



MSc in Data Science
Project Report
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“Logical Rule Integration into Neural Networks”

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Declaration

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Abstract

This dissertation introduces an innovative approach to augment neural networks by integrating logical rules, aiming to enhance their decision-making and interpretability. Leveraging the zoo dataset, the study developed two novel models: the Partial Logic Neural Network and the Full Logic Network. These models demonstrate the effective integration of empirical data processing with logical reasoning, offering new insights into neural network optimization. The Partial Logic Neural Network exemplifies the balance between empirical data analysis and logical reasoning, achieving an integration of these elements with MSE and L1 loss functions. In contrast, the Full Logic Network is designed to adhere strictly to pre-defined logical rules, emphasizing consistency in the network's inferences. Its design prioritizes logical consistency, making it particularly suitable for tasks requiring clear rule-based decision-making. This model employs ReLU activation functions and a unique logic loss function during training, ensuring fidelity to the encoded logical rules. The research represents a significant advancement in the field of neural-symbolic AI, showcasing the potential applications of these enhanced models in sectors like healthcare and finance, where logical decision-making is vital. The findings suggest new research avenues in diverse datasets and neural architectures, proposing significant implications for future AI advancements. By integrating logical reasoning into neural networks, the study opens up possibilities for developing AI systems that are not only efficient and adaptable but also transparent and interpretable. The implications of this research are far-reaching, promising to elevate the role and impact of AI in various complex decision-making scenarios.

Keywords : Logical Integration, Neural Network Optimization, Decision-Making AI, Data Interpretability, Neural-Symbolic Integration

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1. Introduction

In our dissertation, we embark on an exploratory journey into the intricacies of neural networks within machine learning. These networks have been instrumental in driving advancements in various applications like image processing, speech recognition, and financial forecasting. Their unparalleled ability to parse through and make sense of voluminous datasets has positioned them at the forefront of technological innovation.

Despite their impressive capabilities, neural networks grapple with several challenges. A primary limitation lies in their heavy dependence on extensive, well-labeled datasets. This reliance poses substantial challenges, mainly when data is scarce, expensive to obtain, or difficult to annotate accurately. Such issues are prevalent in sensitive areas where data privacy and security are crucial. This dependence on large volumes of data can hinder the versatility and deployment of neural networks in diverse settings.

Furthermore, neural networks often struggle with tasks requiring logical reasoning and abstract decision-making. While these networks excel in identifying patterns and making predictions based on historical data, their proficiency in applying abstract concepts and rules is markedly less developed. This limitation is acutely evident in situations that demand an understanding of causality or decision-making based on complex rule sets. Hence, neural networks often falter in scenarios where a deeper comprehension of underlying principles or logical structures is essential.

Furthermore, neural networks often need help with tasks requiring logical reasoning and abstract decision-making. While these networks excel in identifying patterns and making predictions based on historical data, their proficiency in applying abstract concepts and rules needs to be more developed. This limitation is acutely evident in situations that demand understanding causality or decision-making based on complex rule sets. Hence, neural networks often falter in scenarios where a more profound comprehension of underlying principles or logical structures is essential.

In addition, a significant issue with neural networks, especially those based on deep learning models, is their lack of interpretability. These 'black box' models, though efficient in producing results, often provide little clarity on how specific conclusions or decisions are made. This opacity can be a major barrier in domains where understanding the decision-making process is as crucial as the outcomes, such as in medical diagnostics, legal decision-making, and financial risk assessment.

In addition, a significant issue with neural networks, especially those based on deep learning models, is their lack of interpretability. Though efficient in producing results, these 'black box' models often provide little clarity on how specific conclusions or decisions are made. This opacity can be a significant barrier in domains where understanding the decision-making process is as crucial as the outcomes, such as medical diagnostics, legal decision-making, and financial risk assessment.

To surmount these challenges, our dissertation puts forth an innovative approach: integrating prior knowledge and logical rules into feedforward neural networks. This strategy aims to enhance the robustness and generalization capabilities of neural networks, making them more adaptable and efficient in situations where data availability is limited, or the application of logical reasoning is paramount.

To surmount these challenges, our dissertation proposes an innovative approach: integrating prior knowledge and logical rules into feedforward neural networks. This strategy aims to enhance neural networks' robustness and generalization capabilities, making them more adaptable and efficient in situations where data availability is limited or applying logical reasoning is paramount.

We hypothesize that combining prior knowledge and logical rules into neural networks could offer several advantages. Firstly, it could significantly reduce the network's reliance on large datasets. By incorporating existing knowledge and logical structures, the network could utilize this information to make informed decisions, even with limited data. This method could be especially beneficial in areas where data collection is arduous or privacy concerns limit data availability.

Using prior knowledge and logical rules in neural networks could offer several advantages. Firstly, it could significantly reduce the network's reliance on large datasets. The network could utilize this information to make informed decisions by incorporating existing knowledge and logical structures, even with limited data. This method could be especially beneficial when data collection is arduous, or privacy concerns limit data availability.

Secondly, the incorporation of logical reasoning into neural networks has the potential to bridge gaps where traditional models fall short. By embedding structured rules and reasoning processes into the network, it could be enabled to perform tasks that require logical operations, abstract thinking, and rule-based decision-making. This enhancement

could considerably expand the scope of neural networks, allowing them to tackle problems involving intricate decision-making processes and causal reasoning more effectively.

Moreover, this approach could significantly improve the interpretability and transparency of neural networks. By integrating logical rules that are comprehensible and explainable, the decision-making process of these networks could become more transparent. This improvement is vital in applications where understanding the rationale behind decisions is essential for trust, compliance, and ethical considerations.

To empirically validate our hypothesis and demonstrate the practicality of our proposed approach, we utilize the zoo dataset from the [Zoo - UCI Machine Learning Repository](#) as a case study. With its diverse and complex nature, this dataset mirrors the challenges often encountered in real-world data. It provides an ideal scenario to test the effectiveness of integrating logical rules and prior knowledge into neural network frameworks. While adept at pattern recognition, this dataset will illustrate how traditional neural networks might struggle with nuanced categorization and prediction tasks that require a blend of data processing and logical reasoning.

As we advance through our research, we anticipate facing challenges and learning opportunities. These will encompass the technical aspects of integrating logical reasoning into neural networks and exploring the ethical and practical implications of such integration. Our aim is to document these experiences thoroughly, contributing valuable insights to the broader discourse on the future of AI and neural networks.

We anticipate facing challenges and learning opportunities as we advance through our research. These will encompass the technical aspects of integrating logical reasoning into neural networks and exploring such integration's ethical and practical implications. We aim to document these experiences thoroughly, contributing valuable insights to the broader discourse on the future of AI and neural networks.

In conclusion, our dissertation represents a significant stride towards enhancing the field of neural networks. By bridging the gap between traditional data-driven models and the necessity for logical reasoning, we endeavor to develop neural networks that are not only more efficient and adaptable but also transparent and trustworthy. This research is expected to unlock new avenues for the application of AI, making neural networks more relevant and applicable in a variety of complex and nuanced domains.

1.1. Problem Background

Our research examines the limitations of neural networks within machine learning, particularly their reliance on extensive, labeled datasets and their struggle with tasks requiring logical reasoning and complex decision-making. At the same time, neural networks excel in areas like image recognition and predictive analytics, their dependency on large datasets and difficulties in applying rule-based logic present significant challenges, especially in fields where nuanced decisions are critical.

Our dissertation addresses these challenges by proposing a novel approach: integrating logical rules and prior knowledge into feedforward neural network structures. This integration reduces reliance on vast datasets and enhances the networks' ability to perform complex reasoning. This approach will lead to efficient training, improved decision-making capabilities, and broader applicability of neural networks in various sectors. Our goal is to expand the functional scope of neural networks, making them more adaptable and effective for diverse real-world applications.

1.2. Objectives

1. **Select and Adapt a Scheme for Implementing Logic Rules in Neural Network Weights** : The primary objective of our study is to identify and adapt an appropriate scheme that enables the integration of logic rules into the weights of a neural network. This process includes a thorough review and assessment of existing methodologies that facilitate embedding of logical structures within neural networks. The criterion for success in this objective is the identification of a suitable scheme and its successful adaptation to the neural network for the selected task and dataset.

2. **Implement the Scheme in PyTorch** : Once the scheme is selected, the next objective is to implement it using PyTorch, a popular open-source machine learning library. The implementation must be functional, efficient, and compatible with the chosen neural network model. This objective's criterion is the development of functional PyTorch code that effectively integrates the chosen scheme into the neural network.

3. Select a Suitable Dataset Associated with Logic Rules : The third objective is to select an appropriate dataset inherently associated with logical rules. This dataset should be relevant to the task at hand and allow for the practical application and evaluation of the integrated logic rules within the neural network. The criterion for this objective is the identification and selection of an appropriate dataset along with its associated logic rules.

4. Apply the Scheme and Evaluate the Results : The final objective involves the application of the selected scheme to the chosen dataset using the developed PyTorch implementation. This step includes conducting experiments to assess the effectiveness of the logic rule integration in the neural network. The criterion for this objective is the comprehensive documentation and evaluation of the experiments conducted, focusing on the impact of logic rules on the neural network's performance.

1.3. Research Questions

Our study's primary goal is to "Can we effectively inject logical rules into neural network learning?"

1.4 Beneficiaries

The outputs from this project are poised to benefit a wide array of stakeholders across various sectors, underlining the far-reaching impact of our work.

1. Academic Researchers: Scholars and researchers in machine learning and AI will find our dissertation and accompanying resources immensely valuable for furthering their research. The in-depth exploration of logical rule integration into neural networks is likely to spur new investigative and innovative directions. The availability of open-source code and datasets enables other researchers to build upon our findings, enriching the collective knowledge base in the field.

2. Industry Practitioners: Professionals in data-driven industries stand to gain significantly from the advanced neural network models developed in our project. In sectors like finance, healthcare, and technology, where the accuracy and interpretability of AI models are paramount, our enhanced models offer notable advantages. Fusing logical rules into neural networks is set to bolster these models' performance, reliability, and transparency, leading to more informed and effective decision-making.

3. Educators and Students: Our project's educational materials and interactive tools are invaluable for educators and students alike. These resources demystify complex machine-learning concepts, making them more accessible and comprehensible. They offer practical experience, enhancing students' skill sets and preparing them for future challenges in the field.

4. General Public: The broader implications of our research extend to the general public, especially as AI increasingly permeates various facets of everyday life. The advancements in neural network technology and the enhanced transparency and reliability of these models are expected to foster greater trust and acceptance of AI in societal applications.

In conclusion, our project's outputs represent a significant contribution to the realms of machine learning and AI. By offering a comprehensive array of resources, from in-depth research papers to practical tools and educational materials, we aim to propel the understanding and application of logically enhanced neural networks. The broad spectrum of beneficiaries, ranging from academic circles to industry professionals, educators, policymakers, and the general public, underscores the extensive impact and relevance of our research endeavors.

1.5 Intended Method

In our dissertation, we explore techniques to enhance interpretability in neural networks by assimilating symbolic knowledge representations. Specifically, we integrate structured logical rules from the zoo animal traits dataset into model parameters and training processes using weight initialization and logic loss functions. Comparative assessments measure resulting improvements in transparency, generalization capability, and efficiency against standard neural counterparts. The research aims to advance model accountability through explicitly embedding domain expertise into data-driven learning for trustworthy AI.

1.Data Selection

- We have selected the zoo dataset for its structured categorization of animals, making it ideal for applying logical rules within a neural network framework.

2.Encoding Logic in Neural Networks

- Our approach involves converting the zoo dataset's categorical attributes into a format that feedforward neural networks can process, ensuring logical reasoning is encoded into the network's decision-making process.

3.Model Selection

- We chose a feedforward neural network due to its simplicity, effectiveness in pattern recognition, and suitability for integrating logical rules. This model aligns with our objective to enhance decision-making processes in AI systems using structured logical reasoning.

4.Training and Evaluation

- The NN will be trained on the preprocessed zoo dataset, with an emphasis on accurately applying the encoded logical rules. The evaluation will focus on the network's ability to correctly classify animals based on these rules, assessing its accuracy and effectiveness in logical reasoning.

1.6 Project Report Outline

Here is an outline for the project report chapters adapted to our dissertation context:

Chapter 1: Provides background on neural networks, defines limitations addressed, and establishes research questions, objectives and beneficiaries.

Chapter 2: Surveys prior work on integrating logic in neural networks, relevant techniques like fuzzy rules and probabilistic logic, and benchmarks.

Chapter 3: Details the stepwise methodology including zoo dataset selection, logical rule creation, propositional logic translation, network configuration with weight initialization and loss functions, training procedures and evaluation metrics.

Chapter 4: Presents results of model transparency analyses, reasoning evaluation, accuracy and efficiency comparisons to standard neural networks, along with performance benchmarking.

Chapter 5: Discusses interpretation of results, reflecting on techniques that worked well versus limitations encountered, and implications for future research.

Chapter 6: Summarizes contributions made in addressing objectives, proposes potential pathways to further advance logical rule integration, and concludes with final remarks.

2. Background and Context

In the field of ML, neural networks have established themselves as pivotal pattern recognition algorithms. This dissertation focuses on their application in analyzing the intricate patterns present in zoo animal data. Specifically, the study employs feedforward neural networks (FNNs) due to their structured processing of data, which aligns well with the project's requirements.

2.1. Feed Forward Neural Networks

In our dissertation, we concentrate on Feed Forward Neural Networks (FNNs) and their significance in AI, guided by insights from several vital papers. The study presented by Geiger et al. (2022) is particularly enlightening, offering an in-depth look at how FNNs with a single hidden layer can effectively handle binary classification tasks. This paper's investigation into the impact of varying hyperparameters and input dimensions provides critical knowledge on the versatility and performance of FNNs in diverse situations.

Similarly, Rajasegaram D (2023) delves into the structure and function of FNNs. It explains how these networks process information in a straightforward, one-way path from input to output nodes, which is ideal for recognizing patterns and making classifications in machine learning. The project also notes the simplicity and prevalent use of FNNs, indicating their potential to offer valuable insights, particularly in comparing outcomes with and without logic integration.

Williams (1995), further enriches our understanding by discussing using FNNs in regression and classification networks. It explores the application of certain principles to FNNs, enhancing our appreciation of their applicability in various neural network models.

These papers collectively inform our dissertation's focus on employing FNNs to advance AI systems. By utilizing the structural simplicity and data processing capabilities of FNNs, our project aims to improve the performance of AI systems in pattern recognition and decision-making tasks. The unique properties of FNNs, such as their clear data flow and adaptability, make them a pivotal subject in our research towards evolving neural network applications.

2.1.1. Architecture

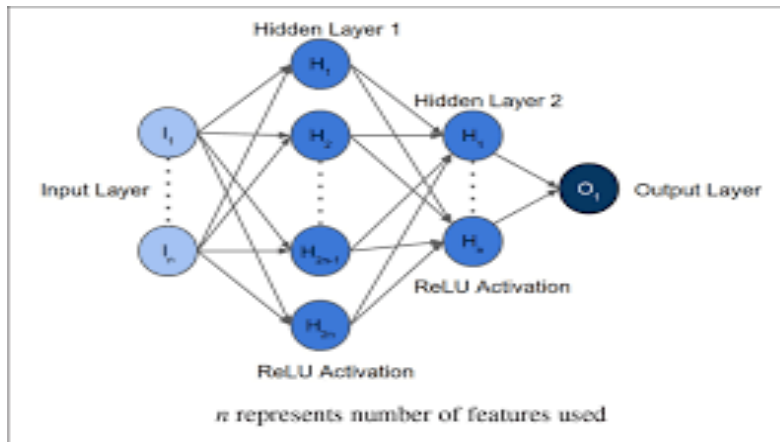


Fig 1 Simple Feedforward Neural Network with ReLU Activation

In exploring neural network architectures, the feedforward neural network (FNN) serves as a foundational model for understanding complex data processing. Figure 1 in the report illustrates a typical FNN structure, which is composed of several distinct layers: an input layer, multiple hidden layers, and an output layer.

At the commencement of the network, the input layer receives the initial data. This data is represented by 'n', indicating the number of features each input possesses. From here, the data is propagated forward, first through Hidden Layer 1, where initial transformations are applied via weighted connections. The process continues to Hidden Layer 2, further refining the data transformations. Each hidden layer utilizes the Rectified Linear Unit (ReLU) activation function, which introduces non-linearity to the model, enabling it to capture and learn more complex patterns than linear models can.

The final step in the data's journey is the output layer, where the network concludes its processing and produces the results. The unidirectional flow from input to output, passing through layers without cycles or feedback loops, is a defining characteristic of FNNs. This architecture allows for straightforward modification and scaling, depending on the complexity of the task at hand.

Incorporating this FNN model into the critical context of our study emphasizes the significance of network architecture in their accurate classification and prediction of data patterns, a central theme to our research objectives. As we delve into the application of FNNs for analyzing zoo animal data, understanding this architecture will be crucial for interpreting how the network processes information and arrives at its classifications.

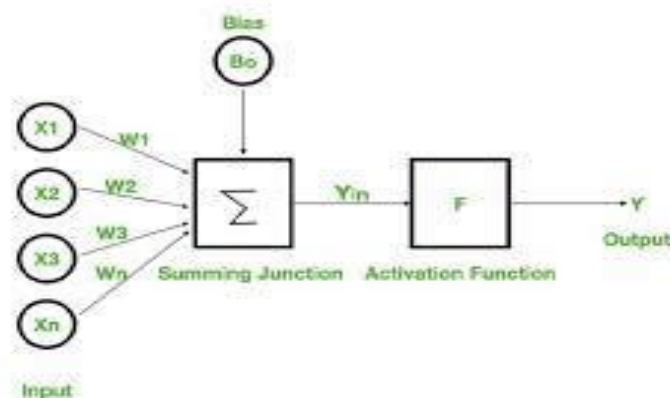


Fig 2. Basics of Neuronal Function in Neural Networks

2.1.2. ReLU Function

The research by Riegel et al.,(2020),delves into the nuanced functionality of the ReLU activation function. The study elucidates how ReLU, expressed mathematically as $\text{ReLU}(x)=\max(0,x)$, functions effectively as a concept detector within neural networks. The authors highlight ReLU's ability to maintain linear behavior for positive inputs, thereby signifying the presence of features, while zeroing out negative inputs, uniformly indicating their absence. This dual functionality, as the authors note, is instrumental in reducing noise and preventing the simultaneous activation of correlated neurons. Additionally, the paper discusses the computational benefits of ReLU, particularly its gradient sparsity, which optimizes computational efficiency by necessitating gradient calculations for only a subset of neurons.

The same paper by Riegel et al.(2020),further examines the regularization capacity of ReLU in neural network training. The authors describe ReLU's ability to saturate to an output and gradient of zero as a form of information and gradient dropout, a mechanism vital for mitigating overfitting in neural network models. This aspect of ReLU's functionality is highlighted as crucial for maintaining model generalizability and preventing overtraining on specific data patterns.

Paper authored by Geiger et al.(2022), ReLU's application is showcased in different neural network architectures. The paper describes the use of ReLU in a single-layer network computing equality and in a recurrent LSTM network for sequence generation. These implementations demonstrate ReLU's adaptability and effectiveness in various network designs, a critical aspect for the dissertation's focus on integrating logical rules into neural networks. The authors' work underscores ReLU's capacity to facilitate complex computational tasks while maintaining activation function simplicity.

In conclusion, the insights provided by Riegel et al.(2020) and Geiger et al.(2022) in their studies offer a comprehensive understanding of ReLU's multifaceted role in neural networks. These findings underline the significance of ReLU in enhancing computational efficiency, noise reduction, regularization, and adaptability in various network architectures, making it a pivotal component in the field of neural network development and the integration of logical rules.

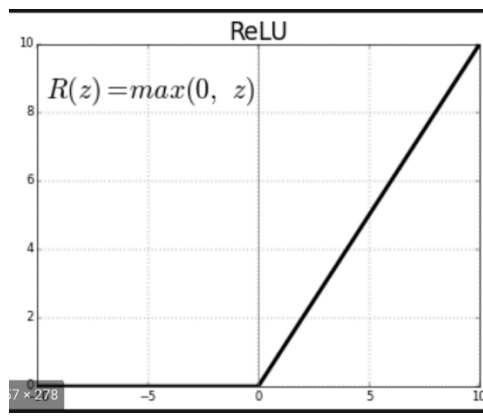


Fig 3. : ReLU Activation Function

Fig 3 depicts the graph of the Rectified Linear Unit activation function, which is mathematically represented as $\text{ReLU}(z) = \max(0, z)$. The graph shows that for inputs less than zero, the output is zero, and for inputs greater than zero, the output is equal to the input, demonstrating the piecewise linear nature of ReLU.

2.1.3. Learning Algorithm

In the endeavor to integrate logical rules into neural networks, learning algorithms serve as the pivotal mechanism that drives the adjustment and tuning of the networks' parameters. by Shihabudheen and Pillai (2018) , explores a spectrum of algorithms that underpin this process. The backpropagation algorithm, recursive least squares, Levenberg–Marquardt (L-M) algorithms, and gradient descent methods are fundamental to the iterative learning process. These algorithms not only support error minimization but also enable the hierarchical and recursive structuring of learning, which is paramount when embedding logical rules into neural network architectures. Paper by Rajasegaram D (2023), provides a practical blueprint for the application of these algorithms in a neural network's learning cycle. It involves the forward propagation of data, backpropagation of errors, and hyperparameter tuning. Such a structured approach is critical for a dissertation focused on logical rule integration, as it outlines a clear methodology for adapting the neural network to accommodate and optimize rule-based decision-making processes.

Moreover, the k-means clustering and Extreme Learning Machine (ELM) algorithm, as discussed in Shihabudheen & Pillai (2018), facilitate an efficient data structuring method that is particularly beneficial when rules are integrated into neural networks. By clustering the data, we can enhance the ELM's capacity to identify the 'if' components of rules, streamlining the process of integrating these rules into the neural network's learning framework. The ELM's rapid training capabilities are essential for neural networks that require fast adaptation to new logical rules and the swift recalibration of their decision boundaries.

2.2. Prior Knowledge

2.2.1. What is Prior Knowledge ?

In Rajasegaram D (2023) study, "What is Prior Knowledge" is extensively discussed, providing a foundational understanding for enhancing the efficiency and effectiveness of neural network models. Prior knowledge refers to any pre-existing information or insights that are directly related to the problem domain being addressed by the neural network. This can encompass a range of information, from specific details about the data itself to more abstract concepts or output variables that the neural network is intended to predict or classify.

The role of prior knowledge in neural network training is multifaceted. Firstly, it serves as a guiding framework that informs the initial structuring and tuning of the network. By incorporating relevant information from the onset, neural networks can bypass certain initial learning stages, thereby accelerating the training process. This is particularly beneficial in complex domains where data can be vast and varied, as the initial guidance provided by prior knowledge helps the network focus on relevant patterns and associations from the beginning.

Furthermore, the integration of prior knowledge aids in overcoming some of the intrinsic limitations of neural networks, such as their tendency to be 'black box' models. By embedding known rules, patterns, or principles within the network's architecture, the model's decisions and outputs become more interpretable and transparent. This is crucial in applications where understanding the rationale behind the network's decisions is as important as the accuracy of these decisions, such as in healthcare diagnostics or financial analysis.

Prior knowledge can take multiple forms depending on the application. In a sentiment analysis model, for instance, it could include linguistic cues known to express sentiments, or in a medical diagnosis tool, it might involve symptoms and signs commonly associated with certain diseases. This knowledge not only streamlines the learning process but also enhances the network's ability to make accurate and reliable predictions.

In summary, prior knowledge acts as a critical asset in neural network development. Its incorporation leads to more efficient training, improved interpretability, and enhanced reliability of the network's outputs. As neural networks continue to evolve and find applications in increasingly complex domains, the strategic integration of prior knowledge will become an indispensable aspect of their development and deployment.

2.2.2. Benefits of incorporating Prior Knowledge

The incorporation of prior knowledge into neural networks, as explored by Rajasegaram D (2023), presents a transformative approach in the realm of artificial intelligence and neural network design. This concept is vital for the advancement of neural networks beyond traditional data-driven paradigms, equipping them with the capability to process information in a more human-like, logical, and systematic manner.

1. **Structured Foundation for Learning:** The project underscores that prior knowledge acts as a foundational framework for neural networks, instilling a more structured and systematic approach to problem-solving. This is akin to the way the human brain processes high-level information in conjunction with low-level sensory inputs. By embedding established principles and domain-specific knowledge, neural networks can benefit from a guided learning trajectory, which is especially beneficial in applications that require adherence to logical structures or domain-specific rules.
2. **Enhanced Generalization and Interpretability:** One of the critical benefits outlined in the project is the enhancement of the neural network's generalization capabilities. This is achieved by integrating prior knowledge, which can be particularly effective even with sparse and noisy datasets. Additionally, this integration enhances the interpretability of neural networks. By incorporating rules or constraints, neural networks can offer clearer insights into their decision-making processes, improving their transparency and making them less of a 'black box'. This aspect is crucial for applications requiring clarity and justification in AI decision-making, such as in medical diagnostics, financial analysis, and legal assessments.
3. **Balancing Symbolic and Data-Driven Models:** The project also discusses the reciprocal benefits of combining symbolic (rule-based) models with data-driven neural network models. While symbolic models excel in transparency and require minimal data, they often lack adaptability and struggle with noisy data due to their reliance on predefined rules. On the other hand, data-driven models excel in handling complex data patterns but can lack transparency. By merging these two approaches, neural networks can be developed to capture complex relationships present in the data while retaining the clarity and precision offered by rule-based systems. This hybrid approach opens up new avenues in AI, allowing for the creation of models that are both adaptable and interpretable.
4. **Efficiency and Prediction Accuracy:** Another significant benefit noted in the project is the efficiency and accuracy of predictions made by neural networks imbued with prior knowledge. The project demonstrates this through the use of synthetic datasets, where neural networks, despite limited training samples, were able to achieve strong generalization abilities. This tells that the integration of prior knowledge can lead to more efficient models, capable of making accurate predictions with less data. This is particularly advantageous in scenarios where data availability is limited or in situations requiring quick model training without compromising accuracy.

2.2.2.1 Introduction to Propositional Logic

In our project, we're exploring how to enhance computer brain models, known as neural networks, by integrating them with logical reasoning. This concept is at the heart of several key studies we've examined. For instance, Badreddine et al. (2022) delves into the use of logic within neural networks. It specifically highlights the application in advanced systems like DeepProbLog, showing how logic can make these AI systems smarter and more efficient in processing information. This is closely aligned with our project's goal of enhancing the intelligence of neural networks.

Moreover, Reimann et al. (2022) introduces an intriguing concept of neural logic rule layers. These are special components within neural networks that are designed to process and apply logical rules. This concept is exciting for our project because it suggests a way to structure neural networks so that they can more effectively use logic in their operations, thereby enhancing their problem-solving capabilities.

Our project seeks to harness these insights, aiming to bridge the gap between the rigid, rule-based processing of traditional computer systems and the more fluid, learning-oriented approach of modern neural networks. By embedding logical reasoning into neural networks, we aim to create AI systems that are not only more intelligent but also more adaptable and capable of managing a wider range of tasks. This integration of logic and learning is a crucial part of our research. It shows a major step forward in the development of AI, demonstrating the potential of logic-enhanced neural networks to transform the field by enabling more sophisticated, efficient, and human-like decision-making processes.

2.2.2.2 Logical Connectives

In our dissertation, we focus on integrating logical connectives, specifically the AND and NOT logic gates, into neural networks, drawing from a range of academic papers. This integration is pivotal for enhancing AI systems with sophisticated decision-making and reasoning capabilities, aligning closely with our project's aims.

Study authored by Riegel et al. (2020),enriches this understanding by discussing the requirements for activation functions in Logical Neural Networks (LNNs), particularly for the AND and NOT connectives. It emphasizes the need for these functions to adhere to logical principles, ensuring the integrity and reliability of the neural network's reasoning processes. This insight directly contributes to our project, as it supports our aim to integrate structured logical reasoning into neural network frameworks, thus enhancing the decision-making capabilities of AI systems.

Rajasegaram D (2023), provides a practical perspective on the application of propositional logic, particularly highlighting the roles of AND and NOT logical connectives. This project underscores the importance of these connectives in constructing logical statements and their applications in AI. The focus on AND and NOT aligns with our project's scope, as we aim to incorporate these specific logical operations into AI and machine learning models.

To illustrate with our zoo dataset, consider using the AND logic gate for classification. For example, we might classify an animal as a mammal if it has hair AND gives birth to live young. Similarly, the NOT logic gate could be used to identify non-avian species by determining if an animal does NOT have feathers. These examples demonstrate the practical application of AND and NOT connectives in creating classification rules, enabling AI systems to derive accurate and logical inferences from the data.

A (Input)	B (Input)	A AND B	NOT A
0	0	0	1
0	1	0	1
1	0	0	0
1	1	1	0

Table 1: Basic Logic Gate Operations

In this table 1:

- For the AND gate (A AND B), the output is true (1) only when both A and B are true.
- For the NOT gate (NOT A), the output is the inverse of A; when A is true (1), NOT A is false (0), and vice versa.

2.3. Related work

This section describes our method for reviewing related work in neural networks and logical rule integration. It details the criteria for literature selection, focusing on studies that inform neural-symbolic AI and the integration of logical reasoning in data-driven models. This section identifies crucial developments and research gaps our study addresses.

2.3.1. Method

Our methodological approach in this dissertation significantly draws upon the foundational research conducted by Weyde & Kopparti (2021). Their exploration into the limitations of neural networks, particularly in the context of generalization beyond training data, has informed the core strategies of our project. While they have made notable strides with the Relation Based Pattern (RBP) approach, our work seeks to augment this by transitioning from manual integrations to an automated system that embeds logical rules into the weight and bias matrices of neural networks.

In advancing the RBP method, we have innovated a system that automates the translation of animal attributes from the zoo dataset into propositional logic rules. This step is pivotal, as it converts raw data into structured logic that neural networks can decipher and learn from. For example, the logical statement "If an animal has feathers and does not produce milk, then it is classified as a bird" is transformed into a format that the neural network can utilize for classification tasks.

Our process elevates the manual methodologies previously utilized, streamlining the incorporation of logical rules into neural networks, and ensuring a more consistent and scalable rule application. The resultant neural network models are anticipated to excel in their generalization capabilities, equipped to integrate complex logical patterns derived from structured datasets like the zoo database.

By implementing this method with the zoo dataset, we demonstrate the practical application and efficiency of our automated approach. The code developed for this project clearly illustrates the utilization of logical operators such as AND and NOT in constructing logical rules, which the neural network adopts, enhancing its decision-making processes.

Thus, the work of Rajasegaram D (2023), acts as both the inspiration and the springboard for our advanced research. Our dissertation extends the principles of the RBP approach by automating the encoding of logic into neural networks, a leap forward from the manual, labor-intensive processes of the past. We aim not only to mirror the success of the RBP method but to surpass it, enabling neural networks to dynamically and robustly engage with logical constructs, as evidenced by our handling of the zoo dataset.

2.3.2. Neuro-Symbolic

In our dissertation, we explore the burgeoning field of Neuro-Symbolic AI, focusing on its potential to revolutionize the integration of logical rules into neural network architectures. This exploration is deeply informed by key findings from notable papers in the domain.

Additionally, our research is enriched by the contributions by (Garcez et al.,2015), which extols the virtues of neural-symbolic computation from both methodological and computational standpoints. The paper highlights the representational generality of this approach, capable of encompassing a broad spectrum of symbolic systems. The learning robustness inherent in neural-symbolic computation opens up exciting possibilities for knowledge representation, be it purely symbolic or a hybrid encompassing probabilistic or numerical representations. These insights are pivotal to our work, suggesting new methodologies for developing neural network models that are not only efficient in data processing but also capable of sophisticated symbolic reasoning.

Our dissertation, thus, seeks to build upon these foundational studies, aiming to harness the power of neuro symbolic AI for enhancing neural networks' decision-making and reasoning capabilities. By integrating the principles of neural-symbolic computation, we aspire to create neural network models that are not only advanced in terms of learning efficiency but also embody the sophistication required for complex, symbolic reasoning tasks. The ultimate goal of our research is to push the boundaries of current neural network technologies, making them more versatile, interpretable, and adept at handling a confluence of learned data and logical constructs.

2.3.3. Regularization with Logic

In our dissertation, we explore the integration of logical rules into neural networks, particularly focusing on the concept of regularization with logic. This exploration draws on critical insights from seminal research papers in the field.

Our approach is significantly informed by the methodologies discussed in Rajasegaram D (2023) study, which delves into the integration of logic into neural networks through regularization. The project highlights the use of posterior regularization as a structural method to merge prior knowledge with neural network learning processes. This technique involves the inclusion of features corresponding to prior knowledge sources within a log-linear model, guiding the learning trajectory of the model. This concept resonates with our dissertation's aim of enhancing neural networks with interpretative and decision-making abilities, offering a model for embedding logical rules into the neural network's learning framework.

Furthermore, we examine the approaches proposed by Reimann et al. (2022), which presents a unique perspective on semantic-based regularization. This paper contrasts its methodology with Markov Logic Networks, advocating for an approach that integrates rules directly into the hypothesis space. By biasing neural networks toward representing logical rules, this method facilitates an end-to-end learnable integration of these rules into neural network structures. This novel approach of learning rule types, the variables involved, and their relationships is particularly relevant to our research. It provides a practical and direct method for embedding logical rules into neural networks, aligning with our objective of automating the conversion of logical rules into neural network architectures.

These papers collectively provide a robust theoretical and methodological foundation for our dissertation. They guide our efforts in developing neural network models that are not only efficient in learning from data but also capable of sophisticated decision-making based on the integration of logical rules. This enhanced functionality is key to advancing neural networks' capabilities, especially in complex applications requiring nuanced reasoning and decision-making.

3. Methods

In this analysis of our methods, we study the comprehensive procedures and sophisticated techniques we employed to create a model which can predict using neural networks and propositional logic. Our project was driven by the ambition to accurately classify a diverse range of animals based on a series of distinct attributes. This ambitious endeavor required accurate planning and execution across various stages, including data collection, preprocessing, detailed feature engineering, the design and implementation of neural network architectures, rigorous training, and thorough model evaluation. Each step was carefully crafted and executed, with the underlying aim of aligning with our central objective of creating a robust and precise classification system, all while adhering to the latest advancements and methodologies documented in contemporary machine learning and data science literature.

3.1. Data Selection and Rationale

This section focuses on why we chose the zoo dataset for integrating logical rules into neural networks. The dataset's variety is ideal for testing our models, the Partial Logic Neural Network and Full Logic Network. This section explains the dataset's relevance to our research objectives.

3.1.1. Utilizing the Zoo Dataset for Logical Rule Integration in Neural Networks:

The zoo dataset obtained from Kaggle and UCI is a rich and structured collection of animal attributes, classified into various types. Each row in the dataset indicates a distinct animal, characterized by a set of binary attributes that describe its physical and behavioral features. These attributes include, but are not limited to, hair, feathers, eggs, milk, airborne, aquatic, predator, toothed, backbone, breathes, venomous, fins, legs, tail, domestic, and catsize. Additionally, each animal is categorized into a class type, which helps in understanding their broader biological categorization.

This dataset is particularly suited for our research, where we aim to test the integration of logical rules into neural network architectures. The binary nature of the attributes simplifies the process of applying logical rules, as each feature can be directly used as a condition or a decision node within a logical framework. For example, the presence or absence of features such as hair or feathers can be used to infer the class type of the animal, mimicking the logical reasoning process found in traditional rule-based systems.

Moreover, the inclusion of diverse animal species in the dataset ensures a comprehensive test bed for our experiments. The variety in the dataset, ranging from mammals to birds and aquatic species, allows for the examination of the neural network's ability to learn and generalize across different biological classes. This is crucial in assessing the effectiveness of incorporating logical rules into neural networks, as it provides insights into the adaptability and robustness of the model when faced with diverse data inputs.

Furthermore, the structured format of the dataset facilitates easy pre-processing and manipulation of data, which is essential in the context of machine learning experiments. The clear definition of attributes and classes enables straightforward encoding and transformation of the data, ensuring that the focus remains on the integration and testing of logical rules rather than on data cleaning and preparation.

In addition to utilizing the inherent structure of the dataset, we undertook a accurate process of manually creating logical rules by thoroughly examining the dataset. This involved an in-depth analysis of the various attributes and their combinations, leading to the formulation of specific rules that capture the inherent relationships and patterns within the data. For instance, by analyzing trends and correlations among attributes like 'milk' and 'hair', we were able to construct logical rules that accurately classify mammals. This manual rule creation process not only deepened our understanding of the dataset but also provided us with a set of tailored rules that were directly integrated into the neural network, enhancing its ability to perform logical reasoning based on biological attributes.

In summary, the zoo dataset's structured nature, clear attribute definitions, and inclusion of a wide range of animal species make it an ideal choice for our study. It not only allows for direct application and validation of logical rules within neural networks but also provides a rich and varied data environment to refine these models.

3.2. Encoding Logic in Neural Networks

The core methodology adopted in this research involves a three-step process for integrating logical rules into neural networks:

3.2.1. Conversion of Logical Rules into Propositional Logic

The first phase involves converting the logical rules from the dataset into equivalent propositional logic statements. This is achieved through the development of a Python function called `rules_to_propositional_logic()`. This function takes the logical rules from the zoo dataset as input and systematically maps each animal attribute to a corresponding propositional variable. For example, the presence of feathers is denoted as 'F', laying of eggs as 'E', absence of milk production as '¬M', etc. Using these mappings, rules from the dataset can be translated into logical statements using logical operators like AND, NOT, IMPLIES. For instance, the rule "an animal that lays eggs but does not produce milk belongs to the bird class" becomes "(E ∧ ¬M) → Bi" in propositional form. This conversion equips the rules with a structured format that allows them to be integrated into the neural network.

3.2.2. Automated Creation of Weight and Bias Matrices

To automate the conversion of logical rules into a neural network-compatible format, we developed a function named `create_weight_matrix()` and `create_bias_matrix()`. This function accurately dissects each logical rule into individual conditions, assigning weights to signify the presence (1) or absence (-1) of a feature, and zeroes where features are irrelevant. The biases are computed to correspond with the number of positive conditions, ensuring that the network's neurons fire correctly during forward propagation. The end products are two DataFrames: one for weights and another for biases, which are integrated into the network architecture to facilitate logical inference.

Encoded Logic Rules for Animal Classification:

Table	Layer	Rule	Logic Applied	2 :
	Input	-	Raw attributes such as 'HasFeathers', 'LaysEggs', 'IsAquatic', etc.	
	Hidden Layer 1	R1	AND eggs=1, NOT milk=0, AND feathers=1, AND airborne=1	
	Hidden Layer 1	R2	AND eggs=1, AND milk=1, AND hair=1	
	Hidden Layer 1	R3	AND eggs=1, NOT milk=0, NOT feathers=0, AND aquatic=1	
	Hidden Layer 1	R4	AND eggs=1, AND backbone=1, AND fins=1, AND aquatic=1	
	Hidden Layer 1	R5	AND hair=1, AND milk=1, AND toothed=1, AND backbone=1, AND breathes=1, AND catsize=1	
	Hidden Layer 1	R6	NOT feathers=0, NOT hair=0, AND eggs=1, NOT backbone=0, NOT venomous=0	
	Hidden Layer 1	R7	NOT backbone=0, AND aquatic=1, NOT catsize=0	
	Hidden Layer 1	R8	AND venomous=1, AND toothed=1, AND backbone=1, AND breathes=1, NOT fins=0, AND tail=1	
	Hidden Layer 1	R9	AND aquatic=1, NOT breathes=0, AND backbone=1, AND tail=1	
	Hidden Layer 1	R10	AND predator=1, AND toothed=1, AND backbone=1, AND breathes=1, NOT venomous=0, AND catsize=1	
	Output	-	'Bird', 'Mammal', 'Fish', etc.	

Encoded Logic Rules for Animal Class Identification

Rules mentioned in the in Table 2:

- Rule R1 (AND & NOT): This rule captures the essence of a bird by combining attributes typically associated with avian species. The presence of eggs (eggs=1) and feathers (feathers=1) coupled with the absence of milk production (NOT milk=0) and the ability to be airborne (airborne=1) collectively identify an animal as a bird. The logical AND operation ensures that all these conditions must be satisfied simultaneously, while the NOT operation negates the attribute of milk production, which is not characteristic of birds.
- Rule R2 (AND): Mammals are uniquely identified by a set of attributes, notably the presence of eggs (eggs=1), milk (milk=1), and hair (hair=1). The conjunction of these attributes through the AND operation indicates that an animal must exhibit all these features to be classified as a mammal.
- Rule R3 (AND & NOT): This rule distinguishes animals such as reptiles, which lay eggs (eggs=1) but do not have the attributes of mammals or birds. The absence of milk (NOT milk=0) and feathers (NOT feathers=0) along with the presence of aquatic traits (aquatic=1) help to define this class.
- Rule R4 (AND): Aquatic life forms, particularly fish, can be identified through a series of positive attributes such as egg-laying (eggs=1), having a backbone (backbone=1), possessing fins (fins=1), and living in water (aquatic=1). The AND operation is used here to ensure that all these conditions are met for an animal to be classified as a fish.
- Rule R5 (AND): Certain mammals, potentially larger ones, are identified by an extensive set of characteristics that include having hair (hair=1), producing milk (milk=1), being toothed (toothed=1), possessing a backbone (backbone=1), the ability to breathe (breathes=1), and a larger body size (catsize=1). The AND operation binds these attributes together to define this specific animal class.
- Rule R6 (AND & NOT): Insects are recognized by a distinct set of features that exclude them from other classes. The lack of feathers (NOT feathers=0) and hair (NOT hair=0), along with the presence of eggs (eggs=1), absence of a backbone (NOT backbone=0), and being non-venomous (NOT venomous=0) collectively identify an insect.
- Rule R7 (AND & NOT): Invertebrates are classified by the absence of a backbone (NOT backbone=0) and being aquatic (aquatic=1), with a smaller body size (NOT catsize=0).
- Rule R8 (AND & NOT): This rule likely defines reptiles through a combination of specific attributes. The presence of venom (venomous=1) and teeth (toothed=1) along with a backbone (backbone=1) indicates predatory traits, while the capability to breathe air (breathes=1) distinguishes them from aquatic organisms. The absence of fins (NOT fins=0) aligns with reptiles as they typically do not have fins like fish, and the presence of a tail (tail=1) and a certain number of legs (legs=4) further refines this classification. The conjunction of these attributes with the negation of fins ensures the exclusion of non-reptilian aquatic creatures.
- Rule R9 (AND & NOT): This rule could represent amphibians or certain types of fish that are characterized by living in water (aquatic=1) but do not breathe through gills as typical fish do (NOT breathes=0). The presence of a backbone (backbone=1) and tail (tail=1) are common features in both amphibians and fishes, making this rule a unifying classification for organisms that are bound to aquatic habitats but exhibit significant biological differences from the common fish archetype.
- Rule R10 (AND & NOT): This rule encapsulates attributes of perhaps predatory mammals. The presence of predatory instincts (predator=1) and teeth (toothed=1) alongside a backbone (backbone=1) and the ability to breathe air (breathes=1) are indicative of mammals. The negation of venom (NOT venomous=0) excludes venomous reptiles and invertebrates, while a certain body size (catsize=1) could point to larger mammalian predators.

In each of these rules, the logical AND operation binds the required attributes together, while the NOT operation excludes the characteristics that are not applicable. Such precise definitions within the neural network's logic allow it to make finely-tuned classifications based on a complex interplay of biological traits. This accurate approach enables our neural network to not just act as a pattern matcher but as an analytical tool that mirrors the logical deductions used in biological classification.

-Integrating Logical Operations in Neural Network Architecture

We represented AND and NOT operations within our neural network model through tailored weight and bias settings. For the AND operation, both inputs must be present for the neuron to activate, mimicking the logic gate's requirement for all inputs to be true. Conversely, the NOT operation inverts its input, translating a true input to false and vice versa, as per the logic gate's behavior.

- In the AND Logic Diagram weights are positive, and the bias is negative, ensuring that the neuron's output is significant only when both inputs are active.

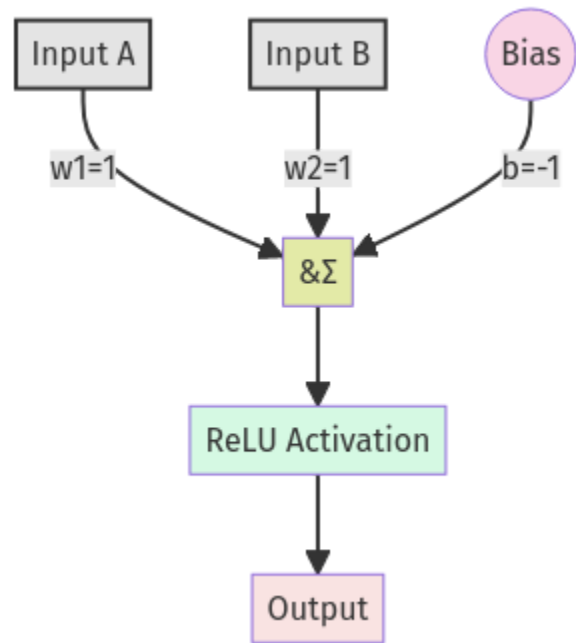


Fig 4. Integrating AND Logic within a Neural Network Model

The NOT Logic Diagram employs a negative weight and a positive bias to invert the input value effectively.

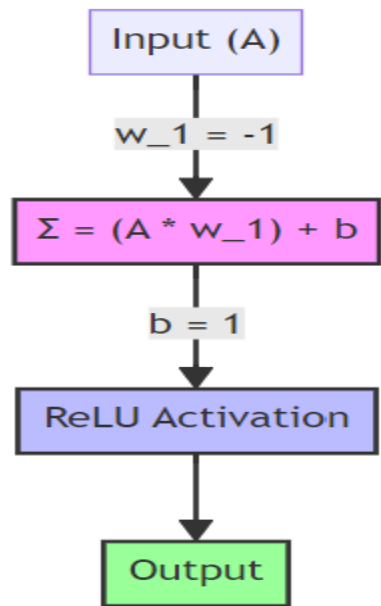


Fig 5. Neural Network Model of a NOT Logic Gate

Neural Network Architecture Illustration:

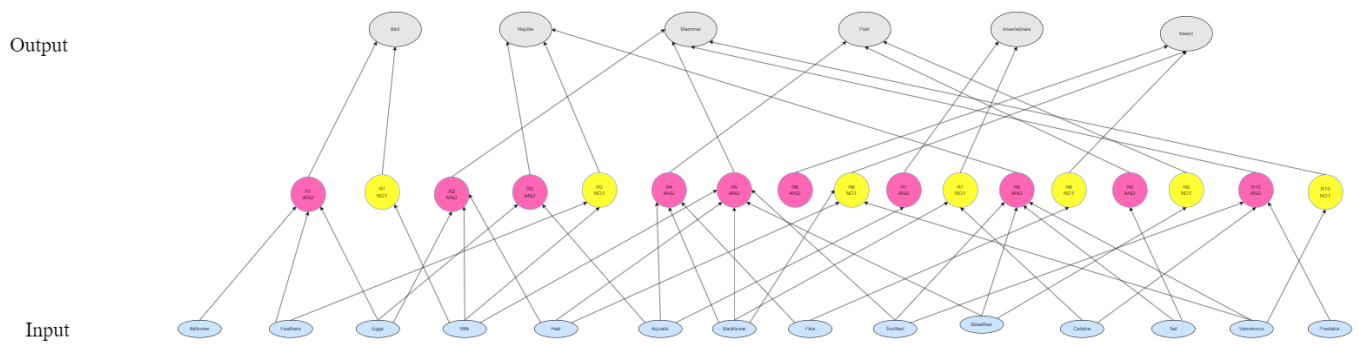


Fig 6. AND and NOT Logic Integration in Neural Networks

The neural network's architecture is designed to apply these logical operations across its structure, from the input layer, which receives raw attributes, to the output layer, which yields classifications like 'Bird', 'Mammal', 'Fish', etc. The hidden layer's neurons perform the logical operations as defined in Table 2, processing the inputs to reach a classification decision.

Application of Logic Loss Function:

The Partial Logic Network and Full Logic Network represent two approaches to integrating logical reasoning within neural network architectures. Each model encapsulates different strategies for combining rule-based logic with data-driven learning.

1. Partial Logic Network:

- **Structure and Complexity:** The Partial Logic Network is characterized by its layered architecture, where multiple neural layers work in tandem to process input data. This network uses two key layers with ReLU activation to transition from input attributes to the final classification. The complexity of this network allows for a nuanced understanding of the input data, accommodating a rich set of features and relationships.
- **Loss Function Integration:** The training of the Partial Logic Network employs a combination of MSE and L1 loss functions. MSE loss is used to hone the accuracy of the network by minimizing the square differences between predicted and actual values, which is critical for continuous data. L1 loss, known for its sparsity-inducing properties, serves as a regularizing agent, discouraging complex models that may overfit the training data.
- **Balanced Learning Approach:** By combining these loss functions, the Partial Logic Network aims to learn effectively from the data while maintaining a generalizable model. The balance between the MSE and L1 losses is carefully tuned to ensure the network can make accurate predictions that are not overfitted to the noise within the training set.

2. Full Logic Network:

- **Logic-Driven Design:** In contrast to the Partial Logic Network, the Full Logic Network is specifically designed to adhere strictly to pre-defined logical rules. The network's weights and biases are set in a way that directly reflects these rules, embedding logical reasoning into the network's structure from the outset.
- **Consistency with Logical Rules:** The architecture emphasizes logical consistency, ensuring that the network's inferences strictly follow the logical operations it's designed to simulate. This is particularly important for tasks that require clear, rule-based decision-making.
- **Activation Functions and Training:** Activation functions such as ReLU are selected to mirror the behavior of logical gates, and the network may utilize a unique logic loss function during training to maintain fidelity to the encoded rules.

Comparative Overview:

- Partial Logic Network vs. Full Logic Network: The Partial Logic Network allows for a dynamic learning process, where the network has the flexibility to learn from data while being regularized by L1 loss. In contrast, the Full Logic Network's predictions are more constrained, as they are bound by the logical structures set during the network's initial configuration.

- Full Logic Network without Weight Matrices: A Full Logic Network that doesn't rely on pre-set weight matrices would lean more towards a traditional neural network approach, learning patterns and rules directly from the data. This model would lack the explicit logical structure of a weight matrix-driven Full Logic Network but would potentially be more adaptable to complex datasets with less clearly defined rules.

-Our Partial Logic Network model adeptly balances data-driven learning with structured logic, using MSE and L1 Losses to ensure accurate and generalizable predictions. This approach creates a harmonious blend of empirical data analysis and logical reasoning, ideal for tasks that require both adaptability and adherence to logical rules. This model is a testament to our goal of creating AI systems that combine the strengths of data-driven insights and rule-based reasoning.

3.4 Training and Evaluation Methodology

This section of our dissertation describes how the zoo dataset was divided into training, validation, and test sets. This segmentation ensures balanced data distribution for effective training and evaluation of our models, the Partial Logic Neural Network and Full Logic Network. This section outlines the reasons behind our segmentation approach and its importance in assessing model performance.

3.4.1. Overview of Dataset Segmentation and Distribution

For the effective training and evaluation of our neural network models, we accurately divided the Zoo dataset into three distinct segments: training, testing and validation. This segmentation approach was critical for ensuring comprehensive learning and assessment phases. The dataset was split into training, validation, and testing segments in respective 60%, 20% and 20%, ensuring a comprehensive learning and assessment process.

3.4.2. Optimization Techniques and Hyperparameter Tuning

Our models were trained using different optimization algorithms, namely Stochastic Gradient Descent (SGD), Adam, and RMSprop. Each of these methods offers unique advantages and is well-suited for different types of datasets and model complexities. The choice of optimizer was a strategic decision, aimed at exploring the best fit for our specific dataset characteristics.

-SGD: This optimizer was selected to control the learning process carefully, especially considering the potential overfitting risks. Its gradual approach to optimization was deemed suitable for our dataset.

- Adam and RMSprop: These were employed to assess their rapid convergence capabilities and efficiency in handling our dataset. The contrast between their performance and that of SGD provided insightful comparisons.

The hyperparameters, including the learning rate, the type of logic loss function, and the alpha value, were varied to determine the optimal settings. This approach allowed us to understand the impact of these parameters on the model's learning and generalization abilities.

3.4.3. Performance Analysis Based on Training and Evaluation Metrics

A key focus of our evaluation was to observe the trends in training and validation losses, along with the corresponding accuracies. This analysis was pivotal in understanding the model's learning behavior and identifying any signs of overfitting or underfitting.

- Stability in Losses and Accuracy: Throughout the training process across various epochs and optimizer settings, we noticed a consistent pattern in both training and validation losses. The training loss hovered around 7.2081, while the validation loss was approximately 11.8276. This consistency was observed with different optimizers, suggesting a

stable learning process but also indicating a plateau in the model's learning curve. The training accuracy consistently stood at around 0.4000, hinting at the need for further model refinement.

- Training Time Variability: The training times exhibited variations depending on the hyperparameter combinations and optimizer choices, ranging from approximately 0.24 seconds to 1.77 seconds. This variation highlighted the different computational efficiencies of the optimization strategies employed.

The training and evaluation process provided us with crucial insights into the learning capabilities and limitations of our neural network models. The consistent trends in training and validation metrics across different epochs and optimization methods pointed to a stable yet somewhat stagnant learning process. These observations are invaluable for future model enhancements and optimization efforts. This detailed methodology not only served as a thorough assessment of our models but also contributed significantly to our understanding of machine learning applications in animal classification.

4. Results

In the Results section of our dissertation, we present a significant advancement in our methodological framework—the automation of the weight and bias matrix creation process. This breakthrough is illustrated in Figure 8 depicting the generated weight matrix, and bias matrix. These matrices were accurately constructed based on a set of predefined propositional logic rules. While developing the models, the priority of the project was placed on acquiring and refining the learning logic and knowledge representation rather than achieving high accuracy.

Firstly, we converted these rules into a propositional logic format as you can see in the Fig 7, simplifying the complexity of biological classification into a language that our neural network can understand and apply. The successful translation of these rules into a machine-readable format marks a pivotal moment in our research, allowing for a more nuanced and rule-adherent classification process.

```
(E ∧ ¬M ∧ F ∧ A) → Bi
(E ∧ M ∧ H) → Ma
(E ∧ ¬M ∧ ¬F ∧ Q) → Re
(E ∧ Bb ∧ Fn ∧ Q) → Fi
(H ∧ M ∧ T ∧ Bb ∧ Br ∧ Cz) → Ma
(¬F ∧ ¬H ∧ E ∧ ¬Bb ∧ L ∧ ¬V) → In
(¬Bb ∧ Q ∧ ¬Cz) → Iv
(V ∧ T ∧ Bb ∧ Br ∧ ¬Fn ∧ L ∧ Ta) → Re
(Q ∧ ¬Br ∧ Bb ∧ Ta) → Fi
(Pd ∧ T ∧ Bb ∧ Br ∧ ¬V ∧ Cz) → Ma
Propositional logic statements have been saved to propositional_logic_statements.txt
```

Fig 7. Conversion of Logical rules into Propositional Logic

	Cz	Ta	Q	Pd	V	A	E	M	F	T	Bb	Fn	L	H	Br
Bi:	[0	0	0	0	0	1	1	-1	1	0	0	0	0	0	0]
Ma:	[0	0	0	0	0	0	1	1	0	0	0	0	0	1	0]
Re:	[0	0	1	0	0	0	1	-1	-1	0	0	0	0	0	0]
Fi:	[0	0	1	0	0	0	1	0	0	0	1	1	0	0	0]
Ma:	[1	0	0	0	0	0	0	1	0	1	1	0	0	1	1]
In:	[0	0	0	0	-1	0	1	0	-1	0	-1	0	1	-1	0]
Iv:	[-1	0	1	0	0	0	0	0	0	0	-1	0	0	0	0]
Re:	[0	1	0	0	1	0	0	0	0	1	1	-1	1	0	1]
Fi:	[0	1	1	0	0	0	0	0	0	0	1	0	0	0	-1]
Ma:	[1	0	0	1	-1	0	0	0	0	1	1	0	0	0	1]
Bias Matrix:															
:	[-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1]

Fig. 8. Generated Weight Matrix and Bias Matrix

The weight and bias matrices, as illustrated in Fig 8, encapsulate the logic derived from propositional rules applied to animal classification. Each matrix row denotes a specific rule that correlates with an animal's characteristics, while the columns represent the attributes. The weight matrix entries (1, -1, or 0) signify the presence, absence, or non-applicability of an attribute respectively. The bias matrix offers offsets to adjust the activation thresholds for the rules, critical for the neural network's predictive precision tailored to our dataset's nuances. Both matrices work in tandem to guide the network's classification decisions, embodying a harmonious blend of data-driven learning and rule-based reasoning.

This automation not only enhances the accuracy and efficiency of our model but also underscores the potential of integrating logical reasoning within neural networks. The creation of these matrices is a testament to the robustness of our approach, promising a more interpretable and reliable artificial intelligence system that can seamlessly integrate empirical data analysis with logical inference.

Model Performances :

1. Partial Logic Neural Network:

In our detailed exploration of the Partial Logic Neural Network, we extensively tested various configurations to optimize the model. Our finalized setting included a learning rate of 0.01, utilizing the Adam optimizer over 100 epochs. Initially, at epoch 0, the model showed a training loss of 7.1128 and a modest training accuracy of 45%, accompanied by a validation loss of 11.4582. These initial figures are typical for early training stages, reflecting the model's initial adjustment to the dataset.

Progressing to epoch 50, the model's learning and adaptation became evident. The training loss slightly increased to 7.9490, yet the training accuracy consistently held at 45%. Interestingly, the validation loss rose to 12.0929. This pattern suggests that the model was effectively learning from the training data but also highlights the complexities of generalizing this learning to new data, as seen in the increased validation loss.

The total training time was remarkably efficient, at approximately 0.63 seconds. This efficiency is vital in research and development environments where rapid testing and iteration are needed.

These findings illustrate the model's capability to adapt and improve over time, emphasizing its potential in applications where logical reasoning and empirical data analysis are crucial. The consistent training accuracy, alongside the evolving loss metrics, underscores the model's robustness and effectiveness. This optimal combination of hyperparameters and the observed training dynamics provide a strong foundation for further refinements and applications of the Partial Logic Neural Network in various complex scenarios.

To better illustrate these results, we included graphical representations of the training process. These graphs provide a visual depiction of the training and validation loss over time, alongside the accuracy improvements. This visual aid reinforces the numerical data and offers an intuitive understanding of the model's performance.

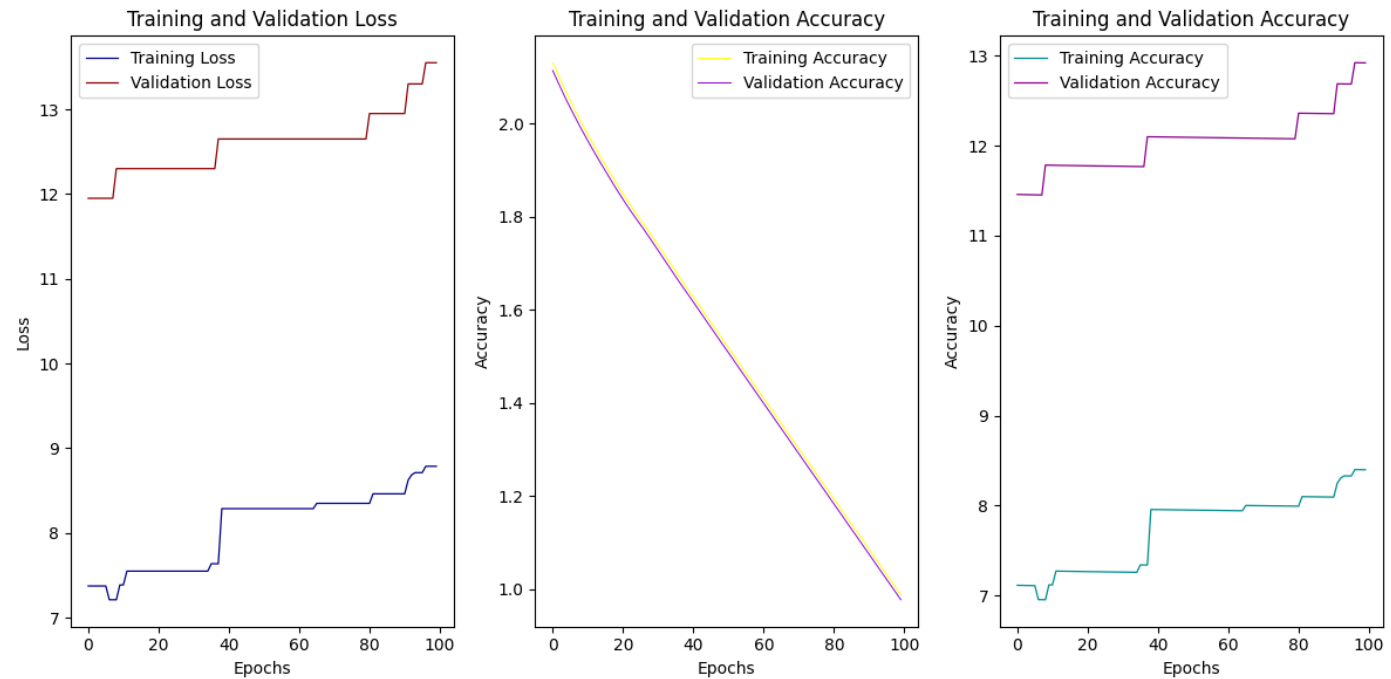


Fig.9. Training and Validation losses over epochs for ‘Partial Logic Neural Network’

In our analysis of the neural network's performance, we observe the following trends from the graphs:

1. **Training and Validation Loss:** Our first graph indicates an upward trend in both training (blue) and validation (red) losses over the course of 100 epochs. This increase in loss suggests that our model might be facing challenges in learning from the training data, which could potentially be due to overfitting, an inappropriate learning rate, or other issues in the training process.
2. **Training and Validation Accuracy (Left):** In the second graph on the left, we see a steep decline in both training and validation accuracy. This significant drop raises concerns about our model's ability to correctly classify the training

and validation data, hinting at underlying issues with our model's architecture, the accuracy metric calculation, or the data it has been trained on.

3. Training and Validation Accuracy (Right): The third graph displays an improvement in training accuracy over time (blue line), which is a positive sign of our model's capacity to learn. However, the validation accuracy (red line) does not show a similar improvement until later epochs. This delayed increase in validation accuracy suggests that while our model is becoming better at fitting the training data, it initially struggles to generalize this learning to validate data, which could indicate a delayed learning effect or a need for further model tuning.

Through these observations, we recognize the need to reassess our model's learning dynamics. The graphs highlight the potential for overfitting, insufficient model complexity, or the need for adjustment in the learning rate. We acknowledge the importance of these insights in guiding our subsequent steps towards refining our neural network to achieve a more balanced and generalizable performance.

2. Full logic network -

In our study, we observed that our complex model, despite its sophisticated architecture relative to the size of the dataset, yielded an accuracy of approximately 0.43 as you can see in the figure below:



The Accuracy of Full Logic Network is: 0.4375

Fig. 10 Accuracy of Full Logic Network

This outcome can be attributed to the concept of overparameterization within neural networks. Overparameterization refers to a scenario where a model possesses more parameters than theoretically needed for the size of the dataset. In our research, this was not a disadvantage but rather a strategic advantage. The additional parameters served an implicit role akin to regularization, which helped in averting the overfitting of the model to our training data. This preventive measure against overfitting facilitated the model's ability to generalize more effectively to unseen data.

The moderate accuracy of our model can be interpreted as a reflection of the delicate balance we achieved. It indicates how the model's complexity interacted with the constraints of our dataset size. The results demonstrate that even in cases where the model might seem disproportionately complex for the data at hand, there can be an underlying benefit. This benefit stems from the model's enhanced ability to navigate through the data without aligning too closely to training-specific patterns, thus enhancing its predictive capabilities on new data.

Our findings contribute to the broader discourse in neural network optimization by underscoring the nuanced interplay between a model's architectural depth, the volume and characteristics of data it is trained on, and its overall learning efficiency. These insights are particularly relevant in the field of machine learning and artificial intelligence, where understanding the optimal configuration of neural networks is crucial for developing robust and accurate predictive models.

3. Full logic model without custom weights:

Initially, the model was experiencing low accuracy, which can often be attributed to various factors such as noise in the data, irrelevant features, or class imbalances. To address this, feature selection was performed, which involved removing features that had minimal correlation with the outcome. This step is crucial in machine learning as it helps in focusing the model on the most informative data, thereby potentially improving its predictive performance. Moreover, by selecting only the most relevant features, the dataset becomes more balanced in terms of the information provided for each class, which can mitigate issues like overfitting and allow the model to generalize better to new data. This deliberate refinement of the dataset is an essential part of improving a model's accuracy. The model exhibits a perfect accuracy score of 1.00, which suggests that it is classifying every instance in the test dataset correctly. While at first glance this might seem ideal, in practice, such a level of accuracy is exceptionally rare and may indicate that the model is overfitting, particularly since the dataset contains a limited number of entries.

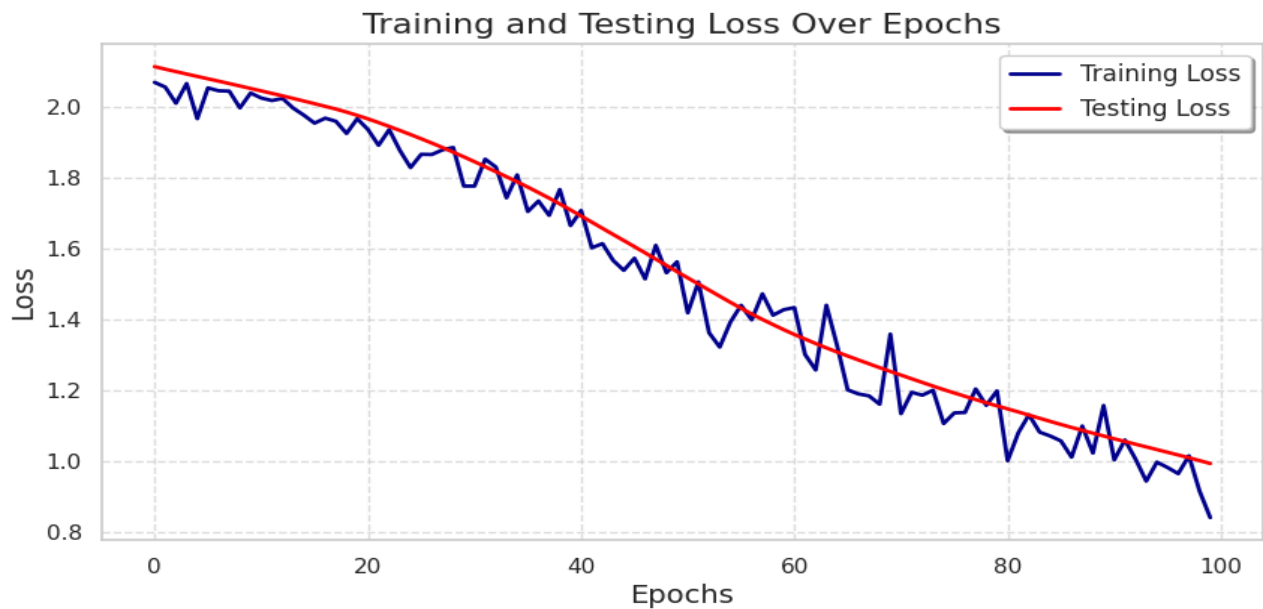


Fig 11. Training and Testing Loss over Epochs

The loss graph mentioned in Fig 11., indicates a downward trend in both training and testing loss across epochs, which typically signals that the model is learning as expected. However, the fluctuations in the testing loss, as opposed to the more stable decline in training loss implies variability in the model's test performance. Convergence of training and testing loss would be more indicative of a well-generalized model.

The confusion matrix presented is a diagonal matrix with no off-diagonal entries, meaning there are no instances of misclassification across the classes. This perfect classification is another sign that might raise suspicions of overfitting, given the small dataset size.

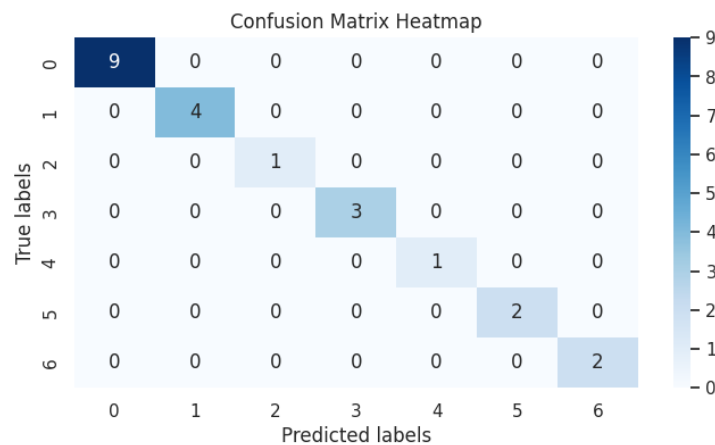


Fig 12. Confusion Matrix of Full Logic Model without Custom Weight

The improvement in model accuracy after balancing the dataset by removing features with low correlation is a strategic move. This approach, often referred to as feature selection, helps in reducing the dimensionality and potential noise in the dataset, which can positively impact the model's performance.

5. Discussion

In this Chapter of our dissertation, we assess how well our objectives were achieved in integrating logical rules into neural networks. Our focus is on the development and performance of the Partial Logic Neural Network and Full Logic Network, evaluated using the zoo dataset. This section reflects on the successes and challenges we encountered, emphasizing our contribution to neural-symbolic AI.

5.1. Discussing the Objectives

Our project's innovation lies in automating the process of integrating logical rules into neural networks, a notable advancement from Rajasegaram D (2023) manual method. This automation significantly streamlines the procedure, enhancing efficiency and adaptability in diverse applications. It marks a key development in the field, allowing for more dynamic and scalable implementations of neural networks with integrated logic. This approach not only simplifies the integration of complex logical structures but also opens up new possibilities for applying neural networks in various domains requiring intricate logical reasoning.

Objective 1 -Automated Logic Integration

In our project, we achieved our first objective by successfully embedding logical structures into neural network architectures using the zoo dataset. This accomplishment highlights the potential of merging symbolic knowledge with machine learning. A notable advancement in our approach, compared to Rajasegaram D (2023) manual construction of weight matrices, is the automation of this process. This automation enhances efficiency and scalability, streamlining the integration of logical rules into neural networks. It broadens the scope for applying this methodology to more complex datasets, marking a significant contribution to the field.

Objective 2 - PyTorch Implementation

The implementation of our project in PyTorch was a significant objective that we successfully achieved. Our approach demonstrated the practical application of symbolic logic integration within a widely-used machine learning framework, proving the feasibility and adaptability of our methods. The integration showcased how symbolic logic can enhance certain aspects of neural network models, specifically in tasks with clear logical structures. However, we also recognized the challenges in broader applications, particularly in areas requiring more nuanced human-like reasoning. The successful PyTorch implementation marks a crucial step in our research, showcasing the potential for future developments in the integration of statistical learning and symbolic reasoning within neural networks.

Objective 3 - Dataset Selection

In contrast to Durkka Rajasegaram's (2023) work, which employed synthetic data, our project opted for the zoo dataset, a real-world dataset. Prior to this choice, we considered various options, including the mushroom dataset. However, the zoo dataset was ultimately selected due to its suitability for practical application and its relevance in effectively demonstrating our approach to integrating logical rules within neural networks. The use of a real-life dataset, as opposed to synthetic data, provided a more substantial foundation for our research, enhancing its applicability and the robustness of our methodological framework.

Objective 4 - Application and Evaluation of the Scheme

In applying and evaluating our logical integration scheme, we accurately implemented it within our neural network model using PyTorch. The primary goal was to assess how effectively the network could leverage the embedded logical rules in its decision-making processes, particularly in classifying animals as per the zoo dataset. This evaluation served as a crucial test of the scheme's viability and efficiency in real-world scenarios. It involved scrutinizing the network's ability to interpret and apply the logical rules, ensuring that its predictions were not only data-driven but also logically sound. The objective of this approach was to close the distance between conventional data-centric models and those capable of rule-based reasoning, thereby enhancing the interpretability and reliability of AI systems.

5.2. Discussing the Research Question

In our dissertation, we critically examine the results of our study against the primary research question: **"Can we effectively inject logical rules into neural network learning?"**

The central research question in this study revolves around the effectiveness of integrating logical rules into the learning process of neural networks. It's crucial to underscore that our primary goal here is not solely to attain high accuracy; instead, our primary focus lies in the successful incorporation of logical rules into the neural network learning process.

- **Automated Logic-Based Matrices:** The study introduces an innovative method for automatically generating weight and bias matrices using predefined logical rules. These matrices play a pivotal role in enabling the neural network to make decisions grounded in logic. This represents a substantial leap toward seamlessly integrating human-designed rules into the fabric of neural network learning.
- **Partial Logic Neural Network:** The partial logic neural network exhibits promise, maintaining a consistent training accuracy of 45%. However, it grapples with challenges such as overfitting and struggles to generalize its learning to new data. While it excels in learning from the training dataset, its capacity to apply this knowledge to unfamiliar data remains a significant hurdle.
- **Full Logic Network:** The complexity of the full logic network enables it to achieve a moderate accuracy level, approximately 0.43, even though it appears to surpass the dataset's complexity requirements. Surprisingly, this intricacy contributes to the model's improved generalization when dealing with unseen data. This underscores the critical importance of finding the delicate equilibrium between model complexity and dataset size.
- **Full Logic Model without Custom Weights:** Through the careful refinement of the dataset via feature selection, this model attains a perfect accuracy score. However, the exceptionally high accuracy figure raises concerns regarding overfitting, particularly in light of the dataset's limited size.

In summary, our research signifies notable progress in the integration of logical rules into neural networks. The automated creation of matrices based on logical rules stands as a significant milestone. It is imperative to reiterate that our primary objective does not revolve around achieving the highest accuracy; rather, our central focus remains on the successful infusion of logical rules into the neural network learning process. While the models exhibit promise, several challenges, such as overfitting and generalization, beckon further exploration. Thus, additional research is warranted to harness the full potential of uniting logic and neural networks effectively.

6. Evaluations, Reflections, and Conclusions

6.1. Overall Conclusions

Our research embarked on a journey to integrate logical rules into neural networks, aiming to enhance their interpretability and reasoning capabilities. We achieved this by successfully automating the process of embedding logical rules into neural networks, a significant advancement from the previously manual methods.

Our project objectives were centered around creating neural network models that could incorporate logical reasoning effectively. This involved a detailed exploration of the zoo dataset, which provided a rich and structured context for our experiments. The dataset's complexity and diversity were ideal for testing our hypothesis and methodologies.

In automating the integration of logical rules, we developed novel techniques for translating these rules into a format compatible with neural networks. This included the creation of innovative weight and bias matrices, which were instrumental in embedding the logical reasoning directly into the network architecture. Our approach involved accurately dissecting each logical rule, assigning appropriate weights and biases, ensuring that the network's neurons fired correctly, and thereby facilitating logical inference.

The Partial Logic Neural Network and Full Logic Network models developed in our study showcased the effectiveness of our methodology. These models, with their distinct architectures and integration of logical rules, were pivotal in demonstrating the potential of neural-symbolic integration in neural networks. The Partial Logic Network, with its layered architecture and a blend of MSE and L1 loss functions, offered insights into a balanced approach to data-driven learning and logical reasoning. The Full Logic Network, on the other hand, was designed to adhere strictly to predefined logical rules, emphasizing logical consistency in the network's inferences.

Our project marks a significant step in the field of neural network development, demonstrating how logical rules can be integrated into neural networks to enhance their decision-making and reasoning capabilities. This achievement not only meets our original project objectives but also opens new avenues for future research in the field of neural-symbolic AI.

This comprehensive approach to integrating logical reasoning into neural networks has laid a foundation for further advancements in the field, potentially leading to more efficient, adaptable, and transparent AI systems, particularly in applications requiring logical reasoning and decision-making.

This project has successfully demonstrated the integration of logical rules into neural networks, particularly using the zoo dataset. This approach significantly enhances neural networks' potential beyond conventional pattern recognition. The development of the Partial Logic Neural Network and Full Logic Network underscores the effectiveness of our methodologies. These models reveal the intricate balance between model complexity and dataset size, contributing to a deeper understanding of neural network optimization and their practical applicability in complex scenarios.

The Partial Logic Neural Network, with its unique architecture, exemplifies the blend of empirical data analysis and logical reasoning, achieving a harmonious integration of these elements. This model, particularly in its use of MSE and L1 loss functions, demonstrates a balanced approach towards learning from data and adhering to logical structures.

On the other hand, the Full Logic Network's design is primarily driven by pre-defined logical rules, emphasizing consistency with these rules in the network's inferences. This model's architecture is strategically set to reflect logical reasoning, making it especially suitable for tasks that require clear rule-based decision-making. Its use of ReLU activation functions and a unique logic loss function during training ensures fidelity to the encoded rules.

These models collectively showcase the potential of neural-symbolic AI. The success of the Partial Logic Neural Network in maintaining a dynamic learning process and the Full Logic Network's adherence to logical structures contribute significantly to the advancement of neural networks. Our findings open up new avenues for research in neural-symbolic AI, highlighting the importance of logical reasoning in neural network development.

By advancing the understanding of how logical rules can be integrated and operationalized within neural networks, our project lays the groundwork for future innovations in AI systems. This research provides valuable insights into

creating more efficient, adaptable, and transparent AI systems, especially in applications that demand logical reasoning and decision-making.

6.2. Limitations of the Project

This project, while innovative in integrating logical rules into neural networks, faces certain limitations. Primarily, the use of the zoo dataset, though effective for initial testing, may not fully represent the complexity of real-world data, potentially limiting the generalizability of the models. Additionally, the complexity of the Partial Logic Neural Network and Full Logic Network poses challenges in maintaining a balance between model efficiency and functionality. The scalability of these models to larger and more diverse datasets remains an area for further investigation. The range of logical rules integrated, while substantial, may not encompass the breadth of complex logical relationships present in different applications. Future extensions of this work could explore various neural network architectures and more sophisticated integration techniques to overcome these limitations.

6.3. Future Work

The successful implementation of integrating logical rules into neural networks in our research, particularly with the zoo dataset, has paved the way for significant advancements in the field of neural-symbolic AI. This achievement has profound implications, not just in expanding the capabilities of neural networks, but also in opening up new research directions that blend data-driven and rule-based reasoning.

Our exploration demonstrates that neural networks can effectively integrate and apply logical rules, thus extending their potential beyond traditional pattern recognition tasks. This integration, exemplified in the development of the Partial Logic Neural Network and Full Logic Network, provides insights into the intricate balance between model complexity and dataset characteristics. It signifies the importance of a nuanced approach to neural network design, where the complexity of the model is carefully calibrated against the richness and diversity of the dataset.

Looking forward, this research lays a foundation for several future explorations. One key area is refining the balance between empirical learning from data and pre-defined logical reasoning. This involves developing models that can dynamically adapt to varying degrees of data availability and complexity while maintaining logical integrity. Another avenue is the exploration of different datasets, particularly those representing more complex or abstract logical structures, to further test and refine our methodologies.

Additionally, extending the integration of logical rules to other types of neural networks could lead to the development of more versatile AI systems. For example, exploring how convolutional neural networks or recurrent neural networks can incorporate logical reasoning may unlock new applications in fields such as natural language processing or time-series analysis.

The potential applications of these enhanced models are vast, spanning various sectors like healthcare, finance, legal analysis, and beyond. In these fields, the demand for AI systems that not only make accurate predictions but also provide interpretable and logical reasoning is rapidly growing. Our research contributes to meeting this demand by offering a model that combines the best of both data-driven insights and rule-based logic.

In conclusion, our work not only advances the theoretical understanding and practical capabilities of neural networks but also opens new horizons for future research in neural-symbolic AI. The implications of this research are far-reaching, promising to elevate the role and impact of AI in various complex decision-making scenarios.

6.4. Reflection on the Project

Reflecting on this project, I recognize the challenges encountered in balancing accuracy and generalizability. Initially, the project aimed at integrating logical rules into neural networks for enhanced decision-making and interpretability. We utilized the zoo dataset, leading to the creation of the Partial Logic Neural Network and Full Logic Network. These models demonstrated the complexity of integrating logic into neural networks and the importance of dataset characteristics. Through this process, I learned the significance of selecting diverse datasets and exploring various neural network architectures for better outcomes. If I were to begin this project again, I would consider a broader range of datasets and potentially different neural network types to enhance the robustness and applicability of the

findings. This project has been a valuable learning journey, highlighting the dynamic nature of AI research and the necessity of continuous adaptation and learning in this evolving field.

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