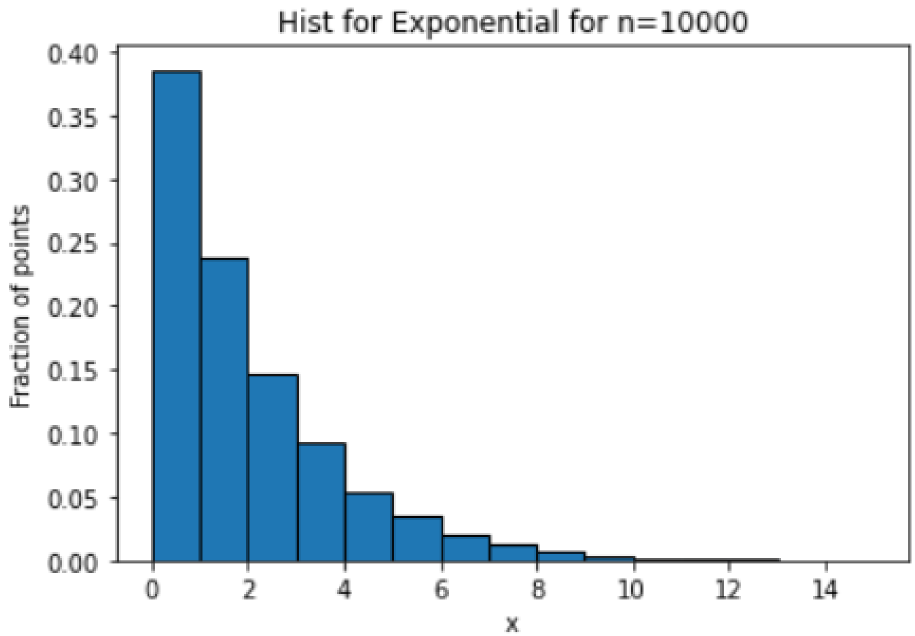

Hist for Normal for n=1000
Fraction of points
x
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
−4
−3
−2
−1
0
1
2
3
4



Exponential Distribution: Here we have to take the parameter λ=0.5 which has been taken. As visible, the plot approaches an exponential distribution with an increase in the number of points taken in the sample. This is true since histogram of the fraction of points or probabilities should follow an exponential type of plot for a sample from an exponential distribution.


Hist for Exponential for n=100
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Fraction of points
0
2
4
6
8
10
12
14
x

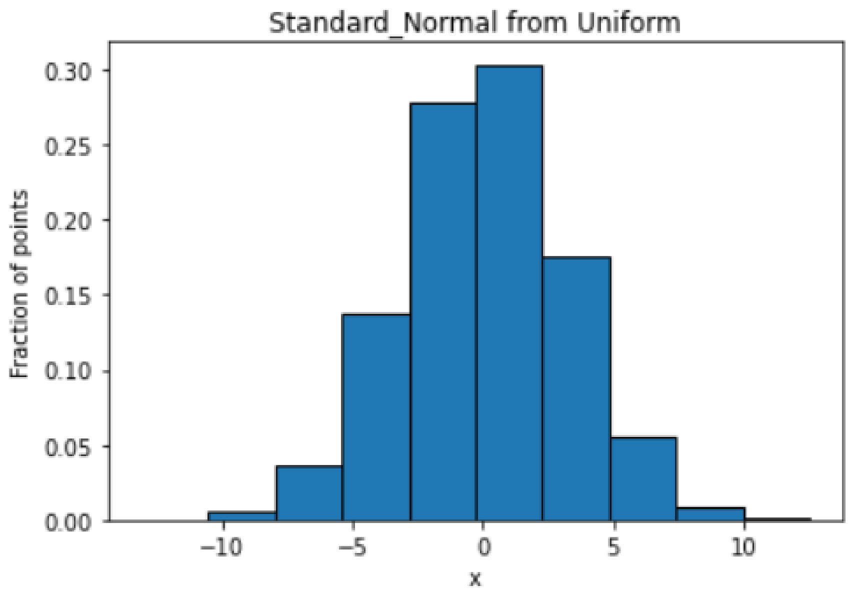

Hist for Exponential for n=1000
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Fraction of points
0
2
4
6
8
10
12
14
x



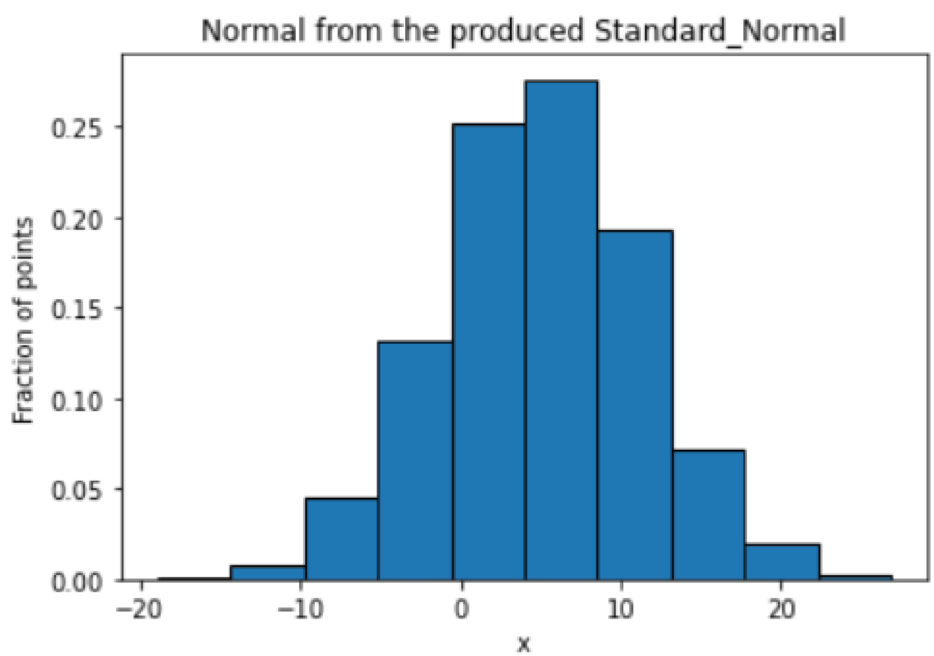
2. Here we used the Central Limit Theorem to generate normal distribution samples. We took 10,000 uniform distribution samples. Now, we found the distribution of means of these samples. Then we normalized this distribution by using the formula: $\sqrt{}(no\_of\_samples) * (samples\_means\_distribution - \mu\_uniform\_distribution)/(\sigma\_uniform\_distribution)$. This normalized distribution follows a standard normal distribution, say X. Now, using $variance * X + mean$, any normal distribution sample with some mean and variance can be produced(Linear operation on gaussian gives a gaussian). An example of the same is shown in the code.

Graphs:-

A. Standard Normal: The normalized distribution given by the equation above follows a standard normal distribution as expected. The histogram for the same is shown below.



B. Normal Distribution with mean=5, variance=4: Using the equation $variance * X + mean$ As mentioned above, we get our normal distribution with the required mean and variance. The histogram for the same is shown below.



3.

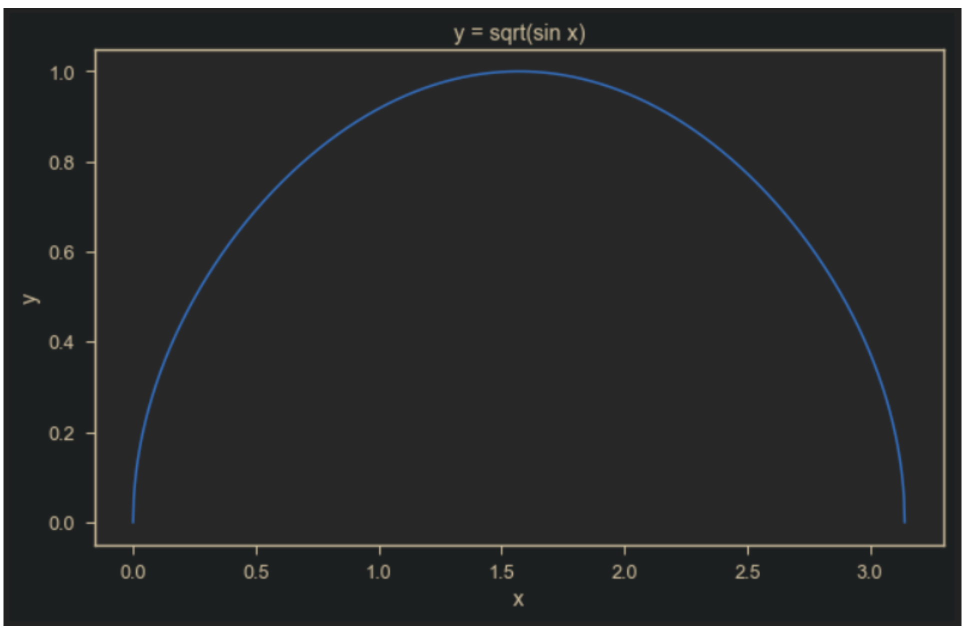
Methodology:

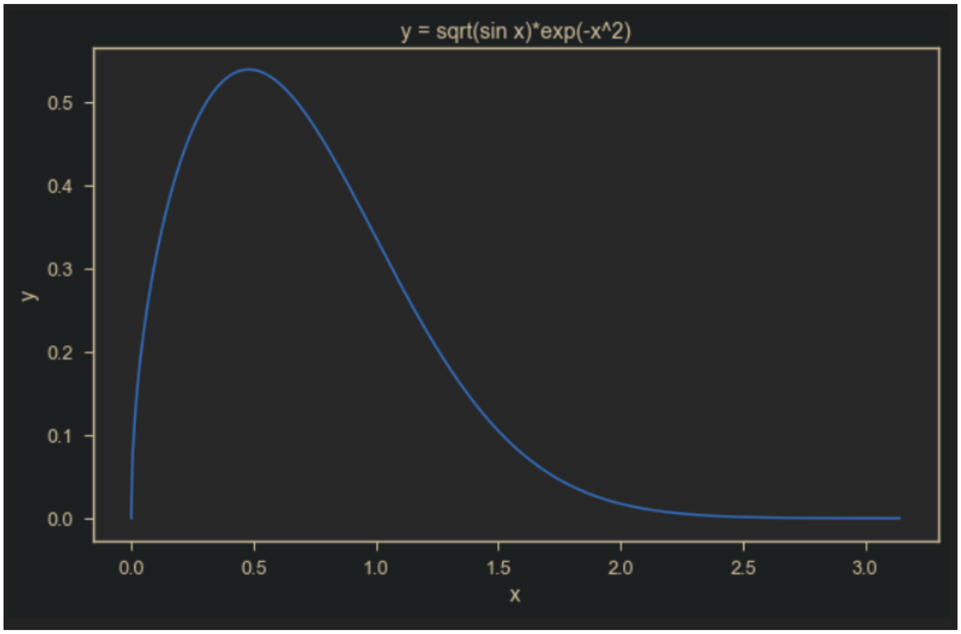
Expected Value Rule for Functions of Random Variables : Let X be a random variable with PDF $p_X(x)$, and let $g(x)$ be a real valued function of X. Then, the expected value of the random variable $g(x)$ is given by : $E[g(X)] = \int_{-\infty}^{\infty} g(x)\, p_X(x)\, dx$.

Strong Law of Large Numbers (SLLN) : The strong law of large numbers states that the sample average converges almost surely to the expected value i.e. $P(\lim_{n\to\infty} \sum_{i=1}^{n} \frac{X_i}{n} = E[X]) = 1$ where X_i is an IID rv.

Plots:

We generated an array of 1000 equally spaced points between 0 and π to be used as values of x. Using functions *f1* and *f2* we generated the values of expressions given in both parts of the question. Plots are shown below:





Area calculation:

Part (a):

Suppose we have a r.v. $X1 \sim U(0,\pi)$

i.e. pdf of $X1$, $f_{X1}(x) = \frac{1}{\pi} \quad \forall \; x \in [0,\pi]$

Consider the function, $g(x_1) = \sqrt{\sin x_1}$

$$E[g(x_1)] = \int_0^{\pi} g(x) f_{x_1}(x)\, dx$$

$$= \int_0^{\pi} \sqrt{\sin x} \, \frac{1}{\pi} \, dx$$

$$\Rightarrow \int_0^{\pi} \sqrt{\sin x} \, dx = \pi \cdot E[g(x_1)]$$

The value of $E[g(x_1)]$ can be found using Law of Large Numbers.

Simulating in code, we found

$E[g(X_1)] \approx 0.76$

$\therefore$ Area $= (\pi)(0.76)$

$= \boxed{2.39}$

Part (b):

Suppose we have a r.v. $X \sim N\left(0, \frac{1}{\sqrt{2}}\right)$ i.e.

pdf of X, $f_X(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} \quad \forall x \in \mathbb{R}$

Consider $X^* = |X|$ which has a pdf,

$$f_{X^*}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \quad \forall x \in [0, \infty)$$

If we truncate the above pdf between 0 & π,

we get a r.v. $X2$ which has a pdf

$$f_{X_2}(x) = \begin{cases} \frac{2}{\sqrt{\pi}} \frac{e^{-x^2}}{F_X(\pi) - F_X(0)} & ; 0 \leq x \leq \pi \\ 0 & ; \text{otherwise} \end{cases}$$

Consider the function, $g(X_2) = \sqrt{\sin(X_2)}$

$$E[g(X_2)] = \int_0^{\pi} g(x)\, f_{X_2}(x)\,.dx$$

$$= \int_0^{\pi} \sqrt{\sin x}\ \frac{2}{\sqrt{\pi}}\ \frac{e^{-x^2}}{F_X(\pi)-F_X(0)}\,.dx$$

$$\Rightarrow \int_0^{\pi} \sqrt{\sin x}\ e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}.\left[F_X(\pi)-F_X(0)\right].E[g(X_2)]$$

The value of $E[g(X_2)]$ can be found by using Law of Large Numbers.

Simulating in code, we found

$$E[g(X_2)] = 0.65$$

$$\therefore \text{Area} = \left(\frac{\sqrt{\pi}}{2}\right)(0.99)(0.65)$$

$$= \boxed{0.57}$$

4.

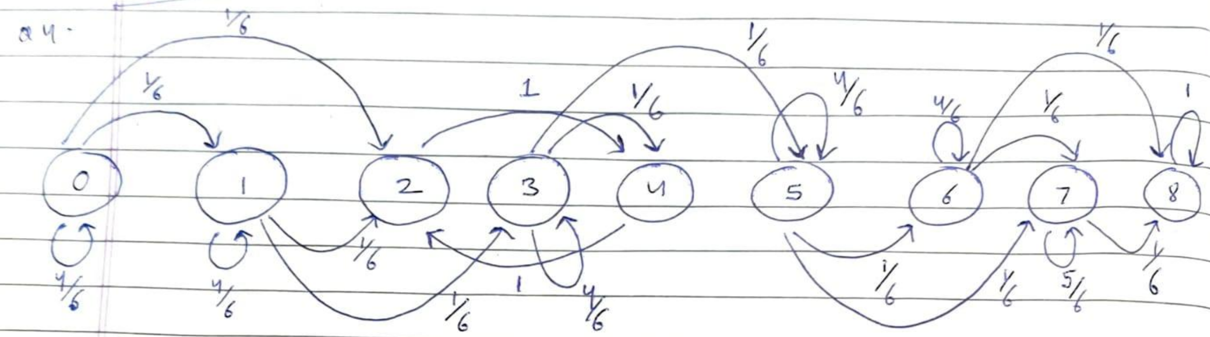
Assumption:

When the pawn is in state 7 and if die rolls a 1 then only transition occurs; otherwise for any other outcome of the die the pawn stays in the same position i.e in state 7.

Part A:

Markov chain:

a 4 -



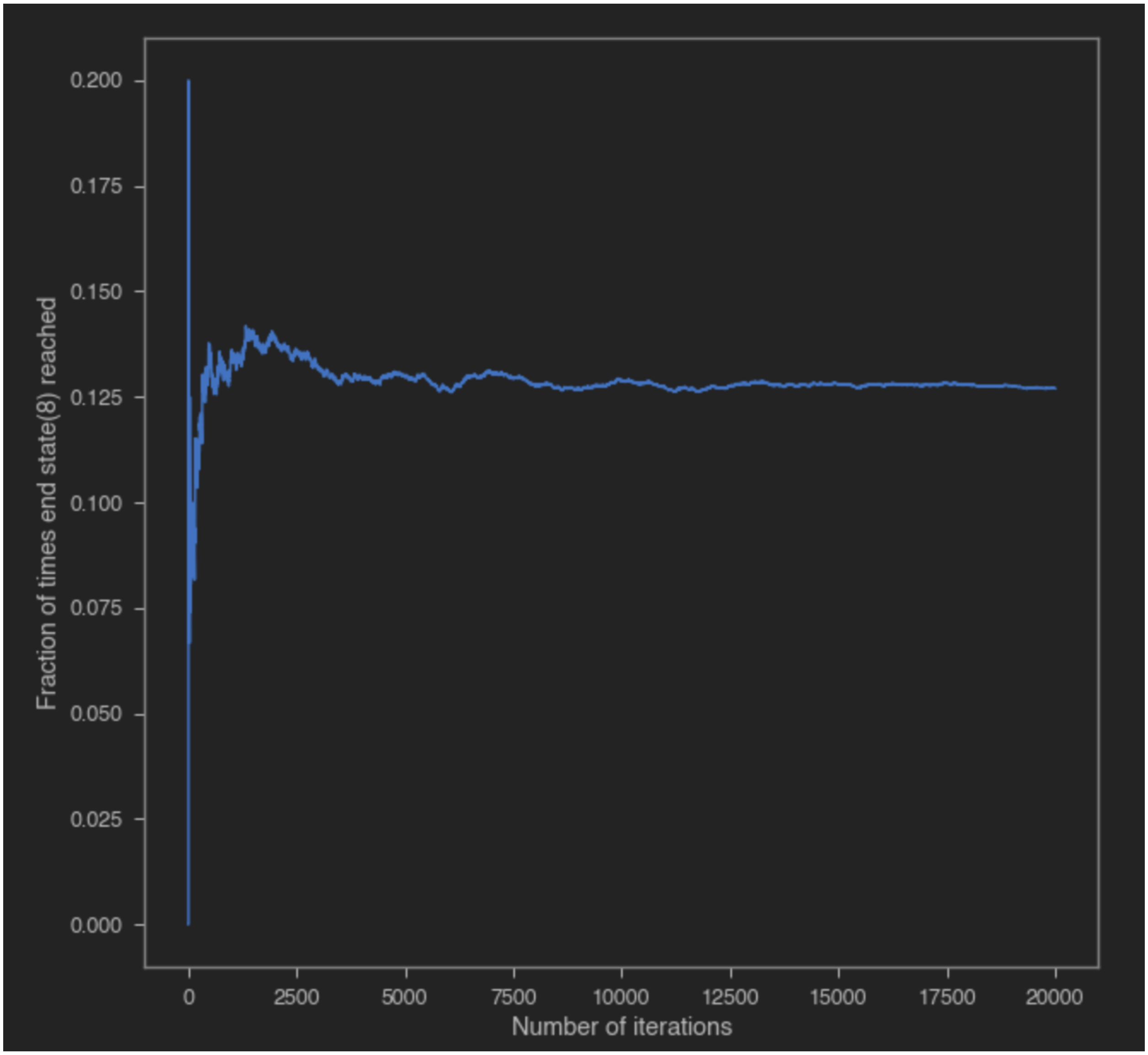
Transition Probability -

$$P = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/6 & 1/6 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4/6 & 1/6 & 1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

State space = $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Part B:

Simulation Results-



Fraction of times end state reached from state 0 vs number of iterations

We can see from the simulation that with a large number of iterations the fraction of times we reach the end state from the state 0 approaches to around **0.125**.

The result obtained using simulations converges to the actual value due to the law **of large numbers.**

The law states that if if there exists n i.i.d random variables $X_1, X_2, \cdots, X_n$ then the average of random variables as the number of variables increases to infinity the average converges towards the actual expected value of the random variable.

Why does the Law of Large numbers work in our case?

We have considered a Bernoulli random variable named **cnt_end** in our code. It outputs 1 if we reach the end state starting from state 0 otherwise we get output as 0.

Now we have considered num_iterations = 20000 so in a way we play the game 20000 times.

With each iteration we get the value of the random variable and we keep on calculating the average for the same. From the simulations we can see that average converges to the expectation of the Bernoulli random variable i.e E[X] = p as the number of iterations increases.

Where p is the probability that we reach the end state i.e 8 if it is given that we start from state 0 because expectation of the Bernoulli random variable is p.So, we prove the law of large numbers and also get our result from the simulation.

Analytically-

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Analytically -

Let a_i → Probability that we ever reach state 8 given we start from state i.

$$a_i = \sum_k a_k P_{ik}$$

a_k → probability that we ever reach state 8 given we start from state k.

P_{ik} → $P(S_t = j \mid S_{t-1} = i)$
(Transition probability from $i \to j$

$a_2 = a_4 = 0$ (as we can never leave 2 or 4 once entered)

$a_8 = 1$ (absorbing state)

Sol.

$$a_0 = \frac{4}{6} a_0 + \frac{a_1}{6} + \frac{a_2}{6}$$

$$a_1 = \frac{4}{6} a_1 + \frac{a_2}{6} + \frac{a_3}{6}$$

$$a_4 = a_2 = 0$$

$$a_3 = \frac{4}{6} a_3 + \frac{1}{6} a_4 + \frac{1}{6} a_5$$

$$a_5 = \frac{4}{6} a_5 + \frac{1}{6} a_7 + \frac{1}{6} a_6$$

$$a_6 = \frac{4}{6} a_6 + \frac{1}{6} a_7 + \frac{1}{6} a_8$$

$$a_7 = \frac{5}{6} a_7 + a_8$$

$$a_8 = 1$$

Back substituting -

$$a_7 = a_8 = 1$$

$$a_5 = a_6 = 1$$

$$a_3 = \frac{1}{2}$$

$$a_1 = \frac{1}{4}$$

$$\boxed{a_0 = \frac{1}{8}}$$

we wanted $\boxed{a_0 = \frac{1}{8} = 0.125}$

Probability that we ever reach state 8 given we start from 0.