Network GUI Documentation

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1 Introduction

This program provides a convenient way to simulate two models for networks of neurons (not to be confused with neural networks), referred to in the program as "Theta" and "Ott-Antonsen." The "Theta" model is a change of variables from the quadratic integrate and fire (QIF) model that removes the discontinuities associated with spiking [1]. For a more gentle introduction, see `http://www.scholarpedia.org/article/Ermentrout-Kopell_canonical_model`. The "Ott-Antonsen" model is a mean-field theory version of the "Theta" model derived using the Ott-Antonsen ansatz [2]. For a more gentle derivation, see section 4.

The menu in the upper left toggles between these two models as well as an option to run both at the same time and compare directly. The table underneath shows the parameters used in the simulation (see section 2) and can be edited directly. The menu in the lower left toggles between graphing options for a given simulation (see section 3). The "Update" button below that reruns the simulation with the current parameters in the table and redraws the current graph.

I originally made this as an aid to my research funded by the Center for Neural Basis of Cognition undergraduate research fellowship in computational neuroscience, and one might say I decided to put all this together instead of "writing a paper," if one were feeling particularly bold. Thus, please bear with me for any possible errors and for the forced use of Matlab. Questions and comments can be submitted to the email above.

2 Explanation of Parameters

Simulation Parameters

- Noise (Theta only): When checked, noise is explicitly added in the form of EPSPs (or IPSPs) independently drawn from a normal distribution at every time step. When unchecked, heterogeneity is added when initializing all of the neurons and drawn from a Cauchy-Lorentz distribution, which gives the eventual convergence to the Ott-Antonsen model (see section 4).
- Ne (Theta only): The number of *excitatory* neurons in the theta network. Needs to be large to see convergence to Ott-Antonsen, and greater than zero to see anything at all.
- Ni (Theta only): The number of *inhibitory* neurons in the theta network. Needs to be large to see convergence to Ott-Antonsen, and greater than zero to see anything at all.
- t0: The starting time of the simulation to be graphed, in milliseconds. Can be used to remove transient behavior. Must be greater than or equal to zero.
- tf: The final time of the simulation to be run and graphed, in milliseconds. Must be greater than t0.
- dt: The time step for integration, in milliseconds. If you see sharp discontinuities/runaway solutions, this should probably be made smaller. If you want the program to run faster you can sometimes make this bigger. Must be greater than zero and a divisor of (tf - t0).

Equation Parameters

Also see section 4 for explicit equations and more information.

- tau_e: The synaptic decay time constant for *excitatory* neurons. Must be greater than zero.
- tau_i: The synaptic decay time constant for *inhibitory* neurons. Must be greater than zero.
- amp: The amplitude of the periodic stimulus. Usually greater than or equal to zero (but won't break anything if it's not).
- beta: The decay time constant of the periodic stimulus. Usually greater than zero.
- omega: The angular frequency of the periodic stimulus. Dividing by 2π and multiplying by 1000 gives the frequency in Hertz. Usually greater than zero.
- I_constant: The constant current going to each *excitatory* neuron. Can be any real number, but is usually small in magnitude.
- I_constant_frac: The fraction of I_constant that goes to each *inhibitory* neuron, that is $I_c^i = I_c^{frac} * I_c$. Can be any real number, but is usually between 0 and 1 inclusive.
- sigma: The amount of noise or the degree of heterogeneity to be added to *excitatory* neurons. Must be greater than zero for Ott-Antonsen to work.
- sigma_frac: The fraction of sigma that is used for each *inhibitory* neuron, that is $\sigma_i = \sigma_{frac} * \sigma$. Must be greater than zero for Ott-Antonsen to work, and is usually less than or equal to 1.
- gee: The synaptic conductance, or connection strength, *from excitatory to excitatory* neurons. Should be greater than or equal to zero to work as intended.
- gei: The synaptic conductance, or connection strength, *from inhibitory to excitatory* neurons. Should be greater than or equal to zero to work as intended.
- gie: The synaptic conductance, or connection strength, *from excitatory to inhibitory* neurons. Should be greater than or equal to zero to work as intended.
- gii: The synaptic conductance, or connection strength, *from inhibitory to inhibitory* neurons. Should be greater than or equal to zero to work as intended.

3 Explanation of Graphing Options

Also see section 4 for explicit equations and more information.

- Theta Model
  - Se/Si: The average of the excitatory/inhibitory synaptic gating variable, analogous to the average electrical output of spiking neurons.
  - Raster plot: The spiking activity of each of the neurons in the network, with excitatory neurons on top. Uses code by Jeffrey Chiou, see `https://www.mathworks.com/matlabcentral/fileexchange/45671-flexible-and-fast-spike-raster-plotting`
- Ott-Antonsen Model
  - Se/Si: The mean excitatory/inhibitory synaptic gating variable, same as in the Theta model
  - Ve/Vi: The mean excitatory/inhibitory membrane voltage
  - Re/Ri: The mean excitatory/inhibitory firing rate

- Theta and Ott-Antonsen: The Se/Si plots of both models for the same parameters can be plotted simultaneously and compared. They should converge when Noise is unchecked, Ne and Ni are sufficiently large (may need to be very, very large to get consistent behavior), and dt is sufficiently small (0.01 usually works best, but can be a little slow). Note that with finite Ne/Ni, some trials will be better matched than others, so it's usually best to run it a few times to check.

4 Ott-Antonsen Derivation and Equations

Let $k \in \{e, i\}$ denote the type of a given neuron, excitatory or inhibitory. The theta model is equivalent to the differential equation in V_k:

$$\frac{dV_k}{dt} = V_k^2 + I_k$$

where

$$I_k(\eta) = I_c^k + S_k + \sigma_k \eta$$

for some noise parameter η and

$$S_k = g_{ke} s_e - g_{ki} s_i$$

The synaptic gating variables are defined to follow

$$\frac{ds_k}{dt} = \frac{1}{\tau_k}\left(-s_k + \frac{1}{N}\sum_{i=1}^{N}\delta(t - t_i)\right) \tag{1}$$

where t_i is the time the ith neuron spikes (the network connection is all-to-all).

Note that $V_k \to \infty$ corresponds to a spike (at which point $V_k \to -\infty$) and that I_k does not depend on V_k. We then consider an infinite number of neurons in the network, where they can be modeled as a continuous distribution. The function that describes how the population voltage changes with time is then

$$f(V, I, s) = V^2 + I(s)$$

Using the Ott-Antonsen ansatz, we take the probability density to be

$$p(V, \eta, t) = \frac{1}{\pi}\frac{a(\eta, t)}{[V - b(\eta, t)]^2 + a(\eta, t)^2}$$

where $p(V, \eta, t)dV$ is the fraction of neurons with membrane potentials between V and $V + dV$, with noise parameter η, at time t (and $dV \to 0$). Additionally, $p(V, \eta, t)$ is a Cauchy-Lorentz distribution with median $b(\eta, t)$ and half-width at half-maximum $a(\eta, t)$. Thus we take $b(\eta, t)$ to be the mean membrane potential for some given η and t.

To give a clearer meaning to $a(\eta, t)$, note that the mean firing rate of the population (with noise parameter η) is the rate at which a single neuron is expected to pass through $V \to \infty$ at time t, or

$$\begin{aligned} r(\eta, t) &= \lim_{V\to\infty} f(V, I, s)p(V, \eta, t) \\ &= \lim_{V\to\infty}\left(\frac{1}{\pi}\frac{(V^2 + I(s))a(\eta, t)}{V^2 - 2Vb(\eta, t) + b(\eta, t)^2 + a(\eta, t)^2}\right) \\ &= \frac{a(\eta, t)}{\pi} \end{aligned}$$

Thus it is possible to obtain two biophysically relevant macroscopic quantities, the mean firing rate (over all values of η):

$$r(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} a(\eta, t)G(\eta)d\eta \tag{2}$$

and the mean membrane potential:

$$v(t) = \int_{-\infty}^{\infty} b(\eta, t)G(\eta)d\eta \tag{3}$$

To find $a(\eta, t)$ and $b(\eta, t)$, we use the continuity equation:

$$\frac{\partial[p(V,\eta,t)]}{\partial t} + \frac{\partial[f(V,I,s)p(V,\eta,t)]}{\partial V} = 0$$

Removing the function arguments and defining $\gamma = \frac{1}{\pi[(V-b)^2+a^2]^2}$

$$\frac{\partial p}{\partial t} = \gamma[\dot{a}V^2 + 2(a\dot{b} - \dot{a}b)V + (b^2 - a^2)\dot{a} - (2ab)\dot{b}]$$
$$\frac{\partial(fp)}{\partial V} = \gamma[(-2ab)V^2 + (2a(a^2 + b^2) - I)V + 2abI]$$

For the continuity equation to be true for all V:

$$\frac{da}{dt} = 2ab \tag{4}$$
$$\frac{db}{dt} = b^2 - a^2 + I \tag{5}$$

Note that a, b are currently functions of arbitrary η, t and are coupled in the following way. Let the complex variable

$$w(\eta, t) = b(\eta, t) + a(\eta, t)i$$

so that $a = Im(w)$ and $b = Re(w)$ and $w^2 = (b^2 - a^2) + (2ab)i$, which looks a lot like Equations 4 and 5. Then these two equations can be written together as

$$\frac{d[w(\eta, t)]}{dt} = w(\eta, t)^2 + I(\eta, s) \tag{6}$$

The last thing we need is a mean field approximation for the synaptic gating variable, s_k. Again we consider an average over the population, so that the continuous version of Equation 1 is

$$\frac{ds}{dt} = -\frac{1}{\tau}(-s + r(t)) \tag{7}$$
$$= -\frac{1}{\tau}\left(-s + \frac{1}{\pi}\int_{-\infty}^{\infty} a(\eta, t)G(\eta)d\eta\right) \tag{8}$$

from Equation 2. This seems like it would be quite a difficult integral to do, especially since we don't even have an explicit form for $a(\eta, t)$. However, we are free to choose a convenient form for $G(\eta)$, and we choose η to be drawn from a Cauchy-Lorentz distribution, that is

$$G(\eta) = \frac{1}{\pi(1 + \eta^2)} = \frac{1}{2\pi i}\left[\frac{1}{\eta - i} - \frac{1}{\eta + i}\right]$$

Then we can use the residue theorem to evaluate the integral.

We choose the contour curve to lie in the positive imaginary axis (the integral over which goes to zero, leaving only the real number line), so that the only residue to be evaluated is at $\eta = i$. Then

$$\frac{1}{\pi}\int_{-\infty}^{\infty} a(\eta, t)G(\eta)d\eta = \frac{1}{\pi}\frac{2\pi i}{2\pi i}Res_{\eta=i}a(\eta, t)G(\eta)$$
$$= \frac{a(i, t)}{\pi}$$

This means that the firing rate $r(t)$ can be evaluated only knowing $a(t)$ at $\eta = i$. Similarly, the mean membrane voltage $v(t)$ from Equation 3 is just $b(t)$ evaluated at $\eta = i$. Notably, the constant current term $I(\eta, s)$ becomes

$$I(i, s) = I_c + S_k(s) + \sigma i$$

so that

$$\frac{dw(i, t)}{dt} = w(i, t)^2 + I_c + S_k(s) + \sigma i$$

We then find $a(i, t) = Im(w(i, t))$ and $b(i, t) = Re(w(i, t))$ and rewrite in terms of $r(t)$ and $v(t)$. Combined with Equation 7 and separating the population by excitatory and inhibitory neurons, we obtain six (6) equations for the entire population. Click stimuli was added in the form of periodic drive to each neuron and is expressed as I_f.

Final Equations

$$
\begin{aligned}
\frac{dr_e}{dt} &= 2r_e v_e + \sigma \\
\frac{dv_e}{dt} &= v_e^2 - r_e^2 + I_c^e + I_f + g_{ee} s_e - g_{ei} s_i \\
\frac{ds_e}{dt} &= \frac{1}{\tau_e}\left(-s_e + \frac{r_e}{\pi}\right) \\
\frac{dr_i}{dt} &= 2r_i v_i + \sigma \\
\frac{dv_i}{dt} &= v_i^2 - r_i^2 + I_c^i + I_f + g_{ie} s_e - g_{ii} s_i \\
\frac{ds_i}{dt} &= \frac{1}{\tau_i}\left(-s_i + \frac{r_i}{\pi}\right)
\end{aligned}
$$

$$
I_f = amp * e^{-\beta(1-\cos(\omega t))}
$$

References

[1] Bard Ermentrout and Nancy Kopell. "Parabolic Bursting in an Excitable System Coupled with a Slow Oscillation". In: *SIAM* 46.2 (1986), pp. 233–253.

[2] Ernest Montbrió, Diego Pazó, and Alex Roxin. "Macroscopic description for networks of spiking neurons." In: *Physical Review X* 5.2 (2015), p. 021028.